Fiscal Policy in a Networked Economy

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Abstract

Policymakers can choose from a variety of fiscal stimulus levers: conditional transfers, unconditional transfers, or direct purchases from certain industries. This choice is complicated by the rich network structures that connect households and industries via directed consumption and input-output linkages. We study this problem in a model with household heterogeneity in MPCs, directed consumption patterns, and exposure to industry and regional shocks. Theoretically, we express fiscal multipliers in terms of estimable sufficient statistics, and we decompose them in terms of three *network effects* on top of a standard Keynesian multiplier. Empirically, we find that targeting fiscal policy is important, but simple. First, optimally targeting fiscal stimulus generates twice as much amplification in GDP as untargeted policy. Second, owing to the empirical absence of two of the three network effects, a simple fiscal policy that targets households based on their MPCs is close to maximally expansionary and optimal.

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1. Introduction

Economic shocks present policymakers with the challenge of designing stimulus programs that best prevent prolonged economic downturns. Most recently, in response to COVID-19, the United States Congress has implemented a broad spectrum of fiscal policies on an enormous scale. This response has included three major common categories of stimulus: undirected transfers (stimulus checks), targeted transfers (expanded unemployment insurance benefits), and targeted spending (industry programs, such as for the airlines). While COVID-19 presents unique challenges, this range of policy responses draws attention to questions that policymakers face during every major recession: which forms of fiscal stimulus are the most effective, whom do they help, and how should they be targeted?

These questions are complicated by the rich networks that make up present-day economies. Economic linkages – through supply chains, regional trade, and heterogeneous employment and consumption relationships – prevent a fiscal planner from conducting policy one household at a time. Rather, policymakers must consider the cascades of expenditure they set off, as expenditures in one industry in one state reach not only its workers but also others in its supply chain, those at firms where workers spend their marginal income, and so on. While such considerations may appear to put optimal policy out of reach, we provide a number of theoretical and empirical results that indicate just the opposite: despite rich economic interconnections, a planner can in many cases design optimal or near-optimal fiscal policy by following simple rules that require only very limited information.

We develop this argument in two parts. The first part of our paper provides a theory of how government spending (and other shocks) propagate through supply chains, employment linkages, and the directed MPCs of heterogeneous households. While these channels interact in complex ways, we show how to decompose all of these interactions into three distinct effects, on top of a baseline Keynesian multiplier. The second part of the paper takes this decomposition to the data and finds that, strikingly, only one term – capturing the heterogeneous incidence of government spending onto households with different MPCs – is quantitatively large. As a result, the optimal policy in a widespread recession simply targets high MPCs, as in much simpler models. This optimal targeting is not only simple, but also quantitatively important, as it results in twice as much policy amplification as untargeted, GDP-proportional expenditure.

Our starting point for this analysis is a semi-structural, general equilibrium model that incorporates heterogeneity among households and firms. On the household side, we allow for heterogeneity in both the magnitude of households' MPCs and their direction toward different goods. On the firm side, we allow for many sectors and regions, linked to one another through an arbitrary input-output structure. Finally, we allow for any pattern of employment of households across the various firms, generating heterogeneous household income processes. Within this rich setting, we study a rationing equilibrium where wages are sticky and thus labor is rationed, so that households can be off their labor supply curves and be involuntarily un(der)employed.[1] This assumption, as well as a focus on the case where an effective lower bound binds, makes our model applicable to severe recessions.

From a micro perspective, the various interconnections between households complicate the problem of a planner designing fiscal policy in this economy (e.g. to minimize involuntary unemployment). Not only do IO linkages across firms cause spending in one industry-region to generate labor income across a range of industries and regions, but also who actually receives this income depends on how firms ration labor among their employees. Moreover, each marginal worker spends some of her additional income on an idiosyncratic bundle of goods, setting off another round of income generation. However, at a macro level, the total effect of any fiscal policy on economic output – or its *fiscal multiplier* – can be decomposed into three distinct effects on top of a baseline Keynesian multiplier. First, the "incidence effect" captures that policies with incidence onto higher-MPC households change output by more. Second, the "bias effect" captures the increase in the multiplier stemming from households that are directly affected by the policy disproportionately directing their marginal spending toward goods produced by high-MPC households. Third, the "homophily effect" captures the amplification that occurs when high (low) MPC households direct their spending to other high (low) MPC households, for instance due to geographic concentration.

Despite these complex interlinkages, there are two striking special cases in which policies can nonetheless be designed using little to no information on the network of economic interconnections between households. Consider a recession-like environment in which the planner's sole motive is to address underemployment and she neglects any disutility costs of labor supply for the underemployed (e.g. because labor is supplied inelastically). We first show that – surprisingly – such a planner can evaluate the optimality of an existing set of fiscal transfers or expenditures without knowledge of how one households' spending translates into another household's income – a consequence of the fact that, at an optimum, the planner is indifferent to the direction of households' spending. Away from the optimum, the question of how to design transfers to *reach* the optimal fiscal policy is more challenging. Our second result shows how this may, nonetheless, be done when the "bias" and "homophily" effects, which capture heterogeneous linkages between households through marginal consumption, are zero. Specifically, the best local improvement to a possibly non-optimal fiscal policy

[1] We provide general, technical results on the existence of equilibria as well as a no-substitution theorem whereby prices are determined independently of demand.

simply targets the highest MPC households.[2]

In order to assess whether the key empirical condition characterizing the second case is satisfied, as well as to gauge the quantitative importance of targeting fiscal policy, the second part of the paper takes our model to the data. We combine several public-use datasets describing 51 US states (plus DC), 55 sectors, and 80 demographic groups to estimate three key empirical objects: the regional input-output matrix describing the input-use requirements of every industry-region pair; the rationing matrix describing how much each demographic-region pair's income changes in response to a one dollar change in production of each industry-region pair; and the directed MPC matrix describing how much each demographic-region pair consumes from each industry-region pair.

The key takeaway from our empirical exercise is that targeted fiscal policy is both simple and important. The main finding underpinning this conclusion is that, empirically, the bias and homophily effects are almost exactly zero for all possible policies. This implies that the aggregate multiplier of any fiscal shock depends *only* on its incidence onto households with higher or lower MPC. Thus, MPC targeting is the maximally expansionary policy. Whereas this policy is easy to implement for transfers, targeting government expenditures to affect the workers with the highest MPCs requires knowledge of the input-output network (as in Baqaee (2015)) and the labor rationing process, both of which shape how changes in demand affects the income of workers. Relative to untargeted government spending, optimally targeted policy is amplified twice as much through the fiscal multiplier. Naive targeting that ignores these margins and solely targets according to the MPC of workers in each industry-region pair is moderately effective but leaves substantial gains on the table.

Related Literature The analysis of this paper builds on and contributes to several distinct strands of literature. Theoretically, our model unifies a range of elements that have all been shown to be important for shock propagation. On the household side, Kaplan, Moll, and Violante (2018) and Auclert (2019) stress that monetary shocks are amplified when the incidence of the shock falls on the households with the highest MPCs. Farhi and Werning (2017), Caliendo, Parro, Rossi-Hansberg, and Sarte (2018) and Dupor, Karabarbounis, Kudlyak, and Mehkari (2018) highlight the importance of regional linkages in amplifying both productivity and demand shocks. A long literature highlights the role that input-output networks play in propagating shocks (see for example Long and Plosser (1987), Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Baqaee and Farhi (2019), Rubbo (2019), Bigio and La‘O (2020)). Moreover, Werning (2015) highlights the theoretical im-

[2]This is true not only in our baseline model, where firms are competitive, but also in an extension with imperfect competition and constant markups.

portance of heterogeneity in income cyclicality in determining the aggregate response to shocks and Patterson (2019) and Guvenen and Smith (2014) demonstrate empirically that the heterogeneous incidence of shocks in the labor market is a potentially meaningful source of shock amplification. While each of these papers focuses on one or two dimensions of heterogeneity, we integrate them in a model rich enough to bring to the data through sufficient statistics. This more reduced-form approach is similar methodologically to Auclert, Rognlie, and Straub (2018), who use intertemporal MPCs to discipline macroeconomic models and study the implications for the financing and timing of fiscal stimulus. Our approach differs in its focus on heterogeneity and the targeting of stimulus in the cross-section.[3] In fact, this approach dates back to the much earlier regional accounting literature which emphasized how demand may spill over across regions (Miyazawa, 1976). We micro-found this literature's focus on fixed prices in an environment with a single factor, sticky wages, and a binding zero lower bound.

The theoretical part of our paper relates most closely to Baqaee (2015). As we do in this paper, Baqaee emphasizes that shocks to an industry affect not only the factors employed in that industry but also those used in producing its inputs, motivating a "network adjustment" to the labor share of each industry. This mechanism also features prominently in our model and we find empirically that it plays a role in shaping optimal policy. In more recent work, Baqaee and Farhi (2018) develop rich macroeconomic models featuring these channels as well as endogenous prices and markups. At the level of generality of their approach, it is hard to disentangle the role that various modelling elements have in shaping amplification. By contrast, we abstract away from price movements but are able to precisely characterize – as well as empirically assess – the channels through which economic linkages affect aggregate shock propagation and how these matter for optimal stimulus policy.[4]

Lastly, this paper also adds to a large empirical literature estimating multipliers from fiscal shocks. Our structural estimates complement reduced-form empirical estimates of open-economy multipliers – we calibrate an aggregate multiplier of 1.30, which is somewhat smaller than, but within the established confidence intervals of, those in Ramey (2011), Nakamura and Steinsson (2014), Chodorow-Reich (2019) and Corbi, Papaioannou, and Surico (2019). While most of the empirical literature has focused on identifying estimates for the fiscal multiplier, a few more recent empirical papers share our focus on uncovering heterogeneity

[3] Other papers adopting the sufficient statistics approach include Wolf (2019), and Koby and Wolf (2019).

[4] Relatedly, Zorzi (2020) studies the interaction of cyclicality in durable consumption and investment with sector-specific employment in a parametric environment. This paper and ours are related insofar as they involve the interaction of directed demand and heterogeneous labor rationing. However, we abstract away from the specific microfoundation of directed demand and take a more reduced form approach that emphasizes richer connections between households and firms.

in fiscal multipliers across space. These empirical papers leverage finer geographical and sectoral data to explore fiscal spillovers. For example, Feyrer, Mansur, and Sacerdote (2017) document geographical spillovers in demand from counties with increased fracking production onto nearby regions. Auerbach, Gorodnichenko, and Murphy (2020) leverage rich data on Department of Defense contracts, finding reduced form evidence for both back-propagation of demand through supply chains and increased demand in other industries through income multipliers. Theoretically, we provide a framework consistent with the evidence presented in these papers and provide more structural estimates detailing the distinct channels through which these spillovers operate.[5]

Outline The rest of the paper proceeds as follows. Section 2 introduces the model and defines the rationing equilibrium. Section 3 derives the multiplier and provides a decomposition characterizing the role of heterogeneity. Section 4 studies optimal fiscal policy, providing conditions under which MPC targeting is optimal. Section 5 introduces the data and methodology we use to estimate the multiplier. Section 6 quantifies the importance of targeted fiscal policy and empirically characterizes the dimensions on which policymakers should target. Section 7 concludes.

2. The Model and Rationing Equilibrium

To explore the propagation of fiscal policies – as well as exogenous demand and supply shocks – we build a semi-structural model. Our goal is to develop a setting that is rich enough to capture many dimensions of household, industrial, and regional heterogeneity, but sufficiently tractable to facilitate a characterization of optimal policy and deliver equations that we can bring directly to the data. In the model, a continuum of heterogeneous households interact in a competitive, multi-sector, multi-region economy over two periods. We consider a rich class of household-level consumption and labor supply functions that accommodate arbitrary preference heterogeneity, household borrowing constraints, and most behavioral frictions, as well as a rich, constant returns to scale input-output structure. We consider a rationing equilibrium, where first period wages are fixed and first period labor supply is determined by exogenous rationing functions rather than by household optimization. This allows households to lie off their labor supply curves and thus enables the model to capture classical involuntary unemployment. In Appendix B, we provide existence results for rationing equilibrium, extend this analysis to an arbitrary number of time periods

[5]Cox, Müller, Pasten, Schoenle, and Weber (2019) use the same procurement data and account for heterogeneity in price stickiness across sectors subject to fiscal shocks, which lies outside of our framework.

and imperfect competition with fixed markups, and compare our rationing equilibrium to a flexible-wage equilibrium.

2.1. Model Primitives

Time is indexed by $t \in \{1, 2\}$. There is a finite set of goods $\mathcal{I}^t$ in each period, each of which is produced by one representative firm j using a vector of intermediates $X_j^t = (X_{j1}^t, ..., X_{j|\mathcal{I}^t|}^t) \geqslant 0$, a single labor factor $L_j^t \geqslant 0$ and a production technology $F(X_j^t, L_j^t, z_j^t)$ that is CRS in inputs and labor, where z_j^t is a vector of parameters that determine the production function. In each period, firms take prices p^t and wages w^t as given and maximize profits. We normalize the wage to one within every period, i.e. $w^t = 1$; intertemporal price comparisons are possible via the real interest rate r^1. Firms choose labor and intermediate inputs to maximize profits in each period:

$$\begin{aligned} p_i^t F_L(X_i^t, L_i^t, z_i^t) &= 1 \\ p_i^t F_{X_j}(X_i^t, L_i^t, z_i^t) &= p_j^t \end{aligned} \tag{1}$$

There is a continuum of households on the interval $[0, 1]$ indexed by i, and a finite set of types N, where each type $n \in N$ has mass $\mu_n > 0$ such that $\sum_{n \in N} \mu_n = 1$. Households consume a vector of goods $c_n^t = \{c_{ni}^t\}_{i \in \mathcal{I}^t}$ in each period t, and they save an amount s_n^1 between periods at a real rate $1 + r^1$; households have no initial savings or debt.[6] Each household n supplies labor l_{ni}^t to each firm i in period t, totalling $l_n^t = \sum_i l_{ni}^t$. Household n therefore has labor income $y_n^t = l_n^t$ in period t. Rather than explicitly microfounding households' decision problems, we simply assume there exist exogenous functions that describe their consumption and labor supply as a function of variables outside their control (see Section 2.2). This allows us to nest non-homothetic preferences, behavioural frictions and borrowing constraints. Households always satisfy their lifetime budget constraint:

$$l_n^1 + \frac{l_n^2}{1 + r^1} = p^1 c_n^1 + \frac{p^2 c_n^2}{1 + r^1} + \tau_n^1 + \frac{\tau_n^2}{1 + r^1} \tag{2}$$

The government levies (possibly negative) lump-sum taxes τ_n^t on households, and it buys G_i^t units of good $i \in \mathcal{I}^t$ subject to running a balanced budget over the two periods. To finance its fiscal spending and tax/transfer programs, the government issues bonds at a real interest

[6] This is without loss, as we can replace initial debt between agents with heterogeneous lump-sum taxes and transfers.

rate of r^1 in the first period. The real lifetime government budget constraint is therefore

$$\sum_{n \in N} \mu_n \left(\tau_n^1 + \frac{1}{1+r^1} \tau_n^2 \right) = p^1 G^1 + \frac{1}{1+r^1} p^2 G^2 \tag{3}$$

So that the government budget constraint continues to hold when prices or the interest rate changes, we assume expenditures are given by an exogenously specified function of real prices, taxes, and a government spending preference parameter θ_G. In particular, $G^t = G^t\left(\varrho, (\tau_n)_{n \in N}, \theta_G\right)$, where ϱ is the price vector (p^1, p^2, r^1) and we assume $G^t(\cdot)$ is such that Equation 3 always holds.

2.2. Rationing Equilibrium

We consider a sticky-wage rationing equilibrium. In this equilibrium concept, first-period wages are exogenously fixed and, consequently, first-period labor is rationed, rather than determined by household optimization. Such an equilibrium notion corresponds well to an environment with wage rigidity of the kind commonly observed in the data (Solon, Barsky, and Parker, 1994; Grigsby, Hurst, and Yildirmaz, 2019; Hazell and Taska, 2019). This concept captures classical involuntary unemployment, wherein – particularly during economic downturns – there are households who would like to work but cannot because firms are unwilling to hire them. Firms, in turn, could lower prices to generate more demand and employ additional workers, but – since they cannot lower wages – would have to do so at a loss. We assume that the same, fixed wage also applies to new hires, so that firms cannot simply fire existing workers and hire under-employed households at lower wages. Fundamentally, this narrative is about households lying off their labor supply curves, which we capture by assuming that in the short run, households do not choose their labor supply but rather have it rationed to them. Of course, even with sticky nominal wages, labor markets may still clear if interest rates are set so that the real inter-temporal price is as in a flexible-price equilibrium. The conduct of monetary policy is therefore a key part of the story; we discuss it more below.

Formally, first period labor is determined by a differentiable rationing function that maps the vector of labor demands $(L_i^1)_{i \in \mathcal{I}^1}$ to a vector of total labor supplied by each household type $l^1\big((L_i^1)_{i \in \mathcal{I}^1}\big)$. The rationing function treats all households within each type identically, and is such that the labor market clears:

$$\sum_{n \in N} l_n^1\big((L_i^1)_{i \in \mathcal{I}^1}\big) = \sum_{i \in \mathcal{I}^1} L_i^1 \tag{4}$$

In the second period, households choose their second-period labor supply and the prices of all goods and wages are set so that all markets clear.

Our extremely reduced-form representation of the rationing function allows us to nest a number of empirically important phenomena. First, although our model only features a single labor type, it may be reinterpreted to accommodate arbitrarily many flexible factors to the extent that their relative wages are completely rigid. For example, each household type may represent a different type of labor; to the extent that a firm marginally demands workers of various types in different proportions, the rationing function will employ them accordingly. The role of relative wage rigidity is to rule out responses of relative wages (and therefore prices) to shocks, which would induce additional margins of substitution by firms and households. Second, our rationing-function approach allows us to accommodate regional migration driven solely by changes in labor demand, since employment is demand- rather than supply-determined, so that the same total income is rationed to each household type in each region regardless of the size or composition of the demographic group in that region. The approach can even accommodate the possibility that labor rationing may respond to migration-induced changes in the prevalence of different groups, so long as the vector of firms' labor demands fully determines workers' incentives to migrate.[7]

We model this labor supply behavior by assuming that households take not only prices but also first period labor income as given, while allowing consumption $c_n^t(\varrho, y_n^1, \tau_n, \theta_n)$ and second period labor supply $l_n^2(\varrho, y_n^1, \tau_n, \theta_n)$ to be given by arbitrary functions of prices, first-period income, taxes, and a preferences parameter θ_n.

All other markets clear in the usual fashion:

$$Q_i^t = F(X_i^t, L_i^t, z_i^t) = \sum_{j \in \mathcal{I}^t} X_{ji}^t + \sum_{n \in N} \mu_n c_{ni}^t + G_i^t, \qquad \sum_{i \in \mathcal{I}^2} L_i^2 = \sum_{n \in N} \mu_n l_n^2 \tag{5}$$

We assume that the nominal interest rate set by the central bank directly pins down the real interest rate that enters into both the government and household budget constraints. We therefore suppose that the central bank sets real interest rates directly, potentially as a

[7]This specification allows us to capture any demand-driven migration mechanism, for example: if there is a drop in demand in region A but not region B, and – in response – workers move from A to B, firms in B may marginally demand more workers of the types initially prevalent in A. This migration would be reflected in both the labor demands in region A and region B. Since the rationing function takes the full vector of labor demands across regions and returns a vector of labor supplies for worker types, a stable rationing function would still capture these dynamics. One set of models not accommodated are those in which amenities are endogenous to the shock and do not depend solely on labor demand.

function of output of any good in any period:[8]

$$r^1 = r^1(Q) \tag{6}$$

A rationing equilibrium is therefore defined as follows:

Definition 1. *A rationing equilibrium is a set of first and second period, agent- and market-level variables* $\{s_n^1, \{c_{ni}^t, l_n^t\}_{t\in\{1,2\}, i\in\mathcal{I}^t}\}_{n\in N}$ *and* $\{r_i^1, \{p_i^t, \{X_{ij}^t\}_{j\in\mathcal{I}^t}, L_i^t, C_i^t, G_i^t\}_{t\in\{1,2\}}\}_{i\in\mathcal{I}^t}$ *that satisfy conditions (1) – (6) given initial conditions.*

The concept of rationing equilibrium we study has a rich intellectual tradition in Keynesian macroeconomics stretching back to Patinkin (1949), Clower (1965) and Barro and Grossman (1971). Indeed, the key idea that price rigidities or other frictions may cause a household to lie off its labor supply curve is a staple of many modern macroeconomic approaches to understanding involuntary unemployment and the business cycle, with our exact formulation via a rationing function being closest to that employed by Werning (2015).

In Appendix B.1, we establish a number of properties of rationing equilibrium that both eliminate any nuisance terms and ensure that our analysis is well-posed. In particular, we provide mild technical assumptions under which – as a consequence of the single labor factor – prices are determined independently from demand (a no-substitution theorem) and an equilibrium exists. We will maintain these assumptions throughout the analysis. Moreover, in Appendix B.5, we compare the rationing equilibrium concept to a benchmark notion of equilibrium with flexible prices. In that setting, the interest rate moves in the first period to clear the labor market while workers remain on their labor supply curves. Importantly, in such a flexible-price equilibrium, household MPCs play no direct part in determining the response of output to a demand shock.

3. The Multiplier

Within the setting outlined in Section 2, we next explore the general equilibrium impact of supply and demand shocks, including government spending and transfer shocks. Our goal is to derive an expression for the general equilibrium multiplier that maps the effect of shocks in partial equilibrium to their general equilibrium impact. This is both of independent interest for understanding shock propagation and a key step toward understanding optimal fiscal policy. Importantly, we derive a representation of the multiplier in terms of sufficient

[8]This specification nests Taylor rules that condition on both sector-level output and inflation as well as money supply targeting via a quantity theory.

statistics that we can both use to understand how network structure in the macroeconomy matters, and take to the data to quantify the role of heterogeneity in shock propagation and the optimal fiscal policy.

3.1. The Output Multiplier in a Networked Economy

Our main results will express the economy's general equilibrium responses to shocks to exogenous parameters as a function of their partial equilibrium effect on goods demand. The partial equilibrium effect on good demand ∂Q is the change in output in response to a shock before prices or incomes have been allowed to adjust. The demand and supply shocks that we consider in our setting are changes in government spending, taxes and transfers, preferences, and technology.

We begin by parameterizing aggregate demand. Recognizing that each household's decisions depend only on real quantities, we can represent type $n \in N$'s Marshallian demand for good $j \in \mathcal{I}^t$ at time $t \in \{1, 2\}$ as $c^t_{nj}(y^1_n, \varrho, \tau_n, \theta_n)$, where $\varrho = (p^1, p^2, r)$, and $\tau_n = (\tau^1_n, \tau^2_n)$. Aggregate consumption demand C^t_j is then given by:

$$C^t_j(\varrho, \tau, \theta) = \sum_{n \in N} \mu_n \; c^t_{nj}(y^1_n, \varrho, \tau_n, \theta_n) \tag{7}$$

where $\theta = (\theta_1, ..., \theta_N)$ and so forth.

To find the partial equilibrium effect of each type of shock, we totally differentiate the goods market clearing condition, given by

$$Q^t = \widehat{X}^t Q^t + C^t + G^t \tag{8}$$

where $\widehat{X}^t$ is the unit-production input-output matrix.[9] We then collect the terms corresponding to changes in demand for goods before accounting for the endogenous response of interest rates and income and for the higher-order effects those responses generate. Doing so yields the following partial equilibrium effect of each shock ∂Q.[10] In particular, given any

[9]More formally, define the unit input demands for any firm i at this price solve the following program:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i, L_i) \text{ s.t. } F(X_i, L_i, z_i) \geqslant 1} p(z) X_i + L_i$$

The input-output matrix then stacks $\widehat{X}_i(z)$ across firms.

[10]The shocks we consider have partial equilibrium effects on demand that are easy to compute. For example, a change in household preferences θ by $d\theta$ has partial equilibrium effect on demand given by $\partial Q = C_\theta(\varrho, \tau, \theta) d\theta$. The effects of the other demand shocks are similarly simple. The partial equilibrium effect of a change in productivity z by dz includes changes in prices and is given by:

$$\partial Q = (C_p + G_p) p_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1$$

combination of shocks to government spending, household preferences, taxes, and/or the structure of production $\partial x \in Span\{d\theta_G, d\theta, d\tau, dz\}$, the partial equilibrium effect on demand is given by:

$$\partial Q^t = \frac{\partial}{\partial x}\left[\widehat{X}^t Q^t + C^t + G^t\right]\partial x \tag{9}$$

where above and for the rest of the text, we assume that derivatives exist as needed.

In a rationing equilibrium, these partial equilibrium shocks propagate through the economy via two fundamental mechanisms. First, as firms ration additional labor demand to workers, households respond to increased income with greater spending on goods, generating an *income multiplier*. Second, as interest rates respond to changing output, households respond with different savings and consumption behavior, generating an *interest rate multiplier*. In deep recessions, the latter effect is likely to be weak, both because the consumption response to interest rates is small and because interest rates may not be able to respond to output in the presence of a zero lower bound (Campbell and Mankiw, 1989; Kaplan, Violante, and Weidner, 2014; Vissing-Jorgensen, 2002). Therefore, for the rest of the analysis, we focus on the income multiplier; for results on the more general case, see Appendix B.2.

Assumption 1. *At least one of the following statements is true:*

1. *The consumption and government responses to real interest rates sum to zero:*

$$C^1_{r^1} + G^1_{r^1} = 0 \tag{10}$$

2. *The central bank response of real interest rates to production is zero:*

$$r^1_Q = 0 \tag{11}$$

Under Assumption 1, the output multiplier takes a particularly interpretable form:

Proposition 1. *For any small shock to parameters $\partial x \in Span\{d\theta_G, d\theta, d\tau, dz\}$, there exists a selection from the equilibrium set such that—under Assumption 1—the general equilibrium response of first period output dY^1 is given by:*

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 \tag{12}$$

where ∂Q^1 is the partial equilibrium change in first-period production associated with ∂x.

Proof. See Appendix A.1. □

This is the key positive formula of the paper and can be understood as a generalization of the traditional Keynesian multiplier $(1 - MPC)^{-1}$ to the case of input-output networks and heterogeneous households. The term

$$C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \tag{13}$$

is the analog of the MPC in the traditional multiplier formula. In this economy, following a demand shock to firms, the term $\left(I - \widehat{X}^1\right)^{-1}$ maps changes in final demand to changes in production via the input-output network. Having pinned down the change in required production, $\widehat{L}^1$ maps these to changes in firms' demand for labor. Next, the rationing function $l^1_{L^1}$ maps those to changes in each household's income. Finally, the directed MPCs of households $C^1_{y^1}$ map those changes to changes in aggregate consumption of each good. The final multiplier is the Leontief inverse of this object as this loop repeats *ad infinitum*.

The crucial difference relative to the traditional Keynesian multiplier is that the structure of production, employment and consumption matters. First, it is important whether shocks load onto low or high MPC households, as studied by Patterson (2019). Moreover, the *interaction* between the input-output network and the directed consumption network matters: the multiplier is largest when it is not only partial equilibrium shocks but also higher order responses that load onto high MPC households, due to those households spending their marginal dollars at firms that hire high MPC workers or at firms that buy inputs from firms hiring high MPC workers, and so forth.[11]

Throughout the rest of the paper, in analogy to the assumption that the aggregate MPC is less than one in the simple Keynesian multiplier, we assume the moduli of $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ and $l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} C^1_{y^1}$ are less than one, which guarantees that the output multiplier is vbvwell defined.[12] We will also always consider the equilibrium selection such that our multiplier formula applies.

[11] This same multiplier expression appears in the regional economics literature on social accounting matrices, dating back to Miyazawa (1976). Our result provides the first fully-microfounded justification of this formula, which receives widespread use in the regional economics literature and applied work to compute expenditure multipliers (such as the BEA's RIMS II system). The connection to the social accounting literature motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households. See Appendix B.6 for a formal description of this interpretation.

[12] We later verify this assumption empirically. Also note that the modulus is less than one whenever all households have MPC less than one.

3.2. Decomposing the Role of Heterogeneity

Several dimensions of heterogeneity interact to produce the multiplier in Proposition 1. In this section, we explain how these dimensions can be understood through three key effects that lead to greater or lesser amplification relative the basic Keynesian case.

To begin, we simplify notation by renormalizing the units of all goods in each period so that all pre-shock, intra-temporal prices are equal to one, i.e. $p_n^t = 1$. For fiscal and demand shocks, which do not affect prices, this is without loss; we therefore restrict to these shocks in this section; we present analogous results for supply shocks in Appendix B.7.

Toward decomposing the role of heterogeneity, we now define the aggregate spending network. First, let $C_{y^1}^1$ be written as the product $\overline{C}_{y^1}^1 \hat{m}$ of a diagonal matrix $\hat{m}$ of MPCs (the column sums of $C_{y^1}^1$) and spending direction $\overline{C}_{y^1}^1$; and define m as the vector of MPCs. Second, define

$$\mathcal{G} \equiv l_{L^1}^1 \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \overline{C}_{y^1}^1 \tag{14}$$

as the map from an additional dollar of spending by one household to the vector of income changes in generates for each other household. Since every dollar spent eventually becomes income, every column of $\mathcal{G}$ sums to one. Lastly, define

$$\partial y^1 \equiv l_{L^1}^1 \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^1 \tag{15}$$

as the partial equilibrium incidence of a shock on labor income.

Lemma 1 first rewrites the generalized multiplier from Proposition 1 in terms of these newly defined terms – ∂y^1 and $\mathcal{G}$. Intuitively, this separates the first loop in the multiplier (the effect of the partial equilibrium demand shock on labor incomes) from all other iterations of the loop (the effect of changing incomes on demand, the effect of those demand changes on income, and so forth).

Lemma 1. *The total change in first-period output due to a partial equilibrium demand shock with labor income incidence ∂y^1 can be expressed as*

$$\vec{1}^T dY^1 = \underbrace{\vec{1}^T \partial y^1}_{\textit{Direct effect}} + \underbrace{m^T \left(\sum_{k=0}^{\infty} (\mathcal{G}\hat{m})^k \right) \partial y^1}_{\textit{Indirect effect}} \tag{16}$$

Proof. See Appendix A.2. □

Lemma 1 clarifies that any shock inducing a partial equilibrium change in labor incomes has two components: a direct effect of increasing GDP and an indirect, or multiplier, effect.

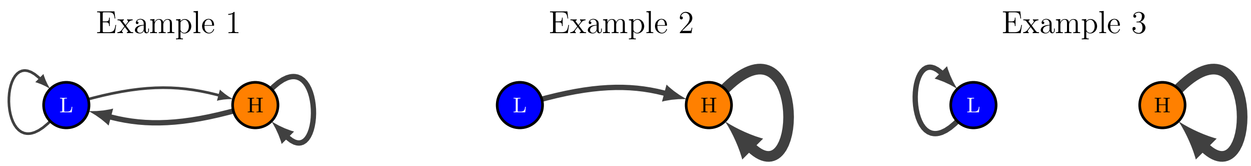

Fig. 1. Example 1: "Neutral" spending-to-income network. Example 2: Typical HH's marginal spending directed toward HHs with higher than own MPC ("bias"). Example 3: Each HH directs marginal spending toward HHs with same MPC ("homophily").

The multiplier effect exactly maps changes in incomes through MPCs and the network of spending to compute all higher-order effects.

The best way to understand the effects of the spending-to-income network $\mathcal{G}$ on amplification is through three examples. In each example below, there are two households: one with low MPC m_L and one with high MPC $m_H > m_L$. We consider a shock ∂y^1 that has incidence $\frac{1}{2}$ on each household's income, so that the incidence-weighted aggregate MPC is $\bar{m} \equiv \frac{m_L + m_H}{2}$. The difference between each example is in the structure of the spending-to-income network $\mathcal{G}$.

Our first example illustrates a neutral case in which network structure is irrelevant. In particular, each household divides its marginal spending equally between the two sectors (see the left panel of Figure 1). In this case, the incidence of spending induced by the income earned in meeting the partial equilibrium demand shock is exactly m times the shock's incidence for each household; similarly for spending induced by income earned in meeting this secondary demand, and so on. Thus, the total change in output is given by the standard $\frac{1}{1-\text{MPC}}$ formula using the incidence MPC, $\bar{m}$.

In the second example, each household instead directs all of its marginal spending to the sector employing the high-MPC household (see the middle panel of Figure 1). Unsurprisingly, this generates higher amplification: the original shock has magnitude 1 and the consumption response of households employed to meet the partial equilibrium demand shock increases output by $\bar{m}$. Then, this spending propagates according to the $\frac{1}{1-\text{MPC}}$ multiplier *at the high MPC,* m_H. The total change in output is then given by $1 + \frac{\bar{m}}{1-m_H}$, which exceeds $\frac{1}{1-\bar{m}}$. Intuitively, the bias of consumption baskets toward higher-MPC households increases amplification.

In the final example, each household directs all of its marginal spending toward itself (see the right panel of Figure 1). In this case, each household's share of the shock incidence propagates separately, at $\frac{1}{1-\text{MPC}}$ with that household's MPC. The total change in output is then:

$$\frac{1}{2}\left(\frac{1}{1-m_L} + \frac{1}{1-m_H}\right) > \frac{1}{1-\bar{m}} \tag{17}$$

where the inequality comes from the fact that $\frac{1}{1-\text{MPC}}$ is *convex* is MPC. Intuitively, since the high-MPC household spends more of its increase in income, it increase output more by directing spending toward the high-MPC household than by directing spending the the low-MPC household. This network homophily increases amplification.

These examples illustrate the three channels by which network structure matters for amplification. First, one must account for the incidence of a shock onto households of higher or lower MPC. Second, the multiplier is higher when households' marginal spending is biased toward households with higher MPCs than their own. Third, homophily in the spending network in the form of correlation between household MPCs and MPCs of households they spend on also generates amplification. Proposition 2 establishes that these three channels capture all of the effects of the spending-to-income network $\mathcal{G}$, to second order in MPCs. Appendix A.3 provides an exact decomposition in terms of Bonacich centralities of $\mathcal{G}$.

Proposition 2. *The total change in first-period output due to a demand shock with unit-magnitude labor income incidence ∂y^1 can be approximated as:*

$$
\begin{aligned}
1^T dY^1 = & \frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\text{Incidence effect}} \\
& + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{next}] - \mathbb{E}_{y*}[m_n]\right)}_{\text{Biased spending direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, m_n^{next}]}_{\text{Homophily effect}}\Bigg) + O^3(|m|)
\end{aligned}
\tag{18}
$$

where y^* is any reference income weighting of unit-magnitude and $m^i_{next} = \left(m^T\mathcal{G}\right)_i$ is the average MPC of households who receive as income i's marginal dollar of spending.

Proof. See Appendix A.3. □

The above proposition holds for all reference partial equilibrium changes in labor earnings y^* of unit size, but naturally the choice of this y^* affects the accuracy of the approximation. In our later empirical analysis, we take y^* as the change in income induced by a GDP proportional demand shock. In this case, we show that the error term accounts for less than 0.3% of the multiplier, so that this approximation is very tight.

It is also natural to consider cases in which bias and homophily are irrelevant for shock propagation. To this end, in Appendix B.8 we discuss how Proposition 2 applies to several important benchmark economies, highlighting cases in which the various alterations to the Keynesian multiplier are zero. One important benchmark is a "homothetic economy" where both consumption and labor rationing functions are homothetic. In this case there can be no bias effect, but heterogeneity in household consumption baskets and sectoral employment can still generate network effects through homophily. A truly "neutral" case occurs when all firms in the economy employ workers at the margin who have the same average MPC as one another. In this case, all of the network adjustment effects are zero and the output multiplier for any shock is simply the Keynesian multiplier evaluated at this average MPC. That is, wherever in the economy a shock strikes, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $l^1_{L^1} = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm's marginal employees have the population average MPC.

Clearly, the conditions required to eliminate the network adjustments are knife-edge. In all other cases, the distribution of shocks does affect aggregate responses, and the IO and directed consumption networks affect both the size and direction of these responses.

4. Optimal Fiscal Policy

So far, we have studied how fiscal and other shocks propagate to affect output and income in general equilibrium. We now turn to our primary motivating question: how should a planner target fiscal stimulus? In this section, we bring the results of Section 3 to the policy problem of a planner who designs government expenditure and transfer policy to maximize welfare. We offer general results – decomposing a planner's motives in the context of regionally and industrially heterogeneous downturns – as well sharper characterizations in important special cases. We draw particular attention to the problem of a planner whose sole motive is to address capacity under-utilization and provide empirically-verifiable conditions under which a simple policy targeting MPCs is optimal.

4.1. Welfare and the Planner's Problem

In previous sections, we have not specified household utility functions, instead simply working with Marshallian demands. In order to analyze welfare, we assume each household n has an additively-separable utility function over consumption, labor supply, and government purchases.[13] At time $t = 1$, households of type n choose consumption but not labor supply, and face a borrowing constraint in the form of a minimum level $\underline{s}_n$ of savings. At time $t = 2$,

[13]Separability between consumption and labor supply ensure that MPCs out of income and transfers are the same. Separability of consumption and labor from government purchases ensures that household decisions do not respond to government purchases directly.

households are unconstrained. The household's problem is therefore:

$$\begin{aligned}
&\max_{\tilde{c}^t,\tilde{l}^t} \sum_{t=1,2} \beta_n^{t-1}\left[u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t)\right] \\
&\text{s.t. } \vec{1}^T\tilde{c}^1 + \frac{\vec{1}^T\tilde{c}^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \leqslant \tilde{l}^1 + \frac{\tilde{l}^2}{1+r^1} \\
&\quad \tilde{l}^1 - \vec{1}^T\tilde{c}^1 - \tau_n^1 \geqslant \underline{s}_n^1 \\
&\quad \tilde{l}^1 = l_n^1
\end{aligned} \tag{19}$$

We assume that the planner is utilitarian, placing some welfare weight λ_n on households of type n. The planner maximizes this objective subject to household optimization, market clearing, labor rationing, supply-determined prices, a budget constraint, and a zero lower bound. We assume the zero lower bound is binding throughout so that the planner simply takes r^1 as given. The planner's problem is:[14]

$$\begin{aligned}
\max_{\{c_{ni}^t, l_n^t, Q_i^t, G_i^t, \tau_n^t\}_{t\in\{1,2\}, n\in N, i\in\mathcal{I}^t}} \quad W \equiv \sum_{n\in N}\mu_n\lambda_n \sum_{t=1,2}\beta_n^{t-1}\left[u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t)\right] \\
\text{s.t. } (c_n^1, c_n^2, l_n^2) \text{ solves Equation 19 given } l_n^1 \\
\text{and all equilibrium conditions hold}
\end{aligned} \tag{20}$$

Below, we will denote the Lagrange multiplier on the government budget constraint by γ and refer to it as the "marginal value of public funds" or MVPF.

4.2. *Optimal Targeting of Fiscal Stimulus*

Our main goal is to answer the question of where the planner should spend marginal dollars so as to have the greatest effect on welfare. To this end, we first decompose the change in welfare due to a small change in either transfers or government expenditure.

Proposition 3. *The change in welfare dW due to a small change in taxes and government expenditure—at a constant interest rate—can be expressed as:*

$$\begin{aligned}
dW = \sum_{n\in N}\mu_n\tilde{\lambda}_n\Bigg[\underbrace{-\Delta_n dl_n^1}_{\text{Address under-emp.}} - \underbrace{\left(d\tau_n^1 + (1-\phi_n)\frac{d\tau_n^2}{1+r^1}\right)}_{\text{Make transfers}} \\
+ \underbrace{\left(WTP_n^1 dG^1 + (1-\phi_n)\frac{WTP_n^2}{1+r^1}dG^2\right)}_{\text{Make expenditures}}\Bigg]
\end{aligned} \tag{21}$$

[14] For a full statement, see Appendix A.4.

where $\widetilde{\lambda}_n$ is the value the planner places on the marginal transfer of first-period wealth to a household of type n, Δ_n and ϕ_n are n's implicit first-period labor wedge and borrowing wedge, and WTP_n^t is the vector of n's marginal willingness to pay for period t government expenditures on each good, in period t dollars.[15]

Proof. See Appendix A.4. □

Equation 21 clarifies three different motives of the planner. First, she seeks to alleviate involuntary un(der)employment by changing the labor allocation so as to provide more employment to households with large negative labor wedges (the underemployed). Second, she may make transfers between households, both in the name of pure redistribution and to help borrowing-constrained households substitute intertemporally. Third, she makes government purchases; here the borrowing wedge enters, as borrowing-constrained households undervalue future purchases due to an artificially high value of wealth in the first period.[16]

This result also clarifies to what extent policy responses should mimic the economic shocks to which they respond. If a negative shock to one industry causes mass unemployment in an industry, larger labor wedges for households employed in that industry imply an increased value of government expenditures there. Spending that directly counteracts exogenous demand shocks can also undo the knock-on effects they induce through worker spending. At the same time, truly optimal policy also involves complex considerations captured by the output multiplier. For instance, it may be more effective to counter a shock in one state by making transfers to high-MPC households out of state – or out-of state firms with the right input demands – than to low-MPC households within the shocked state.

Our next result applies Proposition 3 to consider optimal policy, building on the observation that, at an optimum, the marginal change in welfare with respect to any change in policy must be zero. We show that in two benchmark cases the planner's indifference between transfers to each household and/or expenditure in each sector leads to optimality conditions that can be evaluated without knowledge of the rich interconnections between households.

[15] The change in n's first-period employment, in turn, is given by

$$\widehat{\mu} dl^1 = R^1 \left(I - C^1_{y^1} R^1\right)^{-1} \left(dG^1 - C^1_{y^1}\left(\widehat{\mu} d\tau^1 + \frac{1_{\phi_n=0}\widehat{\mu} d\tau^2}{1+r^1}\right)\right) \tag{22}$$

where $R^1 \equiv l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1}$ is the partial-equilibrium map between output and total employment of each type (across individuals) in the first period.

[16] In Appendix B.9, we provide a further decomposition of these terms for small variations in policy starting at the global optimum, similarly to Werning (2011).

Proposition 4. *The following two statements are true:*

1. *(Optimal transfer policy) Suppose that the marginal social dis-utility of labor supply is constant across all households rationed to on the margin at the optimum.*[17] *Then $dW = 0$ with respect to marginal changes in first-period transfers if and only if, for all $n \in N$,*

$$\gamma = \widetilde{\lambda}_n \left(1 + \frac{m_n}{1 - m_n}(-\Delta_n)\right) \tag{23}$$

   where γ is the marginal value of public funds.

2. *(Optimal expenditure policy) Suppose that the social gains from first-period government expenditure are equal to some $\tilde{v}$ across goods and constraints bounding expenditures above zero do not bind. Then $dW = 0$ with respect to marginal changes in first-period expenditures if and only if, for all $i \in \mathcal{I}^1$,*

$$\gamma = \tilde{v} + \frac{1}{1 - \widetilde{m}_i}\left(-\widetilde{\lambda\Delta}_i\right) \tag{24}$$

   where $\widetilde{m}_i$ is the rationing-weighted average MPC in the production of good i and $\widetilde{\lambda\Delta}_i$ is the rationing-and-welfare-weighted average rationing wedge in the production of good i.[18]

Proof. See Appendix A.5. □

Proposition 4 says that the planner may verify whether the current policy is optimal despite having very partial knowledge about the economy. In the transfer case, the planner only needs information on household level welfare weights ($\widetilde{\lambda}_n$), rationing wedges (Δ_n), and MPCs (m_n) – not the network of marginal spending flows between households. In the expenditures case, the planner needs to know the average MPC and welfare-weighted rationing wedge by industry; these require knowledge of the rationing function linking output to incomes, but not the directed consumption matrix.

The main idea underlying Proposition 4 is that—at an optimum—the social value of additional spending by any household is independent of how that spending is directed. This is clearest in the case of transfers: For any household employed in order to produce marginally-demanded goods, the social value of their employment is equal to the value of a transfer to that household, less the dis-utility of labor. Since (by assumption) the dis-utility of labor is constant across households, and since—at an optimum—the value of transfers must also

[17] Formally, if $\left[R^1 C^1_{y^1}\right]_{n,0} \neq \vec{0}$ then $\widetilde{\lambda}_n(1 + \Delta_n) = const$, where $R^1 \equiv l^1_L \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$.

[18] Formally, $\widetilde{m}_i \equiv \left(m^T R^1\right)_i$ and $\widetilde{\lambda\Delta}_i \equiv \left(\widetilde{\lambda}^T \widehat{\Delta} R^1\right)_i$.

be constant across households, it follows that the social value of additional employment is constant across households. Since the planner is indifferent over the direction of household spending, she targets solely based on the magnitude of that spending—i.e. household MPCs—as well as household welfare weights. A similar argument applies in the case of government expenditures.

4.3. Fiscal Stimulus Targeting Aggregate Underemployment

While the previous proposition clarified the information that a social planner needed to evaluate the optimality of any given policy, it does not define how to implement the optimal policy. To provide sharper answers to this question of what sort of stimulus to use and where to target it, we specialize to an environment where the planner is concerned only with output and the absolute amount of underemployment—ignoring redistribution, the direct value of government purchases, borrowing wedges, and potential disutility costs of labor for underemployed households. We view this as a sensible assumption in the context of a severe depression, where underemployment is widespread and not concentrated in particular demographic groups or regions.

Assumption 2. *The planner's objective satisfies the following conditions:*

1. *The planner is indifferent between households, i.e.* $\widetilde{\lambda}_n = 1$
2. *Government purchases have no intrinsic value, i.e.* $WTP_n^t = 0$
3. *Borrowing constraints do not bind, i.e.* $\phi_n = 0$
4. *All un(der)employed households have no marginal disuility of labor, i.e. if* $\Delta_n < 0$ *then* $\Delta_n = -1$

Moreover, all households n to which labor is rationed on the margin are un(der)employed.

Our next Proposition shows that these assumptions simplify the planner's motives considerably, so that she simply maximizes aggregate income. This makes the analysis of optimal fiscal policy policy tractable as maxmimally expansionary fiscal policy and optimal fiscal policy coincide.

Proposition 5. *Under Assumption 2, the welfare change from a change in expenditures is proportional to the resulting change in output, whereas the welfare change from a change in transfers is equal to the resulting change in income. Formally,*

$$dW = \vec{1}^T \frac{dY^1}{dG} dG + \vec{1}^T \frac{dl^1}{dy^1} \left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1} \right) \tag{25}$$

where the first-period output multiplier $\frac{dY^1}{dG^1}$ *is as in Proposition 1,* $\frac{dY^1}{dG^2} = 0$*, and* $\frac{dl^1}{dy^1}$ *is the first period income multiplier,* $\frac{dl^1}{dy^1} = \left(1 - l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1} C^1_{y^1}\right)^{-1}$.

Proof. See Appendix A.6 □

Key to Proposition 5 is the observation that its assumptions imply there is zero social cost of production; all marginal production is done by underemployed households, who are indifferent to working more. For these households, earned income is as good as a pure transfer. In the case of government expenditure shocks, the total change in income is equal to the total change in output. In the case of lump-sum transfer shocks, income changes both directly and through earnings generated by the change in the output; this generates the difference in multipliers.

In Appendix B.10, we show that this result carries over directly to environments with non-zero markups in the first period.[19] Intuitively, profit owners can be thought of as providing capital services with completely elastic supply. This allows us to treat capital owners "as if" they simply supply labor and are rationed to in proportion to firms' markups. The only modification required to accommodate this broader interpretation is that the output and income multipliers must be extended to include capital income.[20]

Our final policy result provides an answer to our motivating questions—whether to conduct stimulus using transfers or government spending, and what households or sectors to target—in the case where the "consumption network effects" of Proposition 2 are negligible. While this assumption may appear strong *prima facie*, we verify it empirically in Section 6. This sharp condition has stark implications for optimal policy.

Corollary 1. *Suppose that, relative to some income incidence* y^**, the bias and homophily effects are zero for all output and transfer shocks. Then, under Assumption 2, the welfare change from a policy is given by*

$$dW = \underbrace{\left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} m\right)^T}_{\text{Consumption multiplier}} \left(\underbrace{l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1} dG^1}_{\text{Spending income change}} \quad \underbrace{-\widehat{\mu}d\tau^1 - \frac{\widehat{\mu}d\tau^2}{1 + r^1}}_{\text{Transfer income change}}\right) \tag{26}$$

That is, dollar-for-dollar, the best policy is the one most effectively targeting household MPC.

[19] We allow for non-zero markups in the second period as well, provided (a) the government encourages second-period profit creation with consumption and input subsidies proportional to markups and (b) the MPC out of future capital income is zero.

[20] This interpretation contrasts sharply with Baqaee (2015), who proposes that a labor-wedge-reducing planner should target the industry with the highest network-adjusted labor share. The difference comes from the fact that Baqaee's model features competitive firms (hence no markups) and efficiently-allocated capital (no capital wedge).

Proof. See Appendix A.7 □

This result holds for two reasons. First, in the absence of network effects, all households direct their consumption in the same way for the purposes of amplification. Second, the planner simply wants to maximize output. As a result, to generate maximal amplification and thereby construct optimal policy, the best thing a planner can do is target households with the highest MPCs. Moreover, a sufficient condition for the absence of bias and homophily effects is that all households' marginal spending is directed to households whose average MPC is equal to the incidence-weighted average MPC corresponding to a uniform output shock. More formally, $m_n^{\text{next}} = \mathbb{E}_{y^*}[m_{n'}]$ for all n, where y^* is the income incidence of a uniform output shock. In our empirical work, we will show that this condition is approximately satisfied, making this MPC targeting result of direct relevance for policymakers.

A final implication of this result is that, for the same amount of spending, transfers weakly dominate government expenditures for stimulus purposes. This is because transfers more directly target MPCs, a household-level variable. At the same time, if it is possible to target MPCs close to as well with expenditures as with transfers,[21] then expenditures are likely to dominate transfers so long as government spending has some direct value.

5. Data and Estimation Methodology

Using our framework, we have so far derived a simple sufficient statistics expression for the generalized multiplier. We also demonstrated theoretically how rich household, industry and regional heterogeneity can interact to potentially amplify shocks and shape optimal policy. We now take our multiplier to the data to quantify the gains from targeting fiscal stimulus and understand how a planner should target such stimulus in practice. To do this, we directly estimate the sufficient statistics that comprise the multiplier using a variety of datasets. In this section, we describe both the datasets we use to estimate these sufficient statistics and the methodology we employ to calculate the components of the multiplier.

First, recall from Proposition 1 that in the case of zero interest rate responsiveness the response of output to the partial equilibrium response of demand ∂Q^1 to any primitive shock is given by:

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 (I - \widehat{X}^1)^{-1}\right)^{-1} \partial Q^1 \tag{27}$$

To estimate the multiplier, we therefore need estimates of three key objects: the regional input-output matrix $\widehat{X}^1$ describing the input use requirements of every region-industry pair, the rationing matrix $l^1_{L^1}\widehat{L}^1$ describing how much each demographic-region pair's income

[21] This is likely if, for example, it is politically untenable to make transfers to only high-MPC groups.

Dataset	Input Output	Rationing	Directed MPC
American Community Survey (ACS)		X	
BEA Make and Use Tables (IO)	X	X	
BEA Regional Accounts (RA)		X	
Consumer Expenditure Survey (CEX)		X	X
Commodity Flow Survey (CFS)	X		X
Consumer Price Index (CPI)		X	X
Internal Revenue Service Statistics of Income (IRS SOI)		X	
Panel Study of Income Dynamics (PSID)		X	X

Table 1: Summary of Datasets Used in the Estimation of Our Sufficient Statistics.

changes in response to a one dollar change in revenue of each region-industry pair, and the directed MPC matrix $C^1_{y^1}$ describing how much each demographic-region pair consumes from each region-industry pair when they receive a one dollar income shock.

In going to the data, we must also account for three empirically-relevant factors that were absent from our baseline model – capital, profit, and foreign income. At a high level, our strategy is to (1) model capital as an input, (2) model profits by assuming constant markups, as in Appendix B.4, and (3) model foreign factors as a type of "labor" with zero MPC, reflecting that payments leaving the economy do not re-enter through income effects.

The following subsections describe in detail how we estimate each of the three components of the generalized multiplier: the input-output, rationing, and directed consumption matrices. Table 1 shows which datasets are used in the estimation of each object. We restrict our attention to the United States in 2012, which is the most recent year for which we have several of the key datasets.

5.1. *The Regional Input-Output Matrix*

The regional input-output matrix $\widehat{X}^1$ is an $(R \times I) \times (R \times I)$ matrix where I is the number of industries and R is the number of regions. The (ri, sj) component of this matrix corresponds to the amount of sector i in region r's good required to produce a single unit of sector j in region s's good. To estimate this object, we must first take a stand on the level of granularity at which to model sectors and regions. Guided by the level at which input-output data are available, we largely follow the BEA's collapsed input-output sector classification, leaving us with 55 sectors which loosely correspond to the 3-digit NAICS classification.[22] Similarly, to take full advantage of the CFS microdata on interstate trade, we set regions at the level of the state (including Washington D.C.), leaving us with 51 regions. This leaves

[22]For full details on the definition of these sectors and for similar details, see the replication files.

us with 2805 sector-regions.

We construct the regional input-output matrix in three steps. First, following others in the literature, we use data from the 2012 BEA make, use, and imports tables to construct the domestic, national input-output matrix, which measures the dollar value of products from industry j that are used by industry i. In constructing this table, we assume that conditional on sourcing a commodity, the commodity is provided by industries in proportion to the amount of that commodity produced by those industries. We also make an adjustment to account for linkages across industries in capital investment. This is necessary as the standard use table accounts only for changes in intermediate goods usage. To impute each industry's expenditure on investment goods, we assume that all industries invest the same fraction of their gross operating surplus (available in the use table) in capital. To compute the direction of this investment toward different industries, we assume that each firm demands the same investment good and compute its industrial composition with the same procedure – using the use, make, and import tables – as we use for inputs. We then add this investment correction to the previously constructed input-output matrix.

Second, we use the 2012 public-use microdata from the Commodity Flows Survey (CFS) to construct a matrix describing how much each state imports from all other states. The CFS is a survey conducted by the US Census Bureau and includes data on 4,547,661 shipments from approximately 60,000 establishments. The data records the location of the shipping establishment, the commodity being shipped, the value of the shipped commodity, and the location to which the commodity is being shipped. The public use microdata file modifies this underlying data by introducing noise and top-coding extremely large shipments. Using this information, we calculate the total value of shipments between each pair of states for each tradable industry using the mapping between commodities and industries outlined in the BEA's make table.[23] For all nontradable industries, we assume that the commodity is sourced entirely within the state.

Finally, we construct the regional input-output matrix by combining the national industry-level input-output with state-by-state trade flows. Specifically, the amount of industry i in state r used by industry j in state s is the product of the share of industry j's inputs that come from industry i and the fraction of sector i goods flowing to s from r (out of all origin states). This yields a matrix describing, for each industry-region pair, how much of each other industry-region pair's production is used to produce a single unit of output.

[23] Caliendo et al. (2018) use a similar methodology to construct their regional input-output matrix.

5.2. *The Directed MPC Matrix*

The directed MPC matrix $C^1_{y^1}$ corresponds to an $(R \times I) \times (R \times N)$ matrix where N is the number of demographic groups. The (ri, sn) component of this matrix maps how a one dollar change in demographic n living in region s's income changes that household's consumption of good i in region r. Again, this first requires us to take a stand on the level of granularity at which to model demographic groups. Guided by the level at which precise estimation of MPCs is possible in the PSID, we set the number of demographic groups at 82, comprising 80 baseline groups (five income groups, four age groups, two gender groups, two race groups) and two dummy groups for the owners of capital and foreigners.[24]

We construct the directed MPC matrix in three steps. First, we construct MPCs for total consumption expenditure for each of our 80 demographic groups using the PSID, CPI and CEX following the methodology in Patterson (2019). Specifically, we follow the procedure of Gruber (1997), using the panel structure of the PSID to estimate the equation:

$$\Delta C_{ht} = \sum_x \left(\beta_x \Delta E_{ht} \times x_{ht} + \alpha_x \times x_{ht}\right) + \delta_{s(h)t} + \varepsilon_{ht} \tag{28}$$

where C_{ht} is household h's consumption at time t, E_{ht} is household h's labor earnings at time t, x_{ht} is a demographic characteristic of the individual, and $\delta_{s(i)t}$ is a state by time fixed effect. Estimating Equation 28 we then obtain the following estimate of the MPC for household h at time t:

$$\widehat{MPC}_{ht} = \sum_x \hat{\beta}_x x_{ht} \tag{29}$$

However, there are two challenges in performing this estimation. The first issues arises as there are a wide range of factors that could simultaneously move income and consumption. To address this, we instrument for changes in labor market earning using transitions into unemployment. This is desirable as such shocks are both large and persistent. Unemployment shocks therefore capture that variation most important to understanding recessions. Indeed, if recessions can be seen as shocks of the same persistence as unemployment, then this MPC is exactly the right object to capture shock propagation in the manner suggested by the model.[25]

[24] Our five income groups correspond to: less than \$22,000, \$22,000-\$35,000, \$35,000-\$48,000, \$48,000-\$65,000 and more than \$65,000. Our four age groups correspond to those 25-35, 36-45, 46-55 and 56-62. Our race groups are black and non-black. Our gender groups are men and women.

[25] While the MPC out of an unemployment shock is relevant for the general equilibrium amplification of shocks, it is potentially not the right MPC for determining the response of consumption to targeted transfers, which is the focus of the optimal policy analysis. We return to this in Section 6, but note here that the MPCs estimated here are close in magnitude and have similar cross-demographic patterns as those estimated using tax rebates or lottery winnings (Parker, Souleles, Johnson, and McClelland, 2013; Fagereng, Holm,

The second issue stems from measurement in the PSID: for most of the PSID sample, only expenditure on food consumption is measured. Using only this measure is problematic as food is a necessity and expenditure on food is likely to be distorted by the provision of food stamps (Hastings and Shapiro, 2018). To overcome this issue, we use overlapping information in the PSID and CEX to impute a measure of total consumption expenditure, following the methodology of Blundell, Pistaferri, and Preston (2008) and Guvenen and Smith (2014). Concretely, we use the CEX to estimate demand for food expenditure as a function of durable consumption, non-durable consumption, demographic variables and relative prices from the CPI. Under the assumption of monotone food expenditure, this function can be inverted to predict total consumption as a function of food expenditure and demographics in the PSID. This procedure generates substantial heterogeneity across households in estimated MPCs (see Figure A1 in Appendix E).

Next, we estimate the consumption basket shares in each of our 55 industries for each of our 80 demographic groups using the CEX and CPI. We first deflate consumption over the 54 measured categories using the CPI and then compute the average consumption basket share of each demographic group. Using a concordance between NIPA goods and our industry classifications, we then map consumption at the household level in each category to the 55 industries used in our analysis.

We use these consumption basket shares and our estimated MPCs to construct an estimate of the directed MPC for each of the 80 demographic groups out of each of the 55 industries. We do this by assuming linear Engel curves of households for each category of consumption. Formally, we estimate the directed MPC of household h at time t as:

$$\widehat{MPC}_{n(ht)i} = \alpha_{n(ht)i} \widehat{MPC}_{n(ht)} \tag{30}$$

where $n(ht)$ is the demographic group of household h at time t – which we from now on suppress when clear from context – and $\alpha_{n(ht)i}$ is the demographic-specific consumption basket weight of good i. Naturally, the imposition of linear Engel curves may be overly restrictive. However, our estimates always lie in the 95% confidence interval of estimates of good-specific MPCs from the PSID in the years in which this is possible (see Figure A2 in Appendix E), suggesting that we are capturing reasonable dimensions of heterogeneity with this assumption.

Finally, we use our estimated state-state gross flows in goods to arrive at the regionally-directed MPCs. Formally, for tradable goods, we assume that all households in a state consume from all other states in proportion to the fractions of imports of that good that

and Natvik, 2019).

originate from those states:

$$\widehat{MPC}_{risn} = \lambda_{irs}\widehat{MPC}_{ni} \tag{31}$$

where λ_{rs} is the fraction of shipments of good i from state s to state r as a function of the total shipments of good i to state r, as we earlier computed to construct the regional input-output matrix.[26] We assume all nontradable goods are consumed within the state.

The procedure above provides the directed MPC entries for the 80 demographic groups. It remains to estimate the directed MPCs for capitalists and foreigners. For foreigners, we simply set all entries to zero. This coincides with the assumption that, of all foreign recipients of income that leaves the US, none spend this income in the US or indirectly cause other spending in the US. For capitalists, we take the MPC out of stock market wealth as estimated by Chodorow-Reich, Nenov, and Simsek (2019) at 0.028. We then allocate this in the direction of the aggregate consumption basket as reported in the BEA use table.

5.3. The Rationing Matrix

The rationing matrix $l^1_{L^1}\widehat{L}^1$ corresponds to an $(R \times N) \times (R \times I)$ matrix where N is the number of demographic groups. The (rn, si) component of this matrix maps how a one dollar change in the production of good i in region s translates to a change in labor income for demographic n in region r.

We construct the rationing matrix in three steps. We first use the ACS to compute, within each state-industry pair, the total labor earnings by each demographic group in 2012. We also use state-level data from the BEA on compensation and output by industry to compute labor shares of value added for each state-industry pair.

Second, we use these two components, along with the estimated demographic group MPCs, to construct the the labor rationing entries for workers. Concretely, we employ the following formula:

$$\left(l^1_{L^1}\widehat{L}^1\right)_{rnsi} = \mathbb{I}[r=s]\frac{y_{inr}}{\sum_n y_{inr}}\alpha_{ir}\beta_i\left(1+\gamma\left(MPC_n - \overline{MPC}_{ir}\right)\right) \tag{32}$$

where y_{inr} is total earnings of demographic n in industry i in region r, Y_{ir} it total output in industry i in region r, α_{ir} is state-by-industry labor share of value added, β_i is the national value added to output ratio in industry i, γ is the correlation between MPCs and earnings elasticities, and $\overline{MPC}_{ir}$ is the earnings-weighted MPC of all workers in industry i in region

[26] This potentially sources too much consumption from outside the state given that the CFS comprises both consumption goods and intermediate goods flows. In section 6, we explore the robustness of this modelling assumption for how consumption is sourced by considering a model with total consumption autarky where all consumption is sourced within the state. This has a very small impact on the results.

r. The indicator function imposes the condition that all labor earnings are received within the state where production occurs. This is the unique functional form that both preserves a constant correlation between MPC and earnings elasticity, of which there is strong evidence from Patterson (2019) and preserves total income received across all demographic groups in each industry-region pair. We set $\gamma = 1.332$, the correlation of MPC with earnings elasticity to aggregate shocks measured in Patterson (2019).[27] While our model can in principle incorporate regional migration in response to shocks, we – by assuming that employment at each firm only depends on its own labor demand – only partially allow for this possibility. In particular, our calibration rules out the possibility that the share of labor each firm rations from each demographic group may depend on changes in the group's share of the population due to migration.

Finally, it remains to allocate those factor payments that are not received by labor. These take two forms: payments made to the domestic owners of capital and payments made to foreign factors. We compute directly payments made to domestic owners of capital via the following procedure. We first compute profits in each region-industry pair. To do this, we compute the domestic profit share of production from the BEA use table and add this to the residual value added in each state-industry pair that is not paid to labor. We then allocate these profits to the capitalist demographic group in each state according to that state's share of dividend income in the IRS SOI data. Finally, we compute payments made to foreigners as the residual of payments made to intermediate producers, payments made to labor and payments made to capitalists.[28]

6. Empirical Analysis of Targeted Fiscal Policy

In this section, we quantify the importance of targeted fiscal policy and characterize the dimensions on which policymakers should target spending. First, we quantify the degree of heterogeneity in fiscal multipliers, demonstrating both that there are substantial gains to be had from targeting fiscal policy effectively and that these gains stem almost exclusively from differences in the initial incidence of the shock on households with different MPCs. Second, having demonstrated that targeting is quantitatively important, we also demonstrate that it is empirically simple and that the social planner can do very well by simply targeting household MPCs. Throughout both sections, to marshal the discussion, we use our estimated

[27] See Patterson (2019) for more details and discussion.

[28] In a small fraction of cases, this leads to a *negative* foreign share of revenues, which is unrealistic. To avoid this, we could alternatively reduce the profit share of revenue in region-industry pairs with high labor shares. Insofar as we use similarly small MPCs for foreigners and capitalists, this alternative calibration would generate similar quantitative results.

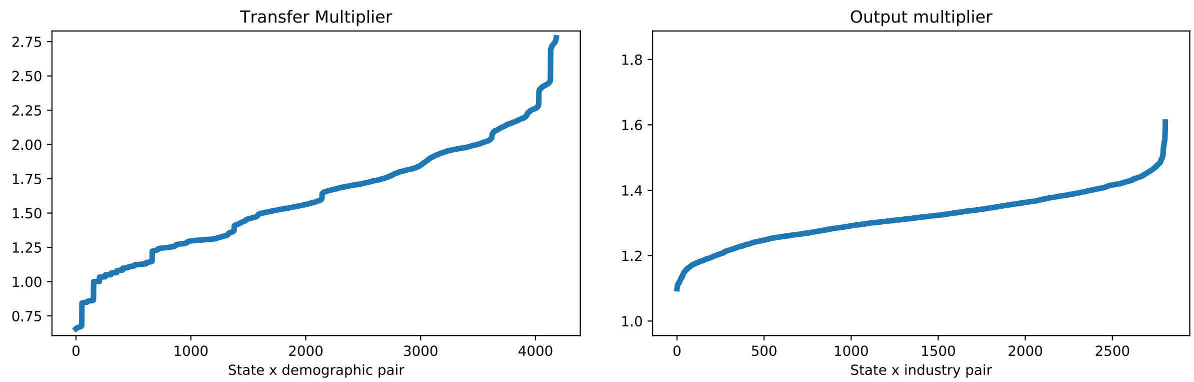

Fig. 2. Left: distribution of transfer multipliers, giving the change in aggregate income from a one dollar transfer to each state-by-demographic group. Right: distribution of output multipliers, giving the change in aggregate income (also, GDP) from one dollar of expenditure on each state-by-industry pair.

sufficient statistics and employ earlier theoretical results detailing the conditions for simple policies to be optimal . Finally, we explore the size of geographic spillovers and the effect of changes in the labor share on the effectiveness of fiscal policy.

6.1. The Importance of Targeting Fiscal Stimulus

We begin our empirical analysis by quantifying the potential gains from targeting each dollar of public spending. To this end, Figure 2 shows the heterogeneity in transfer and output multipliers. The left panel shows the effect on aggregate income of transferring one dollar to a household of a given demographic within a specified state. We uncover wide dispersion in multipliers – the effect on aggregate income of transferring a dollar to a household ranges from slightly below one for some households (some types have negative MPCs) to nearly three dollars for others. The right panel shows the corresponding distribution of output multipliers, showing the effect on total output of spending a dollar in a given industry within a specified state. Again, there is wide dispersion across industry and space with multipliers that differ by a factor of six. Much of the heterogeneity in multipliers – and so the gains from targeting – remain when targeting is forced to be more granular: output multipliers differ by a factor of more than three across industries and a factor of 1.5 across states (see Figure A18 in Appendix E); transfer multipliers differ by a factor of 1.3 across states and display the same heterogeneity across demographic groups (See Figure A19 in Appendix E). Both panels in Figure 2 emphasize that, for a social planner seeking to maximize total output, there are very large gains from targeting fiscal policy to the households or industries with the highest multipliers.

6.1.1. *Decomposing Sources of Heterogeneity in Multipliers*

Recall from Proposition 2 in Section 3.2 that for any change in government spending causing a unit magnitude partial equilibrium change in labor incomes ∂y^1, the total change in GDP is determined by three network adjustments to the basic Keynesian multiplier. Therefore, the dispersion in fiscal multipliers from Figure 2 could be coming from differences in 1) the incidence of the shock, meaning that shocks to some markets load more heavily on agents with higher MPCs, 2) the bias in spending, meaning that some shocks to some markets induce marginal spending that is more directed towards high MPC households, or 3) the homophily of spending, meaning that shocks to some markets induce spending that is more segmented by MPCs. However, we find that — as an empirical fact — all of the heterogeneity across groups in Figure 2 is driven by the differential direct incidence of those shocks onto agents with different MPCs. Aside from being descriptively interesting, this fact is also relevant for designing policy, as Proposition 5 demonstrated that the magnitude of network effects informs the optimality of simple targeting rules.

To understand why only the incidence effect is empirically large, recall from Proposition 2 that, in order for the bias and homophily terms to be large, there must be significant heterogeneity across households in basket-weighted MPCs m_n^{next} and these basket weighted MPCs differ from the benchmark $\mathbb{E}_{y*}[m_n]$. Indeed, if m_n^{next} is homogeneous and $\mathbb{E}_{\partial y^1}[m_n^{\text{next}}] = \mathbb{E}_{y*}[m_n]$, then both the bias and homophily terms are zero as all households effectively direct their consumption in the same way. The left panel of Figure 3 documents that in the data, there is minimal heterogeneity in basket-weighted MPCs, shown by the very shallow slope between basket-weighted MPCs and household MPCs. As a result, the homophily effects are very close to zero. Moreover, the scatterplot demonstrates that basket-weighted MPCs all lie very close to the benchmark average MPC $\mathbb{E}_{y*}[m_n]$. Consequently, bias effects are also very close to zero. Indeed, for any possible shock, the incidence term accounts for more than 99 percent of the multiplier.[29] To drive this point home, the orange line in the right panel of Figure 3 shows multipliers from a counterfactual model without heterogeneous consumption in which the bias and homophily effects are identically zero. As one can see, there is effectively no difference in the full distribution of multipliers when we

[29] Concretely, we construct any feasible ∂y^1 via a linear combination of demand shocks to each sector-region pair. We then compute the bias and homophily effects from each of these shocks and plot the full distribution of bias and homophily terms (see Figures A4 and A5 in Appendix E, respectively). Across the full distribution of shocks, the contributions of the bias and homophily terms range between zero to four tenths of a percent increase in the multiplier – they are empirically negligible for all feasible demand shocks. We also compute the full distribution of error terms arising from the approximation in our decomposition result (Figure A6 in Appendix E) and find that they are uniformly an order of magnitude smaller than the bias and homophily terms. Our approximation is therefore very tight for any feasible shock.

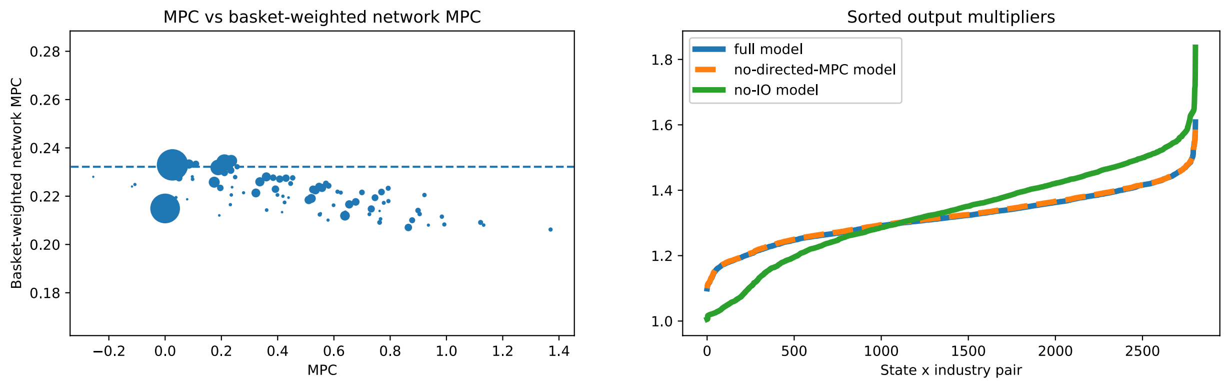

Fig. 3. The left panel shows a scatter of MPCs m_n against basket-weighted MPCs m_n^{next}. The dashed line gives the average MPC $\mathbb{E}_{y*}[m_n]$ for y^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP. The right panel shows the change in GDP for each industry-region pair according to a one dollar demand shock in each pair, sorted by the magnitude of the effect. The full model is the baseline and plotted in blue. No directed MPC assumes that all households direct their consumption in proportion to aggregate consumption. No IO assumes that there is no use of intermediate goods.

impose this condition, demonstrating that it plays no role in shaping the baseline estimates.[30]

The lack of consumption network affects appears to be a real feature of the data, rather than a failure of our estimation approach to capture them. Critically, our estimates of consumption basket shares in the CEX do display substantial variation across households (see Figure A8 in Appendix E), allowing the possibility of large network effects. The lack of estimated consumption network effects then stems from two opposing forces. On one hand, high MPC households disproportionately consume goods produced by low-labor-share industries (see Figure A9 in Appendix E), directing more spending toward capital, the owners of which have low MPCs.[31] On the other hand, our estimates feature substantial within-region non-tradeables demand, with around a third of total labor demand remaining within the state from which consumption originates (see Figure A11 in Appendix E). Moreover, there is spatial heterogeneity in MPCs, with income-weighted MPCs differing by a factor of 1.5 across states (See Figure A16 in Appendix E). Together, these regional forces generate a modest positive homophily effect whereby higher (lower) MPC workers direct their consumption more toward local labor which similarly features high (low) MPC. When combined, however, these labor share and local demand effects – each modest on its own – partially

[30]In Figure A7 of Appendix E we show a scatterplot of the multipliers from these two models. The correlation in multipliers across the two models is nearly perfect.

[31]Conditional on reaching labor, the average MPC of workers producing consumption baskets is homogeneous across the MPC distribution (see Figure A10 in Appendix E), so labor share differences account for the bulk of differences in basket-weighted MPCs stemming from heterogeneous consumption baskets. This finding is also consistent with the empirical patterns in Hubmer (2019).

cancel, so that all types spend on goods baskets produced by households of very close to the average MPC.

Since the heterogeneity in amplification in Figure 2 does not stem from higher order network effects, it must instead come from differences in the incidence of different shocks onto the MPCs of households. For transfers, the initial incidence is immediately apparent and is driven solely by heterogeneity in MPCs in the population. However, for government spending, three distinct factors contribute positively to these differences: First, differences in the demographic composition of the workforce across sectors and regions causes large differences in the average MPCs of workers across firms and regions. Second, differences in the share of labor that each sector directly employs cause large differences in the MPC of the ultimate recipients of factor income. In particular, agents employing lots of capital but little labor pass most factor payments on to the owners of capital who have very low MPC and therefore feature small output multipliers. This is shown in Figure A12 in Appendix E that plots the labor share of each industry-state pair against its output multiplier: there is substantial heterogeneity in labor use and low labor use is associated with a small output multiplier. Third, differences across firms in the covariance of worker MPC and exposure to changes in firm revenue generate additional widening of the distribution of multipliers. This is shown in Figures A14 and A15 in Appendix E where we compare the baseline model with rationing more to agents with higher MPC to a model with rationing to agents uniformly by income, where we observe both an upward shift in the distribution of output multipliers as well as an increase in range.

Conversely, input-output linkages serve an important role in *narrowing* the heterogeneity induced by these differences. This can be seen in the right panel of Figure 3, where the green line corresponds to the model without input-output linkages, which features a much more dispersed distribution of multipliers.[32] The role of input-output linkages in reducing dispersion is intuitive. In the absence of inputs, when the firm directly employing the highest-MPC factors gets an additional dollar of revenue, it spends it all on those high-MPC factors. With inputs, this same firm spends a fraction of its revenue on goods produced by other firms, who in turn direct that money to their (by construction) less-than-highest-MPC factors – effectively diluting the MPC of the initial firm. This dilution effect attenuates the heterogeneity in industry multipliers. Finally, note that in Figure 2, there is much greater heterogeneity in transfer multipliers than output multipliers. Given the importance of the initial incidence in driving the variation, this is natural as transfers more effectively target households with the highest MPCs than expenditures.

[32] See Figure A17 in Appendix E for a scatter plot of the multipliers across both the full model and that without input-output linkages.

	Benchmark Multiplier	Optimal Policy Multiplier
Government Transfers		
Baseline model	1.77	2.78
Government Spending		
Baseline model...	1.30	1.61
... shutting off directed MPCs	1.30	1.61
... shutting off IO network	1.30	1.84
... shutting off MPC-based rationing	1.24	1.49

Table 2: The benchmark multiplier for government spending corresponds to a GDP-proportional government spending shock. The benchmark multiplier for government transfers corresponds to a uniform stimulus cheque policy. The model with IO, directed MPCs and MPC-based rationing is the baseline. No IO assumes that all industries consume no intermediate goods. no directed MPC assumes that all households direct their consumption in proportion to aggregate consumption. When we shut off MPC-based rationing we assume that all households are rationed to in each industry in proportion to their share of income in that industry.

Our finding that IO linkages reduce heterogeneity in output multipliers is distinct from an existing literature that emphasizes the role of IO networks in amplifying economic shocks (Acemoglu et al., 2012; Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2016; Baqaee, 2018; Elliott, Golub, and Leduc, 2020). First and foremost, our finding is not that IO linkages attenuate amplification on *aggregate*, but rather than they reduce the dispersion in amplification across industries. In this sense, we simply have a different focus. Moreover, the key reasons that IO links generate aggregate amplification in the literature—namely, that supply shocks are more powerful when the input share of production is large (a la Hulten) and that supply and demand shocks can cause cascades of firm defaults when production has a fixed cost—play no role in our setting, as we focus on demand shocks and assume production is CRS.

6.1.2. Quantifying Gains from Targeted Spending

We quantify the magnitude of the gains from targeting policy towards the segments of the economy with the highest multiplier by comparing the distribution of multipliers to the multiplier that would exist if either the government distributed a dollar evenly across the population (i.e. untargeted transfers) or if the government simply purchased the bundle of goods across industries and regions in proportion to GDP (i.e. untargeted spending). This second benchmark also gives an estimate for what we term the *aggregate multiplier*, defined as the response of GDP to a GDP-proportional demand shock across industries and regions. We focus first on government spending. Table 2 shows that in the baseline calibration, spending a dollar proportionally to GDP generates an aggregate multiplier of 1.30, a number consistent with the large literature on fiscal multipliers (Ramey, 2011; Chodorow-Reich, 2019). While this number is sizeable, the estimates in Figure 2 demonstrate that had the policymaker instead spent on the state-industry pair with the highest multiplier (which we estimate to be 1.61 in the oil and gas extraction industry in Georgia), the policy would have twice the stimulus effect. For transfer spending, the optimally targeted stimulus (which gives money to black men in South Carolina aged between 25-35 who earn less than $22,000) is 130% more effective than uniform stimulus cheques.

The bottom rows of Table 2 clarify the role that the various dimensions of heterogeneity in the model play in shaping the magnitude of these untargeted benchmarks. Our earlier finding that consumption network effects are unimportant for local shocks carries through and implies that the aggregate multiplier is almost unaffected by the direction of consumption; if one were to assume that all households consumed the same good – one sourced from each household in proportion to its income – then the bias and homophily terms would be exactly zero. The sixth row of Table 2 shows the results under this more restricted setting and reveals that they are almost identical to the baseline estimates.[33]

More surprisingly, the penultimate row of Table 2 shows that accounting for IO linkages is also unimportant for the magnitude of the aggregate multiplier. This is despite the fact that – as we have shown – accounting for IO linkages is important for understanding the cross-section of multipliers. Intuitively, IO linkages reduce the effective MPC of industries with high-MPC workers and increase the effective MPC of industries with low-MPC workers, but have roughly zero effect in the aggregate as these two forces cancel out.

Finally, the bottom row of Table 2 shows the importance of accounting for the fact that high-MPC households are more exposed to business cycle shocks. The first row shows the multiplier in the scenario with income-proportional rationing while the second row shows the case with the empirical incidence of shocks. We find that the income-proportional rationing dampens the output response by approximately 20 percent. This echoes the finding of Patterson (2019), but in a richer model. In the Appendix, we show that accounting for regional vs. national structure (see Table A4 in Appendix E) as well as inter-regional trade (see Table A5 in Appendix E) also has a limited impact on the aggregate multiplier.

6.2. The Simplicity of Targeting Fiscal Stimulus

The previous section demonstrated that the gains from targeting fiscal policy are substantial. The question remains of *how* to target fiscal policy to realize those gains. The next section demonstrates the gains to the government from implementing a very simple targeting rule: targeting based solely on household MPCs is close to optimal.

[33] Table A3 in Appendix E confirms that the bias, homophily, and error terms are small in the case of a GDP-proportional demand shock.

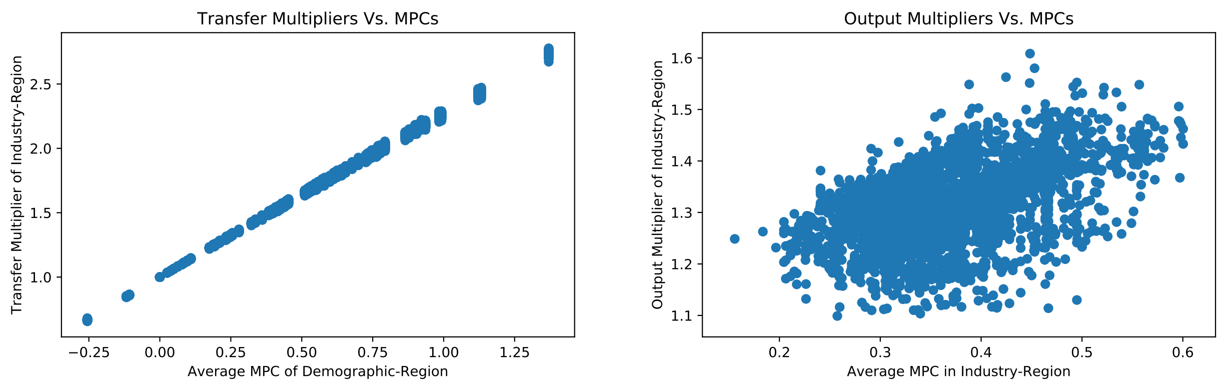

Fig. 4. Left: the effectiveness of targeting transfer stimulus by household MPCs. Right: the effectiveness of targeting expenditure stimulus by the average MPC of workers in each industry-region pair.

6.2.1. Simple MPC Targeting to Maximize Output

Recall from Proposition 5 that – for a planner whose sole goal is to reduce the total extent of factor underutilization – it is optimal to maximize aggregate income.[34] In this case, the policymaker will simply want to structure their spending such that is has the largest multiplier. While identifying the multiplier on each dollar of spending is potentially very complicated, the decomposition results summarized in Figure 3 imply that it is empirically very simple. Figure 3 demonstrated that bias and homophily effects are empirically negligible and that all of the heterogeneity in multipliers stems from the initial incidence of the spending.[35] This is precisely the condition required for Corollary 1, implying that both optimal expenditure policy and transfer policy should be designed simply to target agents with the highest MPCs.

Figure 4 more directly demonstrates the near-optimality of simple MPC targeting. The left panel scatters household MPCs against the resulting transfer multiplier from giving them a dollar, revealing an effectively perfect relationship between the two.[36] Optimal transfer

[34]Proposition 5 is robust to not only the presence of profits (see Appendix Proposition 19) but also to the presence of foreign income, provided the planner is indifferent to foreign factors' disutility of factor supply. To the extent profits and foreign income are small, income-maximization can alternatively be justified through the lens of its effects solely on involuntary unemployment, one of the most often mentioned concerns of policymakers (Elmendorf and Furman, 2008).

[35]In particular, Figure 3 shows us that $m_n^{next} \approx \mathbb{E}_{y*}[m_{n'}]$ for all household types n.

[36]Note that the MPC that we use in Figure 4 is estimated using unemployment as the identifying shock, and therefore captures the consumption response to a potentially persistent shock. The MPC that is better suited for the analysis of fiscal policy would be the MPC out of a transitory shock. If the MPC out of these two shocks are highly correlated across demographic groups, this difference should be less important for the question of which demographic groups to target. While it is hard to test this explicitly, the cross-demographic patterns in MPCs that we utilize here have a correlation of above 0.5 with self-reported MPCs from survey data (Jappelli and Pistaferri, 2014) and have similar patterns as those in response to tax rebates (Parker et al., 2013).

policy is clear – giving cash to households with the highest MPCs is optimal. Even the IO network and industry labor shares are irrelevant to the planner; a policymaker need know only household MPC.

By contrast, optimal expenditure policy targets those sectors such that when their production expands, accounting for the intermediates goods they use and the intermediates used by the producers of those intermediates and so on, the resulting change in labor income ends up in the hands of the highest MPC agents. While this requires no knowledge of the direction of household spending, it does rely on an understanding of the structure of production – through the input-output network and labor rationing. Critically, it is not sufficient for the government to target the sectors *employing* the highest MPC workers. Instead, they should work out the final labor income consequences of their spending and target according to the MPC of the workers receiving that terminal labor income. This echoes results in Baqaee (2015), which emphasizes the need to adjust labor shares for the input-output structure of production. This difference is quantitatively important; the right panel of Figure 4 shows how naively targeting sectors employing the highest MPC workers is effective but leaves a lot of the gains from targeting on the table. To the extent that transfer policy bypasses these complications by directly giving income to households, it is easier to target than government expenditures. The clear caveat is that fiscal expenditure may have direct value. If this is the case, our analysis shows how much stimulus would have to be sacrificed to obtain that direct value, enabling a policymaker with knowledge of the value of direct government purchases to determine which policy to optimally pursue.

6.2.2. MPC Targeting with Dispersion in Underemployment

Although we have focused so far on the case where the social planner seeks solely to increase aggregate income, our framework also provides a flexible toolkit for evaluating the welfare effects of fiscal policy targeting more localized economic downturns. For example, this would be the case with the initial shock to which the policymaker is responding is very concentrated in some areas and underemployment is less widespread. In this case, the planner does not simply wish to maximize aggregate income, but also wants to direct stimulus to those households who are most severely underemployed. In this case, it will no longer be optimal for the planner to simply target based on MPCs, but rather they will target on a combination of MPCs and labor wedges. As an example of how our framework can be applied in this setting, we perform such an exercise using imputed labor wedges during the Great Recession, which, although widespread, had much more severe impacts on certain regions and demographic groups.

Abstracting from any intrinsic value of government spending, Proposition 3 shows that the change in welfare induced by government expenditure is simply the product of the rationing wedges induced by involuntary unemployment and the change in labor income induced by the policy. Moreover, in Appendix C, under standard assumptions on labor supply, we show how the rationing wedge for each demographic group in each state is given by the percentage change in labor hours worked by that group in the Recession relative to the preceding period.[37] To compute the welfare effects of fiscal policy, we can then simply combine changes in hours worked at the state-demographic level in the ACS from 2005-6 and 2009-10, and take the product with the induced spending-to-labor-income map that we have already estimated. This delivers the welfare gain from the stimulus benefit associated with spending one dollar in a specific industry in a specific state in the middle of the Great Recession.

There are three key takeaways from this analysis. First, the output multiplier of fiscal stimulus is strongly positively predictive of its welfare effect, with an estimated R^2 of 69% (see Figure A20 in Appendix E). Thus, on average, targeting stimulus during the Great Recession to the sectors where income flows to the highest MPC households would have been desirable. Second, the average level of labor wedges of workers in a given region and industry is highly predictive of the welfare effect of stimulus targeting that industry and region, with an R^2 of 72% (See Figure A21 in Appendix E). As a result, fiscal policy that directly targets those industries with the greatest underemployment, such as the US government bailout of the auto industry in the Great Recession, is welfare improving, over and above the effect on maximizing output. Third, accounting for just the size of the fiscal multiplier and the average level of wedges in each state-industry pair explains 78% percent of the variation in the welfare effects of stimulus. Thus, in the context of the Great Recession, a planner who simply takes into account the size of multipliers via MPCs and the amount of unemployment in the sectors they are considering directing stimulus toward can realize the bulk of the gains from targeting.

Finally, we note that as the Great Recession induced widespread underemployment, it is perhaps unsurprising that around two thirds of the welfare gains from fiscal stimulus can be explained by the size of fiscal multipliers. In the case of a more localized shock or recessionary episode, heterogeneity in wedges would play a greater role; our framework still facilitates such an evaluation.

[37] In particular, this is true if either (i) all households within a group are homogeneous, have quadratic labor disutility and apply a zero utility discount rate to the future or (ii) all households within a group are probabilistically totally unemployed or fully employed.

6.3. *Discussion*

6.3.1. *Geographic Spillovers*

One focus of the recent empirical literature on fiscal multipliers has been the strength of fiscal spillovers across states. Quasi-random cross-regional variation in fiscal spending has allowed researchers to estimate local fiscal multipliers (Nakamura and Steinsson, 2014; Chodorow-Reich, 2019). The relationship between these local estimates and the national multiplier is complicated by the presence of potentially large local spillovers – research designs using cross-sectional estimates usually recover only the relative effect of spending more in state i than in state j and are unable to directly measure the potential effect that spending in state i has on output in state j.

The regional interlinkages embedded in our model allow us to provide an estimate for the magnitude of these cross-state spillovers. We quantify these spillovers within our model by considering a unit of government spending in each state, which we assume is distributed across industries within the state in proportion those industries' shares of GDP within the state. Averaging across states, we find that total output in the economy increases by 1.3 units in response to 1 unit of additional spending. Of this 30 percent amplification, about 16 percentage points come within the state that received the additional government spending, while 14 percentage points come from from spillovers to other states – firms and households in the shocked state demand more goods and some of those are sourced from other states.[38] The spillover to any given state is small and only about 2 percent as large as the effects within the shocked state. However, each state contributes to the total effect, and overall, the spillovers contribute meaningfully to the overall effect of the shock.

These estimates are in line with some recent empirical evidence estimating the magnitude of these spillovers directly. Specifically, Auerbach et al. (2020) use detailed geographic information on local defense spending and find that large positive spillovers across geographies, suggesting the importance of positive demand spillovers through input-output networks and directed MPCs. They also find that the spillovers are decreasing in the distance between cities. Our results are consistent with this, as our estimated spillovers are largest for the geographically closer states.[39] These estimates suggest that demand spillovers across states are empirically important when evaluating the total effect of localized fiscal spending.

[38]Of course, the shock itself all remains in the shocked state, so that the total change in output within the shocked state is 1.16, on average.

[39]In Appendix Section D, we more formally explore the extent to which our model predicts the cross-state spillovers in response to several identified demand shocks. While the estimates are under-powered, we find evidence suggesting that our structural estimates are qualitatively consistent with cross-state spillovers in response Chinese-import shocks as in Autor, Dorn, and Hanson (2013).

6.3.2. *Fiscal Policy and the Labor Share*

The analysis throughout this paper highlights that the multiplier is not a deep structural parameter of the economy, but rather depends critically on the incidence of the shock in consideration. Similarly, these estimates could change substantially over time as the underlying structure of the economy changes – but we have estimated our multiplier in a single year, 2012. One potentially relevant change in the economy over the past several years is the well-documented decline in the labor share in the US (Karabarbounis and Neiman, 2014; Dorn, Katz, Patterson, and Van Reenen, 2017). More recently, Hazell (2019) provides empirical evidence that this reduction in the labor share has dampened unemployment fluctuations. In this section, we perform a similar exercise in our model, comparing the output and transfer multipliers as industry-specific labor shares change from their 2000 to 2012 levels. Intuitively, if spending is directed away from high-MPC workers and toward low-MPC capitalists, aggregate amplification should fall.

Our methodology is as follows. We assume that, within each year and each industry, the shares of employee compensation in revenue is constant across states. We obtain these shares from the BEA use tables in 2000 and 2012. The aggregate labor share of value added fell from 59.2% in 2000 to 54.9% in 2012; the aggregate labor share of revenue fell from 32.1% to 30.0%. Figure A22 shows the distribution of labor shares of revenue by industry in each year. We maintain our earlier, 2012-based, estimates of demographic-specific consumption baskets and MPCs, demographic employment by region, and input-output network. We allocate the difference in labor income between 2000 and 2012 to a factor with MPC zero; this can be understood as a foreign factor or as profits accruing to MPC-zero shareholders.

Unsurprisingly, the reduction in the labor share leads to a smaller multiplier, as revenues are directed to lower-MPC households. We estimate an aggregate multiplier – i.e. the output response to a shock proportional to the 2012 distribution of output across states and industries – of 1.338 in 2000 and 1.300 in 2012. Figure 5 shows the sorted distributions of output and transfer multipliers across all shocks, for 2000 and 2012. Predictably, the distribution of output multipliers shifts down, as less of the income from a given change in demand flows to workers and more flows to low-MPC factors. Still, the multiplier does not fall for every state-industry pair. Figure A23 shows that a few industries – namely those with sufficiently increased labor shares, such as "apparel and leather and allied products" – have higher multipliers in 2012 than in 2000.

For transfer multipliers, the response to changing labor shares is almost zero. This is because transfers target households of each MPC directly, so that differences in the labor share only affect the incidence of higher-order spending.

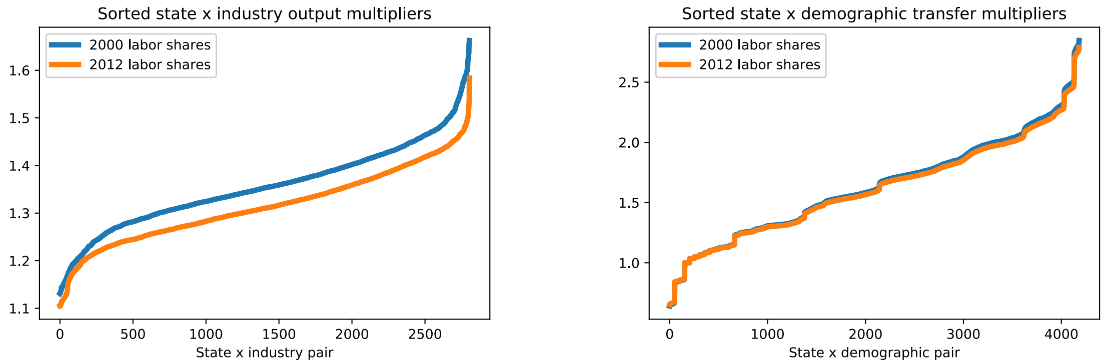

Fig. 5. Multipliers for state-industry-level output shocks and state-demographic-level transfer shocks. Differences in labor shares are more relevant for output shocks.

7. Conclusion

This paper has developed theoretical methods to understand optimal fiscal policy and taken these to the data to both quantify the gains from targeting and characterize the dimensions on which the government should target. We built a Keynesian model with rich household heterogeneity in MPC magnitudes and directions, industrial and spatial linkages, and differential employment sensitivity. All of these elements can be unified into a single reduced-form network that maps the marginal spending of any given household to the marginal income of factor owners producing the goods the household consumes. We provide a novel decomposition to understand the importance of these rich interconnections by providing three corrections to the standard Keynesian multiplier. Critically for policy, in a special case in which spending network effects are absent, the optimal fiscal policy is remarkably simple: even away from the optimum, targeting according to household MPCs is optimal.

Empirically, we find that this special case is not so special – despite a rich regional, input-output and consumption structure, the government can implement near optimal transfer policy by simply targeting households with the highest MPCs. Linkages through the direction of household spending are empirically unimportant, so that the effect of a demand or supply shock on aggregate output only depends on the shock's incidence onto the incomes of households of different MPCs. Indeed, optimal targeted government spending in the model yields twice as much amplification as untargeted stimulus, with targeted transfer spending yielding 130% more amplification than untargeted policy

This is a result with powerful implications for policymakers and researchers. First, governments should understand the costs associated with untargeted fiscal spending. While there may be other important implementation or political constraints that weighed in favor of stimulus checks, the above analysis suggests that the untargeted fiscal policies in the Great Recession and the COVID-19 pandemic left substantial gains on the table. Second, the results suggest that measuring household MPCs and the degree to which they vary along dimensions that are easily observed by the policy maker is a very important research priority.

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Appendices

A. Omitted Proofs

A.1. Proof of Proposition 1

Proof. This proof is a Corollary of Proposition 8, presented in Appendix B.2. From Proposition 8, recall that:

$$dY = \left(I - D\left(I - \widehat{X}\right)^{-1}\right)^{-1} \partial Q \equiv M \partial Q \tag{A1}$$

where:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^1} & \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^1} & \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^2} \end{bmatrix} \tag{A2}$$

Under Assumption 1, this reduces to:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \end{bmatrix} \tag{A3}$$

Simple matrix manipulations show that one may extract just the first $\mathcal{I}^1$ rows:

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 \tag{A4}$$

□

A.2. Proof of Lemma 1

Proof. Starting from Proposition 1 and using that the modulus of $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ is less than 1, we can express:

$$\begin{aligned} dY^1 &= \sum_{k=0}^{\infty} \left[C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right]^k \partial Q^1 \\ &= \partial Q + \overline{C}^1_{y^1} \hat{m} \sum_{k=0}^{\infty} \left[l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \overline{C}^1_{y^1} \hat{m}\right]^k l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^1 \\ \vec{1}' dY^1 &= \vec{1}^T \partial Q + m^T \left(\sum_{k=0}^{\infty} (\mathcal{G}\hat{m})^k\right) \partial y^1 \end{aligned} \tag{A5}$$

where the last line uses the definitions of $\mathcal{G}$ and ∂y^1, and the fact that $\vec{1}^T \overline{C}^1_{y^1} = \vec{1}^T$ (by construction).

Finally, $\vec{1}^T \partial Q^1 = \vec{1}^T \partial y^1$ because $\vec{1}^T \cdot l^1_{L_1} \widehat{L}_1 (I - \hat{X}^1)^{-1} = \vec{1}^T$, since firms earn zero profits.

□

A.3. Proof of Proposition 2

Proof. Let $b \equiv \vec{1}^T(I - \mathcal{G}\hat{m})^{-1}$ be the vector of Bonacich centralities of households in the income-to-spending network; these are well defined as we have assumed the modulus of $\mathcal{G}\hat{m}$ is less than one. Let $(b^{next})^T = b^T\mathcal{G}$ be the row vector with n^{th} entry equal to the average Bonacich centrality of the household to whom n's marginal spending flows.

We begin by providing a lemma that exactly decomposes the general equilibrium change in output, in terms of Bonacich centralities.

Lemma 2. *For any $x \in \mathbb{R}$, the total change in first-period output due to a partial equilibrium demand shock with unit-magnitude labor income incidence ∂y^1 is equal to*

$$\vec{1}^T dY^1 = (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - x\right) + \mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}] \tag{A6}$$

Setting x equal to the $\frac{1}{1-MPC}$ multiplier with the MPC weighted by income y^*, we obtain an exact decomposition in the spirit of Proposition 2.

$$\begin{aligned}\vec{1}^T dY^1 = \; & \underbrace{\frac{1}{1 - \mathbb{E}_{y*}[m_n]}}_{\text{Keynesian multiplier}} + \underbrace{\frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1 - \mathbb{E}_{y*}[m_n]}}_{\text{Incidence effect}} \\ & + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - \frac{1}{1 - \mathbb{E}_{y*}[m_n]}\right)}_{\text{Biased MPC direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}]}_{\text{Homophily effect}}\end{aligned} \tag{A7}$$

Proof. Note that Proposition 1 implies that the change in output resulting from some shock with unit incidence is given by

$$\vec{1}^T dY^1 = b^T \partial y^1 = \vec{1}^T \partial y^1 + b^T \mathcal{G}\hat{m}\partial y^1 \tag{A8}$$

Letting $b^{nextT} = b^T\mathcal{G}$ be the row vector with i^{th} entry equal to the average Bonacich centrality of the household who i's marginal spending flows to. We then have, for any $x \in \mathbb{R}$:

$$\begin{aligned}\vec{1}^T dY^1 &= 1 + \mathbb{E}_{\partial y^1}[m_n b_n^{\text{next}}] = 1 + \mathbb{E}_{\partial y^1}[m_n] \cdot \mathbb{E}_{\partial y^1}[b_n^{\text{next}}] + \text{Cov}_{\partial y^1}[m_n, b_n^{\text{next}}] \\ &= (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{\text{next}}] - x\right) + \text{Cov}_{\partial y^1}[m_n, b_n^{\text{next}}]\end{aligned} \tag{A9}$$

□

We can now prove Proposition 2. First, note that:

$$b_n = 1 + m_n + O(|m|^2) = 1 + \frac{m_n}{1 - \mathbb{E}_{y*}[m_{n'}]} + O(|m|^2) \tag{A10}$$

Plugging this into Equation A7, we have

$$
\begin{aligned}
\vec{1}^T dY^1 &= \frac{1}{1-\mathbb{E}_{y*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1-\mathbb{E}_{y*}[m_n]} \\
&+ \mathbb{E}_{\partial y^1}[m_n]\left(1 + \frac{\mathbb{E}_{\partial y^1}[m_n^{\text{next}}]}{1-\mathbb{E}_{y*}[m_{n'}]} + O(|m|^2) - 1 - \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y*}[m_n]}\right) \\
&+ \mathbb{C}\text{ov}_{\partial y^1}\left[m_n, 1 + \frac{m_n^{\text{next}}}{1-\mathbb{E}_{y*}[m_{n'}]} + O(|m|^2)\right] \\
&= \frac{1}{1-\mathbb{E}_{y*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1-\mathbb{E}_{y*}[m_n]} \\
&+ \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y*}[m_n]}\mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] \\
&+ \left(\frac{1}{1-\mathbb{E}_{y*}[m_n]}\right)\mathbb{C}\text{ov}_{\partial y^1}\left[m_n, m_n^{\text{next}}\right] + O(|m|^3)
\end{aligned}
\tag{A11}
$$

Rearranging, we have Equation 18. □

A.4. Proof of Proposition 3

The full version of the planner's problem, Equation 20, is

$$
\begin{aligned}
\max_{\{c_{ni}^t, l_n^t, Q_i^t, G_i^t, \tau_n^t\}_{t\in\{1,2\}, n\in N, i\in\mathcal{I}^t}} \quad & W \equiv \sum_{n\in N}\mu_n\lambda_n\sum_{t=1,2}\beta_n^{t-1}\left[u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t)\right] \\
\text{s.t. } & (c_n^1, c_n^2, l_n^2) \text{ solves Equation 19 given } l_n^1 \\
& Q^t = \mu^T c^t + \widehat{X}^t(z^t)Q^t + G^t \\
& \widehat{\mu} l^1 = l^1(\widehat{L}^1 Q^1),\ \mu^T l^2 = \vec{1}^T\widehat{L}^2(z^t)Q^2 \\
& \vec{1} \equiv p^t = \left(I - \widehat{X}^t(z^t)\right)^{-1}\widehat{L}^t(z^t)\vec{1} \\
& \vec{1}^T G^1 + \frac{\vec{1}^T G^2}{1+r^1} + \mu^T\tau^1 + \frac{\mu^T\tau^2}{1+r^1} = 0
\end{aligned}
\tag{A12}
$$

Proof. To begin, we define κ_n^t to be n's marginal value of additional expenditure in period t, i.e. for all i, $u_{nc_i}^t = \kappa_n^t$ (recall prices are normalized to one). Therefore,

$$
\begin{aligned}
dW &= \sum_{n\in N}\lambda_n\mu_n\sum_{t=1,2}\beta_n^{t-1}\left(u_{nc}^t dc_n^t - v_n^{t\,\prime} dl_n^t + w_{nG}^t dG^t\right) \\
&= \sum_{n\in N}\lambda_n\mu_n\sum_{t=1,2}\beta_n^{t-1}\left[\kappa_n^t\left(\vec{1}^T dc_n^t - \frac{v_n^{t\,\prime}}{\kappa_n^t}dl_n^t\right) + w_{nG}^t dG^t\right]
\end{aligned}
\tag{A13}
$$

Next note that in the second period, free labor supply implies $v_n^{2\prime} = \kappa_n^2$. In the first, there may be some wedge Δ_n such that $v_n^{1\prime} = \kappa_n^1(1+\Delta_n)$; a positive wedge indicates that n works as if the wage was higher than it is, i.e. oversupplies labor; a negative wedge represents involuntary un(der)employment. In these terms, we have

$$dW = \sum_{n\in N} \lambda_n \kappa_n^1 \mu_n \Bigg[-\Delta_n dl_n^1 + \sum_{t=1,2} \frac{\kappa_n^t}{\kappa_n^1} \beta_n^{t-1} \left(\vec{1}^T dc_n^t - dl_n^t \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \frac{\beta_n w_{nG}^2}{\kappa_n^1} dG^2 \right) \Bigg] \tag{A14}$$

Next, define $\widetilde{\lambda}_n = \lambda_n \kappa_n^1$. Also note that $\frac{\kappa_n^t}{\kappa_n^1}\beta_n^{t-1} = 1$ for $t = 1$. For $t = 2$, we use the modified Euler equation:

$$\kappa_n^1 = \beta_n \frac{1+r^1}{1-\phi_n} \kappa_n^2 \tag{A15}$$

where ϕ_n is a borrowing wedge. $\phi_n \geqslant 0$ is positive when households behave as if interest rates are higher than in reality, i.e. consume more in the future than they would like; this corresponds to borrowing constraints. This gives us

$$dW = \sum_{n\in N} \widetilde{\lambda}_n \mu_n \Bigg[-\Delta_n dl_n^1 + \left(\vec{1}^T dc_n^1 - dl_n^1 \right) + \frac{1-\phi_n}{1+r^1} \left(\vec{1}^T dc_n^2 - dl_n^2 \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1-\phi_n}{1+r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \Bigg] \tag{A16}$$

Differentiating the household's lifetime budget constraint (at constant r^1):

$$\vec{1}^T dc_n^1 - dl_n^1 + \frac{\vec{1}^T dc_n^2 - dl_n^2}{1+r^1} = -d\tau_n^1 - \frac{d\tau_n^2}{1+r^1} \tag{A17}$$

Plugging this in, we have:

$$dW = \sum_{n\in N} \widetilde{\lambda}_n \mu_n \Bigg[-\Delta_n dl_n^1 + \phi_n \left(\vec{1}^T dc_n^1 - dl_n^1 \right) - (1-\phi_n) \left(d\tau_n^1 + \frac{d\tau_n^2}{1+r^1} \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1-\phi_n}{1+r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \Bigg] \tag{A18}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint binds:

$$\underline{s}_n^1 = l_n^1 - \tau_n^1 - \vec{1}^T c_n^1 \implies \vec{1}^T dc_n^1 - dl_n^1 = -d\tau_n^1 \tag{A19}$$

Defining the within-period willingness to pay for government expenditure $WTP_n^t = \frac{w_{nG}^t}{\kappa_n^t}$, we arrive at the final expression:

$$
\begin{aligned}
dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \Bigg[& - \Delta_n dl_n^1 - \left(d\tau_n^1 + (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) \\
& + \left(WTP_n^1 dG^1 + (1 - \phi_n) \frac{WTP_n^2}{1 + r^1} dG^2 \right) \Bigg]
\end{aligned} \tag{A20}
$$

□

A.5. Proof of Proposition 4

Proof. The proof of this result relies on material in Appendix B.9 on characterizing optimal fiscal policy; please consult this section and the results therein before proceeding with reading this proof.

We first prove the result for first-period transfers. At any optimum, we know that Equation A138 must hold for all policy variations $\tau_\varepsilon^1 \in \mathbb{R}^N$ that only vary first-period transfers, keeping other instruments fixed. Taking $\tau_\varepsilon^1 = e_n$, the nth basis vector, we see that:

$$
\left(\widetilde{\lambda}^T - \gamma \vec{1} \right)_n^T = \left(\widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C_{y^1}^1 R^1 \right)^{-1} C_{y^1}^1 \right)_n \tag{A21}
$$

Stacking these over n, we obtain:

$$
\left(\widetilde{\lambda} - \gamma \vec{1} \right)^T = \widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C_{y^1}^1 R^1 \right)^{-1} C_{y^1}^1 \tag{A22}
$$

Since $\{e_n\}$ is a basis and Equation A138 is linear, this equation fully encompasses the optimality condition of Proposition 18 with respect to first period transfers.

We can simplify this system of equations. First, see that:

$$
\begin{aligned}
R^1 (I - C_{y^1}^1 R^1)^{-1} C_{y^1}^1 &= \sum_{k=0}^{\infty} R^1 (C_{y^1}^1 R^1)^k C_{y^1}^1 \\
&= \sum_{k=1}^{\infty} R^1 C_{y^1}^1
\end{aligned} \tag{A23}
$$

Adding $\widetilde{\lambda}^T \hat{\Delta}$ to both sides of Equation A22, we therefore obtain:

$$
\begin{aligned}
\left(\widetilde{\lambda}(1 + \widehat{\Delta}) - \gamma \vec{1} \right)^T &= \widetilde{\lambda} \widehat{\Delta} \left(I + \sum_{k=1}^{\infty} R^1 C_{y^1}^1 \right) \\
&= \widetilde{\lambda}^T \widehat{\Delta} \left(I - R^1 C_{y^1}^1 \right)^{-1}
\end{aligned} \tag{A24}
$$

Which can be rewritten as:

$$
\left(\widetilde{\lambda}(1 + \widehat{\Delta}) - \gamma \vec{1} \right)^T \left(I - R^1 C_{y^1}^1 \right) = \widetilde{\lambda}^T \widehat{\Delta} \tag{A25}
$$

Now, express $R^1 C^1_{y^1} = R^1 \overline{C}^1_{y^1} \hat{m}$. Recognizing that all columns of the spending-to-income matrix $R^1 \overline{C}^1_{y^1}$ sum to one as total spending is equal to total factor income, and—by assumption—that $\widetilde{\lambda}_n(1+\Delta_n)$ is constant across all households n except for those for which the n^{th} row of $R^1 C^1_{y^1}$ is zero, (A25) can be rewritten as:

$$\left(\widetilde{\lambda}(1+\widehat{\Delta}) - \gamma\vec{1}\right)^T (I - \hat{m}) = \widetilde{\lambda}^T \widehat{\Delta} \tag{A26}$$

We therefore have all, for all n, that

$$\widetilde{\lambda}_n(1+\Delta_n) - \gamma = \frac{1}{1-m_n}\widetilde{\lambda}_n \quad \forall n \in N \tag{A27}$$

Which can be simply rearranged to yield the claimed expression:

$$\gamma = \widetilde{\lambda}_n \left(1 + \frac{m_n}{1+m_n}(-\Delta_n)\right) \quad \forall n \in N \tag{A28}$$

We prove the result for first-period government spending in an analogous way. To begin, consider Equation A138 for policy variations $G^1_\varepsilon \in \mathbb{R}^{\mathcal{I}^1}$ that only vary first period expenditure. Again considering each basis vector of $\mathbb{R}^{\mathcal{I}^1}$ and stacking we obtain:

$$0 = \widetilde{\lambda}^T WTP^1 - (\gamma\vec{1}^T + \widetilde{\lambda}^T\widehat{\Delta}R^1) - \widetilde{\lambda}^T\widehat{\Delta}R^1(I - C^1_{y^1}R^1)^{-1}C^1_{y^1}R^1 \tag{A29}$$

This can be rewritten as:

$$\widetilde{\lambda}^T WTP^1 - \gamma\vec{1}^T = \widetilde{\lambda}^T R^1 \widehat{\Delta}(I - C^1_{y^1}R^1)^{-1} \tag{A30}$$

From the assumption the the social gains from government expenditure equal $\tilde{v}$, we have that $\widetilde{\lambda}^T WTP^1 = \tilde{v}$. Moreover, by definition $\widetilde{\lambda\Delta} = \widetilde{\lambda}\widehat{\Delta}R$. Hence (A30) can be rewritten as

$$\tilde{v}\vec{1}^T - \gamma\vec{1}^T = \widetilde{\lambda\Delta}^T \left(I - C^1_{y^1}R^1\right)^{-1} \tag{A31}$$

Next, define $\widetilde{m}_i \equiv \left(m^T R^1\right)_i$ to be the rationing-weighted average MPC in the production of good i and let $\widehat{\widetilde{m}}$ be the corresponding matrix with $\widetilde{m}$ on the diagonal. Moreover, define $\widetilde{C}_{ji} \equiv \frac{\left(C^1_{y^1}R^1\right)_{ji}}{\widetilde{m}_i}$ to be the average direction of consumption of workers producing i, weighted by their MPC and marginal rationing in i's production.[40] Crucially, note that $\widetilde{C}\widehat{\widetilde{m}} = C^1_{y^1}R^1$ by construction and that $\vec{1}^T\widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T\widehat{\widetilde{m}}$:

$$\vec{1}^T\widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T C^1_{y^1}R^1 = m^T R^1 = \widehat{\widetilde{m}}^T \tag{A32}$$

[40]For any i with $\tilde{m}_i = 0$, define $\widetilde{C}_{ji}$ in any way satisfying $\sum_j \widetilde{C}_{ji} = 1$.

The first order condition for expenditures (A31) is therefore equivalent to:

$$(\tilde{v}-\gamma)\vec{1}^T\left(I - C^1_{y^1}R^1\right) = (\tilde{v}-\gamma)\vec{1}^T\left(I - \widehat{\tilde{m}}\right) = \widetilde{\lambda\Delta}^T \tag{A33}$$

But this holds iff and only if:

$$\gamma = \tilde{v} + \frac{1}{1-\widetilde{m}_i}(-\widetilde{\lambda\Delta}_i) \quad \forall i \in \mathcal{I}^1, \tag{A34}$$

completing the proof. □

A.6. Proof of Proposition 5

Proof. Under the proposition's assumptions, Equation 21 reduces to:

$$dW = \mu^T dl^1 - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1+r^1} \tag{A35}$$

Moreover, by Equation 22, we have that:

$$\widehat{\mu} dl^1 = R^1\left(I - C^1_{y^1}R^1\right)^{-1}\left(dG^1 - C^1_{y^1}\left(\widehat{\mu} d\tau^1 + \frac{\widehat{\mu} d\tau^2}{1+r^1}\right)\right) \tag{A36}$$

Combining these equations and rearranging:

$$\begin{aligned} dW &= \vec{1}^T R^1\left(I - C^1_{y^1}R^1\right)^{-1}\left(dG^1 - C^1_{y^1}\left(\widehat{\mu} d\tau^1 + \frac{\widehat{\mu} d\tau^2}{1+r^1}\right)\right) - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1+r^1} \\ &= \vec{1}^T\left(I - C^1_{y^1}R^1\right)^{-1} dG^1 + \vec{1}^T\left[\left(I - R^1 C^1_{y^1}\right)^{-1} R^1 C^1_{y^1} + I\right]\left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1}\right) \\ &= \vec{1}^T \underbrace{\left(I - C^1_{y^1}R^1\right)^{-1}}_{=dY^1/dG^1} dG^1 + \vec{1}^T \underbrace{\left(I - R^1 C^1_{y^1}\right)^{-1}}_{=dl^1/dy^1}\left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1}\right) \end{aligned} \tag{A37}$$

Finally, one may add terms proportional to $\frac{dY^1}{dG^2} = 0$. □

A.7. Proof of Corollary 1

Proof. We first show that, if the bias and homophily effects are zero for all output and transfer shocks relative to some baseline income incidence y^*, then either $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y^*}[m_{n'}]$. We then use this fact to obtain the conclusion of the proposition.

To start, fixing a single type $n \in N$, consider the bias and homophily terms corresponding to a transfer shock with direct incidence $\partial y^1 = \hat{e}_n$ (i.e. only transfering to n).

$$\begin{aligned} \text{bias}^{y^*}_{\partial y^1} &= \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{\text{next}}] - \mathbb{E}_{y^*}[m_{n'}]\right) = m_n\left(m_n^{\text{next}} - \mathbb{E}_{y^*}[m_{n'}]\right) \\ \text{homophily}^{y^*}_{\partial y^1} &= \mathbb{C}\text{ov}_{\partial y^1}[m_{n'}, m_{n'}^{\text{next}}] = 0 \end{aligned} \tag{A38}$$

The assumption that the bias term is zero then implies that either $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y*}[m_{n'}]$.

To apply this fact, recall the definition $m_n^{\text{next}} = m^T R^1 \overline{C}_{y^1}^1$, where $\overline{C}_{y^1}^1$ is the normalized matrix of spending directions, i.e. $C_{y_1}^1 = \overline{C}_{y^1}^1 \widehat{m}$. Our previous observation—that for all n, $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y*}[m_{n'}]$—then implies that $m^T R^1 C_{y^1}^1 = (\vec{m}^{\text{next}})^T \widehat{m} = \mathbb{E}_{y*}[m_{n'}] \cdot m^T$.

Applying this fact to the multipliers in Equation 25, we have

$$\begin{aligned}
\vec{1}^T \frac{dY^1}{dG^1} &= \vec{1}^T \left(I - C_{y^1}^1 R^1\right)^{-1} = \sum_{k=0}^{\infty} \vec{1}^T \left(C_{y^1}^1 R^1\right)^k \\
&= \vec{1}^T + \underbrace{\vec{1}^T C_{y^1}^1}_{=m^T} R^1 + \sum_{k=1}^{\infty} \underbrace{\vec{1}^T C_{y^1}^1}_{=m^T} \left(R^1 C_{y^1}^1\right)^k R^1 \\
&= \vec{1}^T + m^T R^1 + \sum_{k=1}^{\infty} \mathbb{E}_{y*}[m_n]^k m^T R^1 \\
&= \vec{1}^T + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m^T R^1 \\
&= \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T R^1
\end{aligned} \tag{A39}$$

Moreover, we have that:

$$\begin{aligned}
\vec{1}^T \frac{dl^1}{dy^1} &= \vec{1}^T \left(I - R^1 C_{y^1}^1\right)^{-1} \\
&= \vec{1}^T + \underbrace{\vec{1}^T R^1 C_{y^1}^1}_{=m^T} + \sum_{k=1}^{\infty} \underbrace{\vec{1}^T R^1 C_{y^1}^1}_{=m^T} \left(R^1 C_{y^1}^1\right)^k \\
&= \vec{1}^T + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m^T \\
&= \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T
\end{aligned} \tag{A40}$$

Plugging these into Equation 25, we obtain:

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1 + r^1}\right) \tag{A41}$$

Completing the proof. □

B. Additional Results and Extensions

Here we provide results on properties (including existence) of rationing equilibrium (B.1), derive the multiplier with interest rate effects (B.2), extend the baseline model to many periods (allowing for an infinite horizon) (B.3), allow for imperfect competition with fixed markups (B.4), study the structure of the multiplier in a more canonical flexible-wage equilibrium (B.5), provide a network reinterpretation of the multiplier at the zero lower bound (B.6), generalize our decomposition results to account also for supply shocks (B.7), analyze benchmark cases in which the network adjustments to the Keynesian multiplier are zero (B.8), provide first order conditions for optimal policy (B.9), and consider policy in the environment with imperfect competition (B.10).

B.1. Equilibrium Properties

In this Appendix, we ensure our analysis of the multiplier is well-posed and eliminate any nuisance terms that unnecessarily complicate the analysis. To this end, we first provide a no-substitution theorem that ensures prices are technologically determined – and thus independent of demand – and, second, prove the existence of a rationing equilibrium.

The following technical conditions on production technologies and household preferences are sufficient for the no-substitution theorem. Assumption 3 provides basic technical conditions on production and Assumption 4 imposes a simple positivity condition on demand such that there is demand for all goods.

Assumption 3. *For all i and z_i, production $F(X_i, L_i, z_i)$ is continuous, weakly increasing, strictly quasi-concave, and homogeneous of degree one in (X_i, L_i). Further, labor is essential in production, i.e. $F(X_i, 0, z_i) = 0$, and production is strictly increasing in labor. Finally, there exists some $\overline{p} \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i, L_i\}_{i\in\mathcal{I}^t}$ such that for all i, $F(X_i, L_i, z_i) \geqslant 1$ and $\overline{p}X_i + L_i \leqslant \overline{p}_i$.*[41]

Assumption 4. *For any $\varrho, y^1, \tau, \theta$: for each good i there is a household type n for which $c_{ni}^t > 0$.*

Under these two rather weak assumptions, we can show that:

Proposition 6. *Under Assumptions 3 and 4, for a given z^t, there exists a unique p^t consistent with rationing equilibrium, independent of demand.*

Proof. We follow closely the proof technique used in Acemoglu and Azar (2020). We will prove the result for an economy with arbitrary time horizon for maximum applicability. Fix a time period t vector of productivity parameters z. For each i, define the unit cost function:

$$\kappa_i(p) = \min_{F(X_i,L_i,z_i)\geqslant 1,\ X_i,L_i\geqslant 0} pX_i + L_i \tag{A42}$$

The minimum is well defined owing to Assumption 3, which states that F is strictly increasing in labor, CRS, and strictly quasiconcave.

[41] A sufficient but not necessary condition is that every good can be produced using only labor.

We now establish properties of the unit cost function on the domain $p \in \mathbb{R}_+^{\mathcal{I}}$. First, since labor is necessary for production, $\kappa_i(0) > 0$ for all i. Second, by the last part of Assumption 3, there exists $\overline{p}$ such that $\kappa_i(\overline{p}) \leqslant \overline{p}_i$ for all i. Finally, $\kappa_i(p)$ is weakly increasing in p by inspection. These three properties establish that $\kappa(p) \equiv (\kappa_1(p), ..., \kappa_{\mathcal{I}}(p))$ maps $\mathbb{O} \equiv \times_{i=1}^{\mathcal{I}} [0, \overline{p}_i] \to \mathbb{O}$ and is weakly increasing. Moreover, $\mathbb{O}$ is a complete lattice with respect to the following operators:

$$\begin{aligned} p \wedge q &= \big(min(p_1, q_1), ..., min(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \\ p \vee q &= \big(max(p_1, q_1), ..., max(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \end{aligned} \tag{A43}$$

By Tarksi's fixed point theorem, the set of fixed points $\{p \in \mathbb{R}_+^{\mathcal{I}} \mid \kappa(p) = p\}$ is therefore a complete lattice.

In order for p to be consistent with either our flexible-wage or rationing equilibrium, all operating firms must make zero profits. Assumption 4 implies that all firms operate in equilibrium, so $p = \kappa(p)$ is a necessary condition for any equilibrium. It therefore remains to show that κ has a unique fixed point. To this end, we first show that each κ_i is concave. For price vectors p and q and $\lambda \in (0, 1)$, we construct the price vector:

$$p^\lambda = \lambda p + (1 - \lambda) q \tag{A44}$$

By cost minimization,

$$\begin{aligned} \kappa_i(p) &\leqslant p X_i(p^\lambda) + L_i(p^\lambda) \\ \kappa_i(q) &\leqslant q X_i(p^\lambda) + L_i(p^\lambda) \end{aligned} \tag{A45}$$

It follows that:

$$\kappa_i(p^\lambda) = p^\lambda X_i(p^\lambda) + L_i(p^\lambda) \geqslant \lambda \kappa_i(p) + (1 - \lambda)\kappa_i(q) \tag{A46}$$

establishing that each κ_i is a concave function.

Toward a contradiction, suppose κ has more than one fixed point. Then since the set of fixed points is a complete lattice, there must exist distinct fixed points p^*, p^{**} with $p_i^* \leqslant p_i^{**}$ for all i. Now take λ to be given by the following:

$$\lambda = \min_{i \in \mathcal{I}} \frac{p_i^*}{p_i^{**}} \tag{A47}$$

Note that $\lambda \in (0, 1)$ since $p >> 0$ for all fixed points p, since $\kappa_i(0) > 0$ for all i and κ is weakly increasing. We have that $p_i^* \geqslant \lambda p_i^{**}$ for all $i \in \mathcal{I}$ with equality for at least one j by construction. For this j such that $p_j^* = \lambda p_j^{**}$, we then have

$$\begin{aligned} 0 &= \kappa_j(p^*) - p_j^* \\ &\geqslant \kappa_j(\lambda p^{**}) - \lambda p_j^{**} \\ &\geqslant (1 - \lambda)\kappa_j(0) + \lambda \kappa_j(p^{**}) - \lambda p_j^{**} \\ &= (1 - \lambda)\kappa_j(0) \\ &> 0 \end{aligned} \tag{A48}$$

where the first line follows from the zero profit condition, the second line follows from the fact that κ_i is weakly increasing and $\lambda \in (0,1)$, the third line follows from concavity of κ_i, the fourth line follows again from the zero profit condition, and the final line follows from positivity of costs. This is a contradiction. Hence, there must be a unique fixed point at all times t. This implies the stated result and also makes the no-substitution theorem applicable to Appendix B.3 where we extend the baseline model to allow for multiple time periods. $\square$

Proposition 6 establishes that, under Assumptions 3 and 4, a no-substitution theorem holds: given (z^1, z^2), there exist unique, positive prices $p^1(z^1), p^2(z^2) \in \mathbb{R}_+^{\mathcal{I}^t}$ consistent with equilibrium. This result allows us to reduce the number of endogenous price variables in considering comparative statics that keep z^1 and z^2 fixed, allowing us to keep track of just the real interest rate. Implicit in this no-substitution economy is the assumption that good prices respond instantaneously to changes in technology, which is irrelevant in the case of demand shocks.

Moreover, Proposition 6 also implies a simple form for aggregate input demands $X^t(p^t, Q^t)$ in equilibrium. In particular, for any technology z, we define the equilibrium unit input demands as:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i,L_i) \text{ s.t. } F(X_i,L_i,z_i) \geqslant 1} p(z)X_i + L_i \tag{A49}$$

Constant returns to scale imply that aggregate input and labor demands are simply a scaling of these unit input demands. Formally:

Corollary 2. *The aggregate input demand $X^t(p^t, Q^t)$ and labor demand $L^t(p^t, Q^t)$ vectors are given by:*

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{A50}$$

where $\widehat{X}(z^t)$ is the matrix with i^{th} column $\widehat{X}_i(z^t)$ and $\widehat{L}(z^t)$ is the diagonal matrix with i^{th} entry $\widehat{L}_i(z^t)$.

Proof. Fixing z, by Proposition 6, there exists a unique price vector $p(z)$ consistent with equilibrium. The unit input demands for any firm i at this price solve the following program:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i,L_i) \text{ s.t. } F(X_i,L_i,z_i) \geqslant 1} p(z)X_i + L_i \tag{A51}$$

CRS then implies that for a firm producing Q_i units in equilibrium,

$$X_i = Q_i\widehat{X}_i(z) \quad L_i = Q_i\widehat{L}_i(z) \tag{A52}$$

Stacking these equations over $\mathcal{I}^t$ gives

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{A53}$$

$\square$

Proposition 6 implies two additional, useful results. First, the Leontief-inverse matrix always exists. Second, one can use the Leontief-inverse to obtain a useful closed-form expression for the demand-independent prices. This is stated formally in the following corollary:

Corollary 3. *The Leontief-inverse matrix $(I - \hat{X}(z))^{-1}$ exists. Moreover, prices are given uniquely by the following expression:*

$$p(z) = (I - \hat{X}(z)^T)^{-1}\hat{L}(z)\vec{1} \tag{A54}$$

Proof. We first prove that the matrix $(I - \widehat{X}(z))$ is invertible. The zero-profit condition for all i implies that:

$$p(z)X_i + L_i = p_i(z)Q_i \tag{A55}$$

Normalizing by the quantity yields:

$$p(z)\widehat{X}_i(z) + \widehat{L}_i(z) = p_i(z) \tag{A56}$$

Stacking this equation yields the matrix equation:

$$\widehat{L}(z)\vec{1} + \widehat{X}^T(z)p(z) = p(z) \tag{A57}$$

This allows us to solve for the unit labor demands as the unique diagonal matrix such that:

$$\widehat{L}(z)\vec{1} = \left(I - \widehat{X}(z)^T\right)p(z) \tag{A58}$$

Iterating this equation $k \in \mathbb{N}$ times yields:

$$p(z) = \left(1 + \widehat{X}(z)^T + ... + \left(\widehat{X}(z)^T\right)^k\right) \widehat{L}(z)\vec{1} + \left(\widehat{X}(z)^T\right)^{k+1}p(z) \tag{A59}$$

Recall that $\widehat{X}(z)$ is non-negative, $\widehat{L}(z)\vec{1}$ is strictly positive because labor is essential, and $p(z)$ is positive. A necessary condition for $p(z)$ to exist is therefore that $\left(\widehat{X}(z)^T\right)^k \to 0$ as $k \to \infty$. This implies that $\widehat{X}(z)^T$ (and therefore also $\widehat{X}(z)$) has modulus strictly less than unity. It is immediate that the inverse $\left(I - \widehat{X}(z)\right)^{-1}$ exists. From this it follows that:

$$p(z) = \left(1 - \widehat{X}(z)^T\right)^{-1}\widehat{L}(z)\vec{1} \tag{A60}$$

completing the proof. □

Given the above simplifications, throughout the paper we will write $\widehat{X}^t, \widehat{L}^t$ for $\widehat{X}(z^t), \widehat{L}(z^t)$ when z^t is fixed. We write $\widehat{X}$ and $\widehat{L}$ for the block-diagonal matrices composed of $\widehat{X}^1$ and $\widehat{X}^2$, and $\widehat{L}^1$ and $\widehat{L}^2$ respectively.

Having simplified the structure of the model, we proceed to establish that the analysis of equilibrium is well posed by providing regularity conditions under which equilibria exist. To this end, we assume basic continuity properties of demand and that household consumption in the first period is bounded away from fully consuming first period income as income grows large.

Assumption 5. *The primitives satisfy the following properties:*

1. *The consumption and labor functions c_n^t and l_n^1 are continuous in r^1 and y^1.*
2. *For all $n, \varrho, \tau_n, \theta_n$, we have that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in y_n^1.*

3. *For any p, τ, θ: there exists $\overline{y} \in \mathbb{R}_+$ and $\overline{c} < 1$ such that for all $n \in N$, $r^1 \in [\underline{r}, \overline{r}]$, and $y_n^1 > \overline{y}$, we have that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$.*
4. *Interest rates have an upper and lower bound, i.e. $r^1(Q) \in [\underline{r}, \overline{r}]$ and r is differentiable.*

This assumption is extremely mild and satisfied by virtually all standard household problems of which we are aware.[42] With this additional structure we are now able to prove the existence of rationing equilibria for the economy under consideration.

Proposition 7. *Under assumptions 3, 4, and 5, there exists a rationing equilibrium.*

Proof. Fix all exogenous parameters. Note that by Proposition 6, prices p^1 and p^2 are pinned down by technology and so can be taken as given as well.

The outline of the proof is as follows. First, for any interest rate r^1, we will construct a function Ψ_{r^1} that maps vectors of first-period income to vectors of first-period income and show that any fixed point of this map corresponds to an equilibrium with constant r^1. Second, we extend this map to construct a second function Ψ that takes as inputs both a vector of incomes and an interest rate, and we show that any fixed point of this extended map corresponds to an equilibrium of the model. We then apply Brouwer's fixed point theorem to Ψ to show that such a fixed point exists.

First, by Assumption 5 we have the following two facts:

1. For any p^1, p^2, τ, θ: $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in y_n^1 for any n, $r^1 \in [\underline{r}, \overline{r}]$
2. For any p^1, p^2, τ, θ: there exists some $\overline{y} \in \mathbb{R}_+$ and some $\overline{c} < 1$ such that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$ for all n, $y_n^1 > \overline{y}$, $r^1 \in [\underline{r}, \overline{r}]$

Thus, given any vector of incomes y^1, total first period consumption spending C^1 is bounded above:

$$C^1 \leqslant \overline{cy} + \overline{c}\vec{1}' y^1 \tag{A61}$$

Thus, aggregate spending is bounded above by:

$$C^1 + G^1 \leqslant \overline{c}(\overline{y} + \vec{1}' y^1) + \max_{r \in [\underline{r}, \overline{r}]} p^1 G^1(p^1, p^2, r^1, \tau, \theta_G) \tag{A62}$$

where this maximum exists by continuity of $G^1(\cdot)$ in r^1 and compactness of $[\underline{r}, \overline{r}]$. Since $\overline{c} < 1$, it follows that there exists $\overline{Y}$ such that if $y^1 \in Y^1 \equiv \{y^1 \in \mathbb{R}_+^N \mid \vec{1}' y^1 \leqslant \overline{Y}\}$, then aggregate spending—and so, as all spending flows to wages, also the resulting aggregate income—is weakly less than $\overline{Y}$. Formally:

$$\forall r^1 \in [\underline{r}, \overline{r}], y^1 \in Y^1 : l^1 \left(\widehat{L}^1 (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \in Y^1 \tag{A63}$$

This observation allows us to define, for any $r^1 \in [\underline{r}, \overline{r}]$, a function $\Psi_{r^1} : Y^1 \to Y^1$ given by:

$$\Psi_{r^1}(y^1) = l^1 \left(\widehat{L}^1 (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \tag{A64}$$

[42] It is easy to see how Assumption 5 holds if households are utility maximizers whose utility functions satisfy various standard assumptions. Existence and continuity of the consumption and labor functions follow from continuity and quasiconcavity of utility, and from Berge's theorem. Satisfying the lifetime budget constraint follows from non-satiation. Consumption being asymptotically bounded away from first-period income follows from sufficiently decreasing marginal utility.

where recall ϱ denotes (p^1, p^2, r^1) and where the previous argument establishes that $\Psi_{r^1}(y^1)$ is indeed contained in Y^1. Moreover, continuity of $l^1(\cdot), C^1(\cdot)$ and $G^1(\cdot)$ establishes that Ψ_{r^1} is a continuous function.

Second, we define an extended function $\Psi : Y^1 \times [\underline{r}, \overline{r}] \to Y^1 \times [\underline{r}, \overline{r}]$ by setting:

$$\Psi(y^1, r^1) = \left(\Psi_{r^1}(y^1), r^1(Q)\right) \tag{A65}$$

where $Q = (Q^1, Q^2)$ is given by:

$$Q^t = (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G)\right) \tag{A66}$$

and where $r^1(\cdot)$ is the monetary policy function, which recall selects an interest rate in $[\underline{r}, \overline{r}]$.

Third, we now claim that Ψ has a fixed point (y^1, r^1). This follows from Brouwer's theorem: $Y^1 \times [\underline{r}, \overline{r}]$ is a compact, convex domain, and Ψ is continuous because $l^1(\cdot)$ and $r^1(\cdot)$ are continuous, $c_n^t(\varrho, y_n^1, \tau_n, \theta_n)$ is continuous in y_n^1 and r^1, and $G^t(\varrho, \tau, \theta_G)$ is continuous in r^1.

Finally, given a fixed point (y^1, r^1) of Ψ, we can construct a rationing equilibrium as follows: Let p^t be the no-substitution-theorem prices implied by z^t. Let c_n^t, l_n^2, and G^t be given by the relevant functions taking in prices p^t, real rate r^1, and incomes y^1. Let production in each period be:

$$Q^t = (I - \widehat{X}^t)^{-1}(G^t + C^t) \tag{A67}$$

The definition of the consumption, labor supply, and government spending function ensure that household and government budget constraints hold. The construction of Q^t ensures that each goods market clears. Because (y^1, r^1) is a fixed point, first period income is consistent with the rationing function and the first period labor market clears; also because (y^1, r^1) is a fixed point, the interest rate $r^1 = r^1(Q)$ is consistent with central bank policy. Finally, the second period labor market clears by Walras' law. □

As we have established conditions under which an equilibrium exists, our analysis of equilibria going forward will be well-posed. While the fixed-point theorems we use are familiar, we employ a somewhat different strategy to usual existence proofs in (i) leveraging the structure of no-substitution and (ii) clearing markets intertemporally and then constructing intratemporal market clearing from the resulting fixed point interest rate. This provides a common structure to both rationing equilibrium and flexible-wage equilibrium (see Appendix B.5) existence and may be useful to other authors proving equilibrium existence in economies with labor rationing.

B.2. The Output Multiplier with Interest Rate Effects

In Section 3, we assumed with Assumption 1 that the output multiplier was comprised solely of income multiplier effects, either due to interest rates not responding to output or households not responding to interest rates. For completeness, we provide below a version of the multiplier in Proposition 1 that takes these effects into account.

Proposition 8. *There exists a matrix M such that for any small shock to parameters $\partial x \in Span\{d\theta_G, d\theta, d\tau, dz\}$, there exists a selection from the equilibrium set such that the general equilibrium response in output is given by:*

$$dY = M\partial Q \tag{A68}$$

where ∂Q is the partial equilibrium change in production associated with ∂x stacked over time periods. Moreover, the matrix M is given by:

$$M = \left(I - D\left(I - \widehat{X}\right)^{-1}\right)^{-1} \tag{A69}$$

where:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^1} & \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^1} & \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^2} \end{bmatrix} \tag{A70}$$

Proof. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 7, we know that for each ω_n there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit ω. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at e: household budget balance, government budget balance or market clearing. But by Assumption 5, continuity of the fiscal rule, continuity of the interest rate rule and continuity of the rationing function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{A71}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions. Our differentiability assumptions allow us to express

$$\begin{aligned} dQ \ &= \widehat{X} dQ + \widehat{X}_z dz Q + C_{\hat{p}} \hat{p}_z dz + C_{r^1} dr^1 + C_{y^1} dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}} \hat{p}_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G \end{aligned} \tag{A72}$$

Plugging in for dr^1 and $dy^1 = l^1_{L^1} \widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X} dQ + C_{y^1} l^1_{L^1} \widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1}) r^1_Q dQ + \partial Q \tag{A73}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}}) \hat{p}_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1 + (C_\tau + G_\tau) d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. Recognizing that $dY = (I - \widehat{X}) dQ$ and subsituting completes the proof.

□

To understand the form of M, see that D is the mapping from changes in production to changes in both government and private consumption demand. Moreover, see that these changes in demand stem from two sources: direct changes in labor income affecting consumption and changes in interest rates affecting both consumption and government expenditure. In the absence of input-output structure, the multiplier would simply be given by infinite iteration of D in analogy to the canonical Keynesian multiplier: output goes up by ∂Q, this induces a change in demand of $D\partial Q$, which induces a change in demand of $D^2\partial Q$ and so on, yielding the form $(I-D)^{-1}\partial Q$. The presence of input-output linkages changes this in two ways. First, whenever output increases there is a direct effect on intermediate goods demand of $\widehat{X}dQ$, which we have to account for in computing the total change in production from any shock. Second, as we are ultimately interested in changes in final output, not merely production, we have to remove total intermediate goods production. The combination of these two effects results in the Leontief inverse matrix $(I-\widehat{X})^{-1}$ post-multiplying D in the multiplier expression.

B.3. Multiple Time Periods

Consider the benchmark model from Section 2 but instead suppose that $t \in \mathbb{T} = \{1, ..., T\}$, where $T \in \mathbb{N} \cup \{\infty\}$. That is, in each t, firms use a vector of intermediates X_i^t, labor L_i^t and a CRS production technology $F(X_i^t, L_i^t, z_i^t)$. The households have consumption c_n^t and labor supply l_n^t functions that satisfy the dynamic budget constraint:

$$\sum_{t\in\mathbb{T}} \frac{l_n^t}{\prod_{i\leqslant t} 1+r^i} = \sum_{t\in\mathbb{T}} \frac{p^t c_n^t + \tau_n^t}{\prod_{i\leqslant t} 1+r^i} \tag{A74}$$

The government chooses a sequence of lump-sum taxes and spending $\{\{\tau_n^t\}_{n\in N}, \{G_i^t\}_{i\in\mathcal{I}}\}_{t\in\mathbb{T}}$ subject to its lifetime budget constraint:

$$\sum_{n\in N} \mu_n \Big(\sum_{t\in\mathbb{N}} \frac{1}{\prod_{i\leqslant t} 1+r^i} \tau_n^t \Big) = \sum_{t\in\mathbb{N}} \frac{1}{\prod_{i\leqslant t} 1+r^i} p^t G^t \tag{A75}$$

The key difference in defining equilibrium here is the need to specify a rule that decides in which periods we have labor rationing. To this end define a set $\mathcal{T}(\omega) \subseteq \mathbb{T}$ which specifies time indices for which the economy is in a state of labor rationing, where ω is a vector of all exogenous parameters of the model.[43] In periods with rationing $t \in \mathcal{T}(\omega)$, instead of labor market clearing, we have that $l_n^t = l_n^t((L_i^t)_{i\in\mathcal{I}^t})$. An equilibrium of the model is then given by:

Definition 2. *(Dynamic rationing equilibrium) Given parameters ω, a dynamic rationing equilibrium is a set of agent- and market-level variables $\{s_n^t, \{c_{ni}^t\}_{i\in\mathcal{I}^t}, l_n^t\}_{n\in N, t\in\mathbb{T}}$ and $\{r_t, \{p_i^t, \{X_{ij}^t\}_{j\in\mathcal{I}^t}, L_i^t, C_i^t, G_i^t\}_{i\in\mathcal{I}^t}\}_{t\in\mathbb{T}}$ that satisfy the following conditions. (1) Each household n consumes according to its consumption function $c_n^t(\cdot)$ in all periods and supplies labor according to $l_n^t(\cdot)$ in all non-rationing periods i.e. $t \in \mathbb{T}/\mathcal{T}(\omega)$. (2) Firms choose (X_i^t, L_i^t) to*

[43]For example, $\mathcal{T}$ can represent the set of periods in which the effective zero lower bound on real interest rates binds. Insofar as ω is sufficient to determine whether the zero lower bound binds, it is sufficient for it to determine $\mathcal{T}$.

maximize profits for all $t \in \mathbb{T}$ (3) The market for all goods clears for all $t \in \mathbb{T}$ (4) The labor market clears in periods $t \in \mathbb{T}/\mathcal{T}(\omega)$ and is determined by rationing in all periods in periods $t \in \mathcal{T}(\omega)$, i.e. $l_n^t = l_n^t\big((L_i^t)_{i\in\mathcal{I}^t}\big)$ (5) The government spends according to its expenditure function $G^t(\cdot)$.

For our dynamic equilibrium, we can again achieve an analogous Keynesian cross representation to our two period model, as the no-substitution theorem continues to hold. The dynamic fixed point equation for production is given by:

$$Q^t = \widehat{X}^t Q^t + G^t(\{r^t\}_{t\in\mathbb{T}}) + C^t(\{r^t, Q^t\}_{t\in\mathbb{T}}) \tag{A76}$$

Taking a first-order approximation following a partial equilibrium shock ∂Q for both rationing and flexible periods yields:

$$dQ^t = \widehat{X}^t dQ^t + \sum_{\tau\geqslant 1}\Big[\big(G^t_{r^\tau} + C^t_{r^\tau}\big)dr^\tau\Big] + \sum_{\tau\in\mathcal{T}(\omega)}\Big[C^t_{y^\tau} l^\tau_{L^\tau}\widehat{L}^\tau dQ^\tau\Big] + \partial Q^t \tag{A77}$$

Stacking these relations yields the Keynesian cross representation:

$$dQ = \widehat{X} dQ + (G_r + C_r)dr + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A78}$$

where $J_{\mathcal{T}(\omega)}$ is a diagonal matrix with ones on the diagonal

Interestingly, via an appropriate relabelling, there is an heuristic isomorphism between the 2-period model and the T-period model whenever $\mathcal{T}(\omega) = \{t\}_{t=1}^{T_1}$, *i.e.* there is rationing for the first T_1 periods and non-rationing for the subsequent $T_2 = T - T_1$ periods. That is, In the T-period model, the rationing spell maps to the rationing period in the 2-period model. To this end, the formula in Proposition 8 corresponds to a dynamic generalization of the Miyazawa special case.

Proposition 9. *(Dynamic multipliers at the zero lower bound) Suppose that $r^t = \bar{r}^t$ for all $t \in T$. Then the general equilibrium effect on output dY of a partial equilibrium shock ∂Q is generically given by*

$$dY^{\mathcal{T}} = \left(I - C_y^{\mathcal{T}} l_L^{\mathcal{T}} \widehat{L}^{\mathcal{T}}\left(I - \widehat{X}^{\mathcal{T}}\right)^{-1}\right)^{-1} \partial Q^{\mathcal{T}} \tag{A79}$$

where $dY^{\mathcal{T}}$ and $dQ^{\mathcal{T}}$ are $\mathcal{T}\times\mathcal{I}$-length vectors, $\widehat{L}^{\mathcal{T}}$ and $\widehat{X}^{\mathcal{T}}$ are diagonal matrices with entries corresponding to each rationing periods, and where $C_y^{\mathcal{T}}$ is the $(\mathcal{T}\times\mathcal{I})\times(\mathcal{T}\times N)$ matrix of intratemporal marginal propensities to consume, which maps changes in the household income distribution during rationing periods to changes in the consumption of each good during rationing periods.

Proof. The dynamic fixed point equations for market clearing are given in matrix form as:

$$Q^t = \widehat{X}^t Q^t + G^t(\{r^t\}_{t\in\mathbb{T}}) + C^t(\{r^t, Q^t\}_{t\in\mathbb{T}}) \tag{A80}$$

Taking a first-order approximation following a partial equilibrium shock ∂Q for both rationing and flexible periods yields:

$$dQ^t = \widehat{X}^t dQ^t + \sum_{\tau \geqslant 1} \left[\left(G^t_{r^\tau} + C^t_{r^\tau} \right) dr^\tau \right] + \sum_{\tau \in \mathcal{T}(\omega)} \left[C^t_{y^\tau} l^\tau_{L^\tau} \widehat{L}^\tau dQ^\tau \right] + \partial Q^t \tag{A81}$$

Stacking these relations yields the Keynesian cross representation:

$$dQ = \widehat{X} dQ + (G_r + C_r) dr + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A82}$$

where $J_{\mathcal{T}(\omega)}$ is a diagonal matrix with ones on the diagonal. Imposing $r^t = \bar{r}^t$ for all $t \in T$ simplifies this to:

$$dQ = \widehat{X} dQ + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A83}$$

Inverting this system to solve for the total change in production and solving for output:

$$dY = \left(I - C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} \left(I - \widehat{X} \right)^{-1} \right) \partial Q \tag{A84}$$

Applying the selection matrix $J_{\mathcal{T}(\omega)}$ and taking the first $\mathcal{T} \times \mathcal{I}$ rows:

$$dY^{\mathcal{T}} = \left(I - C^{\mathcal{T}}_y l^{\mathcal{T}}_L \widehat{L}^{\mathcal{T}} \left(I - \widehat{X}^{\mathcal{T}} \right)^{-1} \right)^{-1} \partial Q^{\mathcal{T}} \tag{A85}$$

Which is the required expression. □

However, there is a subtle difference in the intuition behind the results in the two cases. In the T-period case, the shocks in each rationing period can influence the level of output in all other periods. As a result, it is no longer sufficient to consider the directed MPC of households, but rather the directed *intertemporal* MPC of households that represents marginal changes in consumption across goods and time. Indeed, if we set the response of the rationing function, the unit labor demands and the input-output matrix to the identity, we recover a $\mathcal{T}$-period version of the multiplier formula provided by Auclert *et al.* (2018):

Corollary 4 (Intertemporal Keynesian Cross)**.** *In the environment of Proposition 9, if the rationing matrix and the input output matrix compose to the identity matrix, i.e.*

$$I = l^{\mathcal{T}}_L \widehat{L}^{\mathcal{T}} \left(I - \widehat{X}^{\mathcal{T}} \right)^{-1} \tag{A86}$$

then the general equilibrium effect on output $dY^{\mathcal{T}}$ in response to a partial equilibrium shocks $\partial Q^{\mathcal{T}}$ is given by:

$$dY^{\mathcal{T}} = \left(I - C^{\mathcal{T}}_y \right)^{-1} \partial Q^{\mathcal{T}} \tag{A87}$$

Proof. Simply imposing the given condition on Equation A79 yields the stated result. □

B.4. Imperfect Competition

In this section we show how to incorporate imperfect competition in the form of fixed markups on marginal costs. We now return to the standard two period model $T = 2$ under rationing equilibrium. However, instead of each sector being populated by a continuum of perfectly competitive firms, we now suppose that for all $i \in \mathcal{I}^t$ there is a single monopolist producing each good, charging a fixed markup of m_i^t over their marginal cost.[44] Of course, firms now have the capability of making profits π_i^t and we must distribute these profits to households in equilibrium. Despite this, we argue that a no substitution theorem still holds and we can obtain analogous multiplier formulae once we augment labor income rationing with profit rationing. To do this, we have to slightly modify Assumption 3:

Assumption 6. *For each t there exists some $\overline{p}^t \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i^t, L_i^t\}_{i \in \mathcal{I}^t}$ such that for all i, $F(X_i^t, L_i^t, z_i^t) \geqslant 1$ and $(1 + m_i^t)(\overline{p}^t X_i^t + L_i^t) \leqslant \overline{p}_i^t$*

Under this modified assumption, we can state and prove the modified no-substitution theorem with markups:

Proposition 10. *Under Assumptions 6 and 4, for a given z^t and m^t, there exists a unique p^t consistent with both flexible-wage and rationing equilibrium, independent of demand.*

Proof. We modify the proof of proposition 6 to accommodate markups. Each firm now sets a price $p_i = (1 + m_i^t)\kappa_i(p)$, where κ_i is i's unit cost function. That is, i prices goods as though it were a competitive firm with production function $\frac{1}{1+m_i^t} F(X_i^t, L_i^t, z_i^t)$. Consider now a modified economy without markups and production functions given by the previously-stated markup-adjusted production functions. Assumption 6 implies that Assumption 3 holds in this modified economy. The result then follows by direct application of Proposition 6. □

Having now established that the no substitution theorem continues to hold, we now proceed to establish our multiplier formulae in this setting. As previously mentioned, the key difference here is the need to apportion firm profits to households. To this end, suppose that profits from each firm are distributed to households according to an exogenous profit rationing function $\Pi^t : \mathbb{R}^{\mathcal{I}} \to \mathbb{R}^N$ satisfying $\sum_{i \in \mathcal{I}} \pi_i^t = \sum_{n \in N} \Pi^t(\pi^t)_n$ for all $\pi^t \in \mathbb{R}^{\mathcal{I}}$. We let $d_n^t = \Pi^t(\pi^t)_n$ represent household n's total dividend income in period t.

As a result of profit distribution, household income is now comprised of rationed first-period labor income, chosen second-period labor income, and distributed (not chosen) dividend income in both periods. We therefore allow household consumption and labor supply functions to depend on d_n^t directly.

We can now state a profit-inclusive Keynesian cross. Note that the only difference to Proposition 8 comes from the need to account for changes in profits, how these are distributed to households as dividends and their directed MPCs out of dividends.[45]

[44] Note that this generalizes the more standard model in which each sector is composed of many differentiated firms, with each firm and household having the same CES aggregator for its demand from the firms making up each sector.

[45] For the sake of generality, we distinguish between aggregate MPC out of dividend and labor income, i.e. $C_d^t \neq C_y^t$. Of course, for utility-maximizing households, these will be the same provided the income arrives in the same period.

Proposition 11. *For any small shock to parameters there exist a pair of rationing equilibria production Q and $Q+dQ$ before and after the shock. If the shock induces a partial equilibrium change in production ∂Q, the general equilibrium change dQ is given to first order by:*

$$dQ = \widehat{X}dQ + (C_r + G_r)r_Q dQ + C_y l^1_{L^1}\widehat{L}^1 dQ^1 + C_\pi \widehat{\Pi} dQ + \partial Q \tag{A88}$$

where here C_π is the matrix of household directed MPCs out of profit income, $\widehat{\Pi}$ is the block diagonal matrix composed of $\widehat{\Pi}^1$ and $\widehat{\Pi}^2$, and where $\widehat{\Pi}^t$ is the diagonal matrix with i^{th} entry $m^t_i p^t_i$, and all quantities are evaluated at the initial equilibrium.

Proof. This proof simply modifies the proof of Proposition 8. It is stated in full for clarity. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 7, we know that for each ω_n there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit ω. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at e: household budget balance, government budget balance or market clearing. But by Assumption 5, continuity of the fiscal rule, continuity of the interest rate rule, continuity of the rationing function and continuity of the profit allocation function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{A89}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions:

$$\begin{aligned} dQ \;&= \widehat{X}dQ + \widehat{X}_z dz Q + C_{\hat{p}}\hat{p}_z dz + C_{r^1} dr^1 + C_{y^1} dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}}\hat{p}_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G + C_\pi \widehat{\Pi} dQ \end{aligned} \tag{A90}$$

Plugging in for dr^1 and $dy^1 = l^1_{L^1}\widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X}dQ + C_{y^1} l^1_{L^1}\widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1}) r^1_Q dQ + C_\pi \widehat{\Pi} dQ + \partial Q \tag{A91}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}})\hat{p}_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1}\widehat{L}^1_z dz Q^1 + (C_\tau + G_\tau) d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. □

B.5. Flexible-Wage Equilibrium

In this Appendix we consider a more standard flexible-wage equilibrium concept. In this context, we derive the multiplier and contrast it to the multiplier obtained in rationing equilibrium.

The notion of flexible-wage equilibrium is standard. The main difference relative to rationing equilibrium is that households now choose their labor supply in the first period. Household behavior can therefore be denoted by Marshallian consumption and labor supply functions $c_n^t(\varrho, \tau_n, \theta_n)$ and $l_n^t(\varrho, \tau_n, \theta_n)$. Firm optimality (Equation 1), household budget balance evaluated at their consumption demand and labor supply functions (Equation 2), and government budget balance (Equation 3) continue to hold. Now the first period labor market must clear in the standard fashion, so that Equation 5 is strengthened to:

$$F(X_i^t, L_i^t, z_i^t) = D_i^t \equiv \sum_{n \in N} \mu_n c_{ni}^t + \sum_{j \in \mathcal{I}^t} X_{ji}^t + G_i^t, \quad \sum_{i \in \mathcal{I}^t} L_i^t = \sum_{n \in N} \mu_n l_n^t \quad \forall i \in I^t, t \in \{1, 2\} \tag{A92}$$

We therefore define a flexible-wage equilibrium as:

Definition 3. *A flexible-wage equilibrium is a set of first and second period, agent- and market-level variables* $\{s_n^1, \{c_{ni}^t, l_{ni}^t\}_{t \in \{1,2\}, i \in \mathcal{I}^t}\}_{n \in N}$ *and* $\{r^t, p_i^t, \{X_{ij}^t\}_{j \in \mathcal{I}^t}, L_i^t, C_i^t, G_i^t\}_{t \in \{1,2\}, i \in \mathcal{I}^t}$ *that satisfy conditions (1), (2), (3), and (A92) given initial conditions.*

This flexible-wage equilibrium provides a baseline specification against which we will compare the rationing equilibrium results. Note that in flexible-wage equilibrium the real interest rate adjusts flexibly to clear the labor market; it is not controlled by a central bank. This owes to the fact that while the central bank could set the nominal rate, prices would adjust to maintain the real rate.

The no-substitution theorem used in the analysis of rationing equilibrium directly carries over to the environment with a flexible-price equilibrium.

Proposition 12. *Under Assumptions 3 and 4, for a given* z^t*, there exists a unique* p^t *consistent with flexible-wage equilibrium, independent of demand.*

Proof. The proof follows exactly that of Proposition 6. □

Moreover, it can be established that a flexible-wage equilibrium exists under some mild technical conditions. In particular, we need to make some continuity and boundedness assumptions on consumption and labor supply:

Assumption 7. *The consumption and labor functions* c_n^t *and* l_n^t *are continuous in* r^1*. Moreover, for all n,* $\lim_{r^1 \to -1} \sum_{i \in \mathcal{I}^1} c_{ni}^1(\varrho, \tau, \theta) \to \infty$*,* $\lim_{r^1 \to -1} l_n^1(\varrho, \tau, \theta)$ *is bounded,* $\lim_{r^1 \to \infty} \sum_{i \in \mathcal{I}^1} c_{ni}^1(\varrho, \tau, \theta)$ *is bounded, and* $\lim_{r^1 \to \infty} l_n^1(\varrho, \tau, \theta) \to \infty$*.*

With this additional structure we are now able to prove existence of flexible-wage equilibria for the economy under consideration.

Proposition 13. *Under Assumptions 3, 4 and 7, there exists a flexible wage equilibrium.*

Proof. We prove the existence of an equilibrium by defining a fixed point map for $1 + r^1$, the gross real interest rate, such that at any fixed point the savings market clears. Given such an interest rate, we then explicitly construct an equilibrium.

Fix all exogenous parameters. Recalling that technology pins down prices and labor and input usage, we use the notation $p = p(z)$, $\widehat{X} = \widehat{X}(z)$, and $\widehat{L}^1 = \widehat{L}^1(z)$. To ensure the object over which we will construct the fixed point map lies in a compact set, we define the following transformation:

$$\tilde{r}^1(1+r^1) = \frac{1+r^1}{2+r^1} \tag{A93}$$

where $\tilde{r}(0) = 0$, $\tilde{r}(\infty) = 1$, $\tilde{r}$ is continuous and invertible. We now define a correspondence $\Phi : [0,1] \rightrightarrows [0,1]$ by

$$\Phi(\tilde{r}^1) = \begin{cases} [0,1], & p^1(C^1(\tilde{r}^1) + G^1) = l^1(\tilde{r}^1) \\ \{1\}, & p^1(C^1(\tilde{r}^1) + G^1(\tilde{r}^1)) > l^1(\tilde{r}^1) \text{ or } \tilde{r} = 0 \\ \{0\}, & p^1(C^1(\tilde{r}^1) + G^1(\tilde{r}^1)) < l^1(\tilde{r}^1) \text{ or } \tilde{r} = 1 \end{cases} \tag{A94}$$

where here $C^1(\tilde{r}^1)$ is shorthand for $C^1(p^1, p^2, r^1, \theta, \tau)$, with the same conventtion for $l^1(\tilde{r}^1)$ and $G^1(\tilde{r}^1)$. Notice that Φ is non-empty-valued and convex-valued. In order to apply Kakutani's theorem, it suffices to show that Φ is UHC. To show that Φ UHC, it suffices to show that for any selection $\phi \in \Phi$:

$$\lim_{\tilde{r}^1 \to 0} \phi(\tilde{r}^1) = 1 \quad \lim_{\tilde{r}^1 \to 1} \phi(\tilde{r}^1) = 0 \tag{A95}$$

To this end, by Assumption 7, see that as $\tilde{r}^1 \to 0$, $p^1 c_n^1 \to \infty$ while l_n^1 is finite for all types $n \in N$; meanwhile, government expenditures are (always) weakly positive. Thus, as $\tilde{r}^1 \to 0$, it must indeed be that $p^1(C^1 + G^1) > l^1$. Now suppose that $\tilde{r}^1 \to 1$. Again, by Assumption 7, it must be that $p^1 c_n^1 \to 0$ while $l_n^1 \to \infty$ and so $p^1(C^1 + G^1) < l^1$; here we use that first-period government expenditures are bound by the government budget constraint. We have therefore established that Φ is UHC. Thus, Kakutani's fixed point theorem implies that Φ has a fixed point. That is there exists $1 + r^1 \in [0, \infty]$ such that:

$$p^1\big(C^1(\varrho, \theta, \tau) + G^1(\varrho, \theta, \tau)\big) = l^1(\varrho, \theta, \tau) \tag{A96}$$

Also note that by construction of Ψ, the resulting fixed point $1 + r^1$ is finite and strictly positive.

Using this r^1, we will now construct a flexible-price equilibrium, i.e. a set of first and second period, agent- and market-level variables:

$$\{s_n^1, \{c_{ni}^t, l_{ni}^t\}_{t\in\{1,2\}, i\in\mathcal{I}}\}_{n\in N} \quad \text{and} \quad \{r^1, p_i^t, \{X_{ij}^t\}_{j\in\mathcal{I}}, L_i^t, C_i^t\}_{t\in\{1,2\}, i\in\mathcal{I}} \tag{A97}$$

satisfying the conditions of Definition 3. We set within-period prices $p^t = p^t(z^t)$. For all t, n, let c_n^t, l_n^t, and G^t be given by the household consumption and labor and government expenditure functions at real interest rate r^1. Let firms produce quantities $Q^t = (1 - \widehat{X}^t)^{-1}(C^t + G^t)$, demand inputs $X^t = \widehat{X}^t Q^t$, and demand labor $L^t = \widehat{L}^t Q^t$.

We now verify that the equilibrium conditions hold: household and government budget constraints follow by assumption on the consumption, labor, and expenditure functions. Firm optimization holds since firms make zero profits at the no-substitution theorem prices, so long as they demand inputs and labor optimally, according to $\widehat{X}^t$ and $\widehat{L}^t$. $Q^t = (1 - \widehat{X}^t)^{-1}(C^t + G^t)$ and $X^t = \widehat{X}^t Q^t$ imply the goods market clears. Labor supplied equals $p^t(C^t + G^t)$, by—for $t = 1$—the selection of the interest rate, and by—for $t = 2$—combining the household and government budget constraints with this condition at $t = 1$; labor demanded equals $\vec{1}^T \widehat{L}^t(1-\widehat{X}^t)^{-1}(C^t+G^t)$. Using firms' zero profit condition to substitute for $\widehat{L}^t$, labor demand can be rewritten as $(p^t)^T(1-\widehat{X}^t)(1-\widehat{X}^t)^{-1}(C^t+G^t)$, so the labor market clears. This completes the proof that a flexible-price equilibrium exists. □

We now obtain a representation of the partial equilibrium effect on demand of any shock to primitives. We begin by parameterizing aggregate demand. Recognizing that each household's decisions depend only on real quantities, we can represent type $n \in N$'s Marshallian demand for good $j \in \mathcal{I}^t$ at time $t \in \{1,2\}$ as $c^t_{nj}(\varrho, \tau_n, \theta_n)$, where $\varrho = (p^1, p^2, r)$, and $\tau_n = (\tau^1_n, \tau^2_n)$. Aggregate consumption demand C^t_j is then given by:

$$C^t_j(\varrho, \tau, \theta) = \sum_{n \in N} \mu_n \; c^t_{nj}(\varrho, \tau_n, \theta_n) \tag{A98}$$

where $\theta = (\theta_1, ..., \theta_N)$ and so forth. We define aggregate labor supply $L^t(y^1\varrho, \tau, \theta)$ and government expenditure analogously.

To find the partial equilibrium effect of each type of shock, we totally differentiate the goods market clearing condition:

$$Q^t = \widehat{X}^t Q^t + C^t + G^t \tag{A99}$$

We then collect the terms corresponding to changes in demand for goods before accounting for the way that direct changes in Q^t cause higher-order, "multiplier" effects. Doing so yields the following partial equilibrium effect of each shock:

Proposition 14. *The following shocks have partial equilibrium effects on aggregate demand given by:*

1. *A change in government preferences θ_G by $d\theta_G$:*

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G) d\theta_G \tag{A100}$$

2. *A change in household preferences θ by $d\theta$:*

$$\partial Q = C_\theta(\varrho, \tau, \theta) d\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\theta d\theta \tag{A101}$$

3. *A change in taxes or transfers by $d\tau$:*

$$\partial Q = C_\tau(\varrho, \tau, \theta) d\tau + G_\tau(\varrho, \tau, \theta_G) d\tau - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\tau d\tau \tag{A102}$$

4. *A change in productivity z by dz:*

$$\partial Q = \left(C_p + G_p - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_p\right) p_z dz + \left(\widehat{X}_z + (C_{r^1} + G_{r^1})(L_{r^1})^{-1} \widehat{L}_z\right) dz Q \tag{A103}$$

Proof. We consider each shock case by case. For each, we totally differentiate the goods market clearing condition and group all terms that have no dependence on resulting changes in equilibrium output. To this end, recall that the goods market clearing condition is given by:

$$Q = \widehat{X}Q + G + C \tag{A104}$$

In the flexible-wage case, total differentiation of this system of equations yields:

$$dQ = \widehat{X}dQ + \widehat{X}_z dzQ + C_p p_z dz + C_{r^1}dr^1 + C_\tau d\tau + C_\theta d\theta + G_p p_z dz + G_{r^1}dr^1 + G_\tau d\tau + G_{\theta_G}d\theta_G \tag{A105}$$

Similarly, we can expand the labor market clearing conditions to write

$$\widehat{L}dQ + \widehat{L}_z dzQ = L_p p_z dz + L_{r^1}dr^1 + L_\tau d\tau + L_\theta d\theta$$

Substituting for dr^1, we obtain

$$\begin{aligned} dQ =& \widehat{X}dQ + \widehat{X}_z dzQ + C_p p_z dz + C_\tau d\tau + C_\theta d\theta + G_p p_z dz + G_\tau d\tau + G_{\theta_G}d\theta_G \\ & (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\left(\widehat{L}dQ + \widehat{L}_z dzQ - L_p p_z dz - L_\tau d\tau - L_\theta d\theta\right) \end{aligned} \tag{A106}$$

We now consider the partial equilibrium effect of each type of shock by zeroing all general equilibrium effects through changes in output and by zeroing all other shocks:

1. A change in government preferences θ_G by $d\theta_G$:

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G)d\theta_G \tag{A107}$$

2. A change in household preferences θ by $d\theta$:

$$\partial Q = C_\theta(\varrho, \tau, \theta)d\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_\theta d\theta \tag{A108}$$

3. A change in taxes or transfers by $d\tau$:

$$\partial Q = C_\tau(\varrho, \tau, \theta)d\tau + G_\tau(\varrho, \tau, \theta_G)d\tau - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_\tau d\tau \tag{A109}$$

4. A change in productivity z by dz:

$$\partial Q = \left(C_p + G_p - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_p\right)p_z dz + \left(\widehat{X}_z + (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\widehat{L}_z\right)dzQ \tag{A110}$$

□

Having understood how primitive shocks map into partial equilibrium changes in demand, we now explore how these shocks map into changes in output in general equilibrium. To do this, we examine the impact of a small shocks or, equivalently, the impact of large shocks to first order. We represent the general equilibrium mapping of any of these shocks by a matrix that we label the multiplier. Our strategy is simply to totally differentiate the market clearing conditions in matrix form.

Proposition 15. *For any small shock to parameters, there exist a pair of flexible-wage equilibria with production $Q = (Q^1, Q^2)$ and $Q + dQ$ before and after the shock. Assume*

$L^t_{r^1} \neq 0$. *Then if the shock induces a partial equilibrium change in production* ∂Q*, the general equilibrium change* dQ *is given to first order by:*

$$dQ = \widehat{X}dQ + \begin{bmatrix} C^1_{r^1} + G^1_{r^1} & 0 \\ 0 & C^2_{r^1} + G^2_{r^1} \end{bmatrix} \begin{bmatrix} \left(L^1_{r^1}\right)^{-1} & 0 \\ 0 & \left(L^2_{r^1}\right)^{-1} \end{bmatrix} \widehat{L}dQ + \partial Q \tag{A111}$$

where all quantities above are evaluated at the initial equilibrium. Moreover, the impact on ouput is generically given by:

$$dY = \left(I - \begin{bmatrix} C^1_{r^1} + G^1_{r^1} & 0 \\ 0 & C^2_{r^1} + G^2_{r^1} \end{bmatrix} \begin{bmatrix} \left(L^1_{r^1}\right)^{-1} & 0 \\ 0 & \left(L^2_{r^1}\right)^{-1} \end{bmatrix} \widehat{L}\left(I - \widehat{X}\right)^{-1}\right)^{-1} \partial Q \tag{A112}$$

Proof. The existence of two nearby equilibria is a consequence of UHC of the equilibrium set in the parameters. We formally show that the equilibrium set is UHC. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 7, we know that for each ω_n there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit ω. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at e: household budget balance, government budget balance or market clearing. But by Assumption 7, and continuity of the fiscal rule, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

We can relate these two nearby equilibria by totally differentiating the goods market clearing conditions. Stacking the vectors that represent the two periods, we have:

$$\begin{aligned} dQ =& \widehat{X}dQ + \widehat{X}_z dzQ + C_p p_z dz + C_{r^1}dr^1 + C_\tau d\tau + C_\theta d\theta \\ &+ G_p p_z dz + G_{r^1}dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G \end{aligned} \tag{A113}$$

Similarly, we can expand the labor market clearing conditions to write $\widehat{L}^t dQ^t + \widehat{L}^t_{z^t} dz^t Q^t = L^t_p p_z dz + L^t_{r^1} dr^1 + L^t_\tau d\tau + L^t_\theta d\theta$.[46] Substituting for dr^1, we have:

$$\begin{aligned} dQ =& \widehat{X}dQ + (C_p + G_p)p_z dz + (C_\tau + G_\tau)d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G + \widehat{X}_z dzQ \\ &+ (C_{r^1} + G_{r^1})(L_{r^1})^{-1}(\widehat{L}dQ + \widehat{L}_z dzQ - L_p p_z dz - L\tau d\tau - L_\theta d\theta) \\ =& \left(\widehat{X} + (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\widehat{L}\right) dQ + \partial Q \end{aligned} \tag{A114}$$

[46]While using this condition for both $t = 1$ and $t = 2$ might appear to over-determine dr^1, the two determinations are actually equivalent by Walras' law.

where:

$$
\begin{aligned}
\partial Q = & \Big(C_p + G_p - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_p\Big) p_z dz \\
& + \Big(\widehat{X}_z + (C_{r^1} + G_{r^1})(L_{r^1})^{-1} \widehat{L}_z\Big)\, dzQ \\
& + \Big(C_\tau + G_\tau - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\tau\Big) d\tau \\
& + \Big(C_\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\theta\Big) d\theta + G_{\theta_G} d\theta_G
\end{aligned}
\tag{A115}
$$

□

A classical dichotomy holds in our flexible-wage model, such that the expressions above do not depend on monetary policy. Also notice that matrices are block diagonal: we can solve out each period in isolation. To fix ideas, consider an increase in first-period government spending (matched by a decrease in the second period), and suppose that substitution effects dominate. Then the exogenous increase in demand for goods (a) causes firms to demand more inputs and (b) increases the real wage in period 1 through an increase in the real interest rate, dampening consumption demand in period 1; this generates some "second-order" change in first-period demand. The equilibrium effect on first-period production occurs in the limit as these higher-order responses die out (which they do, if we start from a stable equilibrium). Here, the second period is implicit in households' choices, but we need not consider it directly.

We now compare the flexible-wage multiplier to the generalized Keynesian multiplier. There are three main differences between the two multipliers. First, the two multipliers tell very different stories for how the interest rate responds to shocks. In the flexible case, interest rate changes are mediated through the labor market while in the rationing case they are determined through monetary policy.

Second, whereas in the flexible-wage case income is determined according household labor supply, in the rationing case income is determined by exogenous rationing functions not chosen by households. This implies that some households may want to supply more labor while others want to supply less; the equilibrium is in general inefficient. Insofar as preexisting employment relationships determine which of these households are which, they are essential for understanding how shocks propagate in the economy. Crucially, the fact that—in rationing equilibrium—households do not choose their labor supply opens the door to shocks that have extremely disparate impacts on the wealth of different households.[47] Indeed, in the rationing case as consumers have both endogenous and exogenous labor income sources, there are two channels through which consumers respond to shocks. In the flexible-wage case they respond only through changes to their endogenous labor income.

Finally, notice that in the flexible-wage case, the first-period effect of a pure demand shock can be assessed without referring to the second period; in the rationing case, it is necessary to consider intertemporal transmission channels. In the former, the interest rate is determined simply by labor market clearing condition within either period (the two are equivalent). In the latter, the interest rate—which affects first-period consumption—is determined by the

[47]This in principle possible in flexible-wage equilibrium (if households have very different labor supply elasticities) but far less realistic.

endogenous policy response of the central bank, which in turn depends on the second-period shock, as well the amount of income that agents earn in the first period.

The consequence of these three differences is that demand shocks propagate very differently in the rationing price case and the flexible-wage case. These formulae therefore indicate that shock propagation hinges strongly on the level of price rigidity in the economy, even down to the relevant channels that need to be considered. For example, labor supply elasticities are crucial in understanding the output response under flexible-wages but irrelevant in the rationing case and consumers' MPCs out of income are important in understanding the rationing case but play no part in determining the response under flexible-wages.

B.6. A Network Interpretation of the Multiplier

The multiplier formula in Proposition 1 that forms the backbone of our analysis in this paper also appears in the regional economics literature on social accounting matrices dating back to Miyazawa (1976). Our result therefore provides the first formal economic analysis that provides a microfoundation for this formula which receives widespread use in the regional economics literature and applied work to compute expenditure multipliers (such as the BEA's RIMS II system). This relationship motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households.

Formally this corresponds to an input-output matrix given by:

$$\widehat{X}^1 = \begin{bmatrix} \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} & \widehat{X}^1_{\mathcal{I}^1 N} = C^1_{y^1} \\ \widehat{X}^1_{N\mathcal{I}^1} = l^1_{L^1}\widehat{L}^1 & 0 \end{bmatrix} \tag{A116}$$

The multiplier at the zero lower bound can then be expressed as:

$$\begin{aligned} \begin{bmatrix} dQ^1_{\mathcal{I}^1} \\ dQ^1_N \end{bmatrix} &= \left(I - \begin{bmatrix} \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} & \widehat{X}^1_{\mathcal{I}^1 N} \\ \widehat{X}^1_{N\mathcal{I}^1} & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} \partial Q^1_{\mathcal{I}^1} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \left(I - \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} - \widehat{X}^1_{\mathcal{I}^1 N}\widehat{X}^1_{N\mathcal{I}^1} \right)^{-1} & \cdots \\ \cdots & \cdots \end{bmatrix} \begin{bmatrix} \partial Q^1_{\mathcal{I}^1} \\ 0 \end{bmatrix} \end{aligned} \tag{A117}$$

One sees immediately that this recovers our generalized Keynesian cross of Proposition 1. We can therefore think of households as firms who, in order to supply labor, demand a consumption bundle as inputs. On top of the assumption that households only interact through firms, this representation also relies on the assumption that households do not choose their labor supply in the first period; this makes them analogous to firms, who must meet market demand.

B.7. Network Decompositions for Supply Shocks

We now derive network decompositions of the multiplier as in Section 3.2 that are valid for both demand and supply shocks, extending the earlier analysis. To this end, we see that changes in GDP when we consider a supply shock have two distinct components:

$$d(GDP) \equiv d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{A118}$$

Where it is without loss to redefine units of consumption goods and evaluate at an initial equilibrium with $p^{1T} = \vec{1}$. Propositions 2 and 17 already decomposed the first term $\vec{1}^T dY^1$. To achieve our decomposition for supply shocks, we therefore need only compute $dp^{1T}Y^1$. To this end, we can employ Corollary 3, where we derived prices in closed-form as a function of z:

$$p^1(z) = \left(1 - \widehat{X}^1(z)^T\right)^{-1}\widehat{L}^1(z)\vec{1} \tag{A119}$$

It follows that the change in GDP can then be decomposed as before but with a new term which depends only on the IO matrix and labor shares and not labor rationing or household consumption. This is stated formally below:

Proposition 16. *The total change in first-period output due to a shock with unit-magnitude labor income incidence ∂y^1 can be approximated as:*

$$\begin{aligned} d(p^{1T}Y^1) = &\frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\textit{Incidence effect}} \\ &+ \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{next}] - \mathbb{E}_{y*}[m_n]\right)}_{\textit{Biased spending direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}}\Bigg) \\ &+ \underbrace{d\left[\left(1-\widehat{X}^1(z)^T\right)^{-1}\widehat{L}^1(z)\vec{1}\right]^T Y^1}_{\textit{Price Effect}} + O^3(|m|) \end{aligned} \tag{A120}$$

where y^* is any reference income weighting of unit-magnitude and m^i_{next} is the average MPC of households who receive as income i's marginal dollar of spending.

Proof. Recall that we have:

$$d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{A121}$$

Which we can always take as:

$$d(p^{1T}Y^1) = \vec{1}^T dY^1 + dp^{1T}Y^1 \tag{A122}$$

through an appropriate renormalization of the initial units of the goods. By Proposition 2, we have that:

$$1^T dY^1 = \frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\text{Incidence effect}} + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{\text{next}}] - \mathbb{E}_{y*}[m_n]\right)}_{\text{Biased spending direction effect}} + \underbrace{\mathbb{C}\text{ov}_{\partial y^1}[m_n, m_n^{\text{next}}]}_{\text{Homophily effect}}\Bigg) + O^3(|m|) \quad \text{(A123)}$$

We now need only compute the term $dp^{1T}Y^1$. To this end, from Corollary 3 we have that:

$$p^1(z) = \left(1 - \widehat{X}^1(z)^T\right)^{-1}\widehat{L}^1(z)\vec{1} \quad \text{(A124)}$$

Differentiating yields:

$$dp^{1T}Y^1 = \underbrace{d\left[\left(1 - \widehat{X}^1(z)^T\right)^{-1}\widehat{L}^1(z)\vec{1}\right]^T Y^1}_{\text{Price Effect}} \quad \text{(A125)}$$

Adding the two terms yields the claimed expression and completes the proof. □

B.8. Special Cases Where Network Effects in Propagation Vanish

In the main text, we briefly discussed two important cases where network effects in shock propagation vanish. Here, we more formally state the results and provide more detailed discussion of the results.

Proposition 17. *The following statements are true:*

1. *(No incidence or bias effects) Suppose that consumption preferences and labor rationing are homothetic, that no households are net borrowers in period* 1*, and that there is no government spending.*[48] *Then, for a GDP-proportional, unit-magnitude demand shock, the incidence and bias effects are zero, so that we have:*

$$\vec{1}^T dY^1 = \frac{1}{1-\mathbb{E}_{y^1}[m_n]}\Bigg(1 + \underbrace{\mathbb{C}ov_{y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}}\Bigg) + O^3(|m|) \quad \text{(A126)}$$

   where y^1 is the vector of first-period incomes.
2. *(No incidence, bias, or homophily effects) Suppose that all industries have a common rationing-weighted average MPC, m.*[49] *Then the incidence, bias, and homophily effects are zero, so that for any reference weighting y^* that can be induced by a demand shock,*

[48] By homothetic labor rationing, we mean that marginal and average rationing of income are equal. Formally, if we let $\mathcal{L}^1 \equiv \widehat{L}^1\left(I - \widehat{X}^1\right)^{-1} Y^1$ be the vector of first-period firm-level labor bills, then we require that $y^1 = l^1_{L^1}\mathcal{L}^1$.

[49] Formally, $\sum_{n\in N}(l^1_{L^1})_{ni} m_n = m$ for all $i \in \mathcal{I}^1$.

the change in output corresponding to any unit-magnitude demand shock is:[50]

$$\vec{1}^T dY^1 = \frac{1}{1 - \mathbb{E}_{y*}[m_n]} = \frac{1}{1-m} \tag{A128}$$

Proof. We prove the two claims separately:

1. Recalling that $(m^{\text{next}})^T \equiv m^T \mathcal{G}$, and the shock satisfies $\partial y^1 \propto y^1$, the following are equivalent:
$$m^T \mathcal{G} y^1 - m^T y^1 = 0 \iff \mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] = 0 \tag{A129}$$
It therefore suffices to show that $\mathcal{G} y^1 = y^1$.
Plugging in the definition of $\mathcal{G}$, we have $\mathcal{G} y^1 = l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \overline{C}^1_{y^1} y^1$. Since each household saves zero on net, y^1 is equal to total spending. Homotheticity of consumption implies that $\overline{C}^1_{y^1} y^1$, then, is the vector of total consumption of goods; since there is no government spending, this equals total output, Y^1. Finally, homotheticity of rationing implies that $l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} Y^1 = y^1$.
2. Recall by Proposition 1 that when either $C_{r^1} + G_{r^1} = 0$ or $r^1_{Q^1} = 0$, the general equilibrium effect on income of a partial equilibrium shock is given by:
$$dY^1 = \left(I - C^1_y l^1_{L^1} \widehat{L}^1 (1 - \widehat{X}^1)^{-1}\right)^{-1} \partial Q^1 \tag{A130}$$
We wish to investigate whether there exists some $m \in (0,1)$ such that the following holds for all ∂Q:
$$\vec{1}^T dY^1 = \frac{1}{1-m} \vec{1}^T \partial Q^1 \tag{A131}$$
First, we note a simple fact of linear algebra. Suppose an invertible matrix M has columns summing to some constant m. This is equivalent to:
$$\vec{1}^T M v = m \vec{1}^T v, \quad \forall v \tag{A132}$$
It is then true that for any v:
$$m \vec{1}^T (M^{-1} v) = \vec{1}^T M (M^{-1} v) = \vec{1}^T v \tag{A133}$$
Thus, M^{-1} has columns summing to $\frac{1}{m}$.
Second, note that the desired result (A131) holds if and only if
$$\left(I - C^1_y l^1_{L^1} \widehat{L}^1 (1 - \widehat{X}^1)^{-1}\right)^{-1} \tag{A134}$$

[50]Formally, saying that y^* can be induced by a demand shock says that there exists a ∂Q^* such that:

$$y^* = l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^* \tag{A127}$$

sums to $\frac{1}{1-m}$. This is equivalent, by the first observation, to the claim that each column of:

$$C_y^1 l_{L^1}^1 \widehat{L}^1 (1 - \widehat{X}^1)^{-1} \tag{A135}$$

sums to m.

It remains to show that this claim is equivalent to the condition provided in the statement of the Proposition. Namely, we must show that

$$\vec{1}^T C_y^1 l_{L^1}^1 \widehat{L}^1 (1 - \widehat{X}^1)^{-1} = m\vec{1}^T \iff \vec{1}^T C_y^1 l_{L^1}^1 = m\vec{1}^T \tag{A136}$$

Multiplying each side by $(I - \widehat{X}^1)(\widehat{L}^1)^{-1}$—which exists since labor is essential in production—reveals that (A136) holds if $(I - \widehat{X}^1)(\widehat{L}^1)^{-1}$ has columns summing to one.

By our earlier linear algebra observation, this holds if and only if $\widehat{L}^1(1 - \widehat{X}^1)^{-1}$ has columns summing to one. This can be seen by recalling the no-profit condition

$$p^1 = (I - (\widehat{X}^1)^T)^{-1} \widehat{L}^1 \vec{1}, \tag{A137}$$

using our normalization $p = \vec{1}$, and taking the transpose of both sides.

□

The first part of the proposition shows how, even in a "homothetic economy," heterogeneity in household consumption baskets and sectoral employment can generate network effects through homophily. This happens even at the same time as homotheticity eliminates the bias effect by ensuring that each household's marginal consumption is proportional to its initial consumption, so that the income-weighted average of marginal consumption is proportional to output. Still, when households with different MPCs direct their spending toward different goods, the households employed to produce the goods consumed by higher-MPC households experience a greater change in income – not from the initial, uniform shock, but from the economy's response to it. Insofar as these households have different MPCs from the average, homophily is still possible. This mechanism generate non-neutrality for the multiplier, even if the economy and the shock considered are "neutral" in all other aspects. Aggregate neutrality requires (to second order in MPCs) that the economy feature exactly zero correlation between households' MPCs and the MPCs of the households they spend on.

The second part of the proposition imposes that each firm's marginal employees have the same average MPC as one another. This eliminates the incidence, bias, and homophily effects, leaving only the classical Keynesian multiplier. That is, wherever in the economy a shock strikes, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $l_{L^1}^1 = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm's marginal employees have the population average MPC.

B.9. Optimal Policy at a Global Optimum

In the main text, we focused primarily on small changes in welfare corresponding to small changes in policy. In this section, we specialize to the case of small changes in policy *at an optimum*. Thus, the corresponding changes in welfare are second order.

Our first result decomposes the first-order condition for optimal government spending and transfers into five distinct mechanisms. This is closely related to Proposition 3 in the main text, which considers the change in welfare away from the global optimum.

Proposition 18. *Suppose taxes τ^{1*}, τ^{2*} and expenditures G^{1*}, G^{2*} solve the planner's problem. Now consider a change in policy $\tau^t = \tau^{t*} + \varepsilon \tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$, indexed by ε. The following first-order condition holds:*

$$\begin{aligned} 0 = & \underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \widehat{\Delta} R^1)\right) G^1_\varepsilon}_{\text{Opportunistic government spending}} + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G^2_\varepsilon}{1 + r^1}}_{\text{Short-termist government spending}} \\ & - \underbrace{(\widetilde{\lambda} - \gamma \vec{1})^T \widehat{\mu} \left(\tau^1_\varepsilon + \frac{\tau^2_\varepsilon}{1 + r^1}\right)}_{\text{Pure redistribution}} + \underbrace{\widetilde{\lambda}^T \frac{\widehat{\phi} \widehat{\mu} \tau^2_\varepsilon}{1 + r^1}}_{\text{Relaxation of borrowing constraints}} \\ & - \underbrace{\widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C^1_{y^1} R^1\right)^{-1} C^1_{y^1} \left(R^1 G^1_\varepsilon - \widehat{\mu} \tau^1_\varepsilon - \frac{1_{\phi_n = 0} \widehat{\mu} \tau^2_\varepsilon}{1 + r^1}\right)}_{\text{Keynesian stimulus (alleviation of involuntary unemployment)}} \end{aligned} \tag{A138}$$

where γ is the marginal value of public funds.

Proof. The planner takes prices and—locally—the interest rate as given. Goods and labor market clearing and first-period rationing determine the change in first-period employment as a function of G^1_ε and τ^1_ε. We are left with the following first-order condition of the planner's problem:

$$0 = dW + \gamma \left[\mu^T \tau^1_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1 + r^1} - \vec{1}^T G^1_\varepsilon - \frac{\vec{1}^T G^2_\varepsilon}{1 + r^1} \right] \tag{A139}$$

where dW is as in Equation 21. This gives an expression for the change in welfare in terms of τ_ε, G_ε, and l^1_ε, the change in first-period employment. By Equation 12, $\widehat{\mu}\ l^1_\varepsilon = R^1(I - C^1_{y^1} R^1)^{-1} \partial Q^1$, where $R^1 \equiv l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ and $\partial Q^1 = G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \tau^1_\varepsilon - C^1_{y^2} \widehat{\mu} \tau^2_\varepsilon$. For borrowing-constrained households, $C^1_{y^2} = 0$; they would already like to substitute additional consumption toward the first period but are constrained not to do so. Other households are

Ricardian, implying $C^1_{y^2} = \frac{C^1_{y^1}}{1+r^1}$. Plugging in for dW, and using matrix notation, we have

$$
\begin{aligned}
0 = \widetilde{\lambda}^T \Bigg[& -\widehat{\Delta} R^1 (I - C^1_{y^1} R^1)^{-1} \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) \\
& - \left(\widehat{\mu} \tau^1_\varepsilon + \frac{\widehat{\mu}(I - \widehat{\phi}) \tau^2_\varepsilon}{1+r^1} \right) + \left(\widehat{\mu} WTP^1 G^1_\varepsilon + \widehat{\mu}(I - \widehat{\phi}) \frac{WTP^2}{1+r^1} G^2_\varepsilon \right) \Bigg] \\
& + \gamma \left(\mu^T \tau^1_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1+r^1} - \vec{1}^T G^1_\varepsilon - \frac{\vec{1}^T G^2_\varepsilon}{1+r^1} \right)
\end{aligned}
\tag{A140}
$$

Now, observe that the term on the first line can be rewritten:

$$
\begin{aligned}
& R^1 (I - C^1_{y^1} R^1)^{-1} \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) \\
= & R^1 \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) \\
= & \left(R^1 G^1_\varepsilon + \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) C^1_{y^1} R^1 G^1_\varepsilon \right) \\
& - R^1 \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1} \right) \\
= & R^1 G^1_\varepsilon + R^1 \left(I - C^1_{y^1} R^1 \right)^{-1} C^1_{y^1} \left(R^1 G^1_\varepsilon - \widehat{\mu} \tau^1_\varepsilon - \widehat{\mu} \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1} \right)
\end{aligned}
\tag{A141}
$$

Substituting this back in and rearranging, we obtain Equation A138. □

To better understand the form of the implied optimal policy, we discuss each term of Equation A138 in turn. The opportunistic government spending term is as in Werning (2011) and Baqaee (2015). It augments the standard first-order condition for government spending with a labor-wedge term, reflecting that the social cost of additional government purchases is lower than the market cost when they are produced using underemployed labor. The second term is also an augmented version of the standard expression for government spending—this time in the second period. The borrowing wedge reflects that households with binding borrowing constraints implicitly discount the future at a higher-than-market rate; the planner must account for this when deciding whether to make purchases on their behalf.

The third term of Equation A138 is a standard, pure redistribution term, weighing the private benefits of transfers against the social cost (the MVPF). The fourth term augments this, when there are borrowing constraints. In particular, taxes in the second period are less costly to borrowing-constrained households, since they discount the future more heavily than the market rate indicates.

Finally, the last line captures the value of stimulus brought on by changes in income—those corresponding to pure income transfers via taxes and labor market income earned by government employees producing expenditures.[51] $C^1_{y^1}$ maps income changes to changes in consumption. Then the output multiplier $(I - C^1_{y^1} R^1)^{-1}$ maps this partial equilibrium change in consumption to the general equilibrium change in output. Finally, R^1 maps the change in output to the change in labor supplied to meet income-induced demand changes, loading onto the labor wedges.

B.10. *Optimal policy with imperfect competition*

In this section, we extend the optimal policy results of section 4 to the more general environment with constant, non-zero markups. As is section 4 we normalize prices p^t_i to one throughout, without loss of generality.

To highlight as clearly as possible the parallels to the case without profits, we make two important assumptions. First—although in the first period, profit-creation is uninternalized by households—we assume that the government incentivizes second-period profit-creation with Pigouvian subsidies funded lump-sum by shareholders.

Assumption 8. *There is an ad-valorem subsidy s^2_i on the purchase of i (for consumption or production), set equal to the profit rate m^2_i. It is funded directly by an additional lump-sum, second-period tax $\hat{\tau}^2_n$ defined by $\mu_n \hat{\tau}^2_n = \sum_{i \in I} \left(\widehat{\Pi}^2_{ni} \Big/ \sum_{n' \in N} \widehat{\Pi}^2_{n'i} \right) s^2_i Q^2_i$.*

Second, we assume that the MPC out of future profits is zero. This is a rather weak assumption, as the MPC out of even *current* capital income is small empirically.

Assumption 9. *For all households n, $C^1_{\pi^2} = 0$.*

B.10.1. *Planner's problem*

We begin by defining the household's problem. It is the same as Equation 19 in section 4.1, except that households now also receive profit income.

$$
\begin{aligned}
&\max_{\tilde{c}^t, \tilde{l}^t} \sum_{t=1,2} \beta^{t-1}_n \left[u^t_n(\tilde{c}^1) - v^t_n(\tilde{l}^t) + w^t_n(G^t) \right] \\
&\text{s.t. } p^1 \cdot \tilde{c}^1 + \frac{p^2 \cdot \tilde{c}^2}{1+r^1} + \tau^1_n + \frac{\tau^2_n}{1+r^1} \leqslant \tilde{l}^1 + \frac{\tilde{l}^2}{1+r^1} + \pi^1_n + \frac{\pi^2_n - \hat{\tau}^2_n}{1+r^1} \\
&\quad\;\; \tilde{l}^1 + \pi^1_n - p^1 \cdot \tilde{c}^1 - \tau^1_n \geqslant \underline{s}^1_n \\
&\quad\;\; \tilde{l}^1 = l^1_n
\end{aligned}
\tag{A142}
$$

Note that this microfoundation implies $C_y = C_\pi$. That is, additional income from rationed labor has the same effects on consumption as additional income from profits.

As in section 4, we study the policy problem of a planner at the zero lower bound. Formally, the planner's problem is the same as in Equation 20 except that household behavior solves Equation A142 and aggregate variables evolve according to Equation A88 with $r_Q = 0$.

[51] If second period expenditures are held constant, then the net income transfer is zero, i.e. this term operates solely through redistribution to different households (who may spend differently).

B.10.2. Policy changes away from the optimum

This section considers changes in welfare due to small changes in not-necessarily-optimal policies, as in section 4.2. The only difference now is the presence of profits.

This setup in mind, we now consider the change in welfare induced by changes in transfers and government expenditure, analogously to Proposition 3.

Lemma 3. *Under assumptions 8 and 9, the change in welfare dW due to a small change in taxes and government expenditure—at a constant interest rate—can be expressed:*

$$\begin{aligned} dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \Bigg[& -\Delta_n dl_n^1 + d\pi_n^1 - \left(d\tau_n^1 + (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) \\ & + \left(WTP_n^1 dG^1 + (1 - \phi_n) \frac{WTP_n^2}{1 + r^1} dG^2 \right) \Bigg] \end{aligned} \tag{A143}$$

where $\widetilde{\lambda}_n$ is the value the planner places on the marginal transfer of first-period wealth to a household of type n, Δ_n and ϕ_n are n's implicit first-period labor wedge and borrowing wedge, and WTP_n^t is the vector of n's marginal willingness to pay for period t government expenditures on each good, in period t dollars. The changes in first-period employment and profits, in turn, are given by

$$\begin{aligned} & \hat{\mu} dl^1 = l_{L^1}^1 \hat{L}^1 \left(1 - \hat{X}^1 \right)^{-1} dY^1, \quad \hat{\mu} d\pi^1 = \hat{\Pi} \left(1 - \hat{X} \right)^{-1} dY^1, \\ & dY^1 = \left(I - C_{y^1}^1 \left(l_{L^1}^1 \hat{L}^1 + \hat{\Pi}^1 \right) \left(I - \hat{X}^1 \right)^{-1} \right)^{-1} \partial Q^1 \end{aligned} \tag{A144}$$

Proof. We follow the same steps as the proof of Proposition 3 (see Appendix A.4) up to the substitution of the budget constraint, which now includes profits. With profits, differentiating the household's lifetime budget constraint (at constant r^1) gives:

$$p^1 dc_n^1 - dl_n^1 - d\pi_n^1 + \frac{p^1 dc_n^2 - dl_n^2}{1 + r^1} = -d\tau_n^1 + \frac{d\pi_n^2 - d\hat{\tau}_n^2 - d\tau_n^2}{1 + r^1} \tag{A145}$$

Note that since $\sum_{n' \in N} \hat{\Pi}_{n'i}^2 = m_i^2 = s_i^2$:

$$d\hat{\tau}_n^2 = \frac{1}{\mu_n} \sum_{i \in I} \left(\hat{\Pi}_{ni}^2 \Big/ \sum_{n' \in N} \hat{\Pi}_{n'i}^2 \right) s_i^2 dQ_i^2 = \frac{1}{\mu_n} \hat{\Pi}_{ni}^2 dQ_i^2 = d\pi_n^2 \tag{A146}$$

Plugging in the change in the differentiated budget constraint, we have:

$$\begin{aligned} dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \Bigg[& -\Delta_n dl_n^1 + \phi_n \left(p^1 dc_n^1 - dl_n^1 \right) + (1 - \phi_n) \left(d\pi_n^1 - d\tau_n^1 - \frac{d\tau_n^2}{1 + r^1} \right) \\ & + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1 - \phi_n}{1 + r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \Bigg] \end{aligned} \tag{A147}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint $\underline{s}_n^1 = l_n^1 + \pi_n^1 - \tau_n^1 - p^1 c_n^1$ implies $p^1 dc_n^1 + d\tau_n^1 = dl_n^1 + d\pi_n^1$. Defining the within-period willingnesses to pay $WTP_n^t = \frac{w_{nG}^t}{\kappa_n^t}$, we arrive at the final expression:

$$\begin{aligned} dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \Bigg[& -\Delta_n dl_n^1 + \left(d\pi_n^1 - d\tau_n^1 - (1-\phi_n) \frac{d\tau_n^2}{1+r^1} \right) \\ & + \left(WTP_n^1 dG^1 + (1-\phi_n) \frac{WTP_n^2}{1+r^1} dG^2 \right) \end{aligned} \tag{A148}$$

Finally, the expressions for $dl, d\pi, dY$ come from rearranging Equation A88 under assumption 9 and using $dY = (1 - \widehat{X})dQ$. □

Studying Equation A143 reveals a key insight: Under assumptions 8 and 9, the change in welfare due to a change in taxes and expenditures is the same as in an *as-if* economy without profits but where share-holders supply labor with a wedge -1. This labor supply wedge corresponds to complete under-employment; share-holders—who experience no marginal disutility of holding shares—would continue to be willing to hold shares until profits-per-revenue reached zero. Just like labor suppliers, share-holders do not choose their income but rather take it as given. This *as-if* representation of profits as under-employed labor allows us to carry over all of the results from Section 4 with minimal alterations.

Proposition 19. *Under assumptions 2, 8, and 9, the welfare change from a change in expenditures is proportional to the resulting change in output, whereas the welfare change from a change in transfers is proportional to the resulting change in income. Formally,*

$$dW = \vec{1}^T \frac{dY^1}{dG} dG + \vec{1}^T \frac{d(l+\pi)^1}{dy^1} \left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1} \right) \tag{A149}$$

where $\frac{dY^1}{dG^1} = (1 - C_{y^1}^1 R^1)^{-1}$ *and* $\frac{dY^1}{dG^2} = 0$ *are first-period output multipliers and* $\frac{d(l+\pi)^1}{dy^1} = (1 - R^1 C_{y^1}^1)^{-1}$ *is the first-period income multiplier; here* $R^1 = \left(l_{L^1}^1 \widehat{L}^1 + \widehat{\Pi}^1 \right) \left(I - \widehat{X}^1 \right)^{-1}$.

Proof. Reinterpret profit income as labor supply with wedge -1, as discussed above. The proof then follows from Appendix A.6. □

The key here is that assumption 2's imposition that all marginal labor supplies have a labor supply wedge of -1 exactly matches with the shareholders' implicit labor supply wedge of -1. Indeed, both are indifferent to supply more of their factor. As a result, there is zero social cost to additional employment of either factor, so the optimal policy simply maximizes output.

As without markups, the output-maximizing policy is simply MPC-targetting when "network effects" are not present:

Corollary 5. *Suppose that all households' marginal spending is directed to households whose average MPC is equal to the incidence-weighted average MPC corresponding to a uniform* *output shock.*[52] *Formally,* $m_n^{next} = \mathbb{E}_{y*}[m_{n'}]$ *for all* n*, where* $m_i^{next} \equiv \left(mR^1 C^1_{y^1}\widehat{m}^{-1}\right)_i$ *and* $R^1 = \left(l^1_{L^1}\widehat{L}^1 + \widehat{\Pi}^1\right)\left(I - \widehat{X}^1\right)^{-1}$*. Then, under assumptions 2, 8, and 9, the welfare change from a policy is given by:*

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1 + r^1}\right) \tag{A150}$$

Dollar-for-dollar, the best policy is the one most effectively targeting household MPC.

Proof. Again, the proof follows from Appendix A.7 after reinterpreting profit income as labor supply with wedge -1. □

B.10.3. First-order conditions for optimal policy

The same *as-if* representation of profits as under-employed labor also allows us to carry over results from section B.9 to the case of imperfect competition.

Proposition 20. *Suppose taxes* τ^{1*}, τ^{2*} *and expenditures* G^{1*}, G^{2*} *solve the planner's problem. Now consider a change in policy* $\tau^t = \tau^{t*} + \varepsilon\tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$*, indexed by* ε*. Then, under assumptions 8 and 9, the following first-order condition holds:*

$$\begin{aligned}
0 = &\underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \check{\Delta}\check{R}^1)\right) G^1_\varepsilon}_{\text{Opportunistic government spending}} \quad + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G^2_\varepsilon}{1 + r^1}}_{\text{Short-termist government spending}} \\
&\underbrace{- (\widetilde{\lambda} - \gamma\vec{1})^T \widehat{\mu}\left(\tau^1_\varepsilon + \frac{\tau^2_\varepsilon}{1 + r^1}\right)}_{\text{Pure redistribution}} \qquad \underbrace{+ \widetilde{\lambda}^T \frac{\widehat{\phi}\widehat{\mu}\tau^2_\varepsilon}{1 + r^1}}_{\text{Relaxation of borrowing constraints}} \\
&\underbrace{- \widetilde{\lambda}^T \check{\Delta}\check{R}^1 \left(I - \check{C}^1_{y^1}\check{R}^1\right)^{-1} \check{C}^1_{y^1} \left(\check{R}^1 G^1_\varepsilon - \widehat{\mu}\tau^1_\varepsilon - \frac{1_{\phi_n = 0}\widehat{\mu}\tau^2_\varepsilon}{1 + r^1}\right)}_{\text{Keynesian stimulus (alleviation of involuntary unemployment)}}
\end{aligned} \tag{A151}$$

where γ *is the marginal value of public funds,* $\check{R}^1 = \begin{bmatrix} l^1_{L^1}\widehat{L}^1 \\ \widehat{\Pi}^1 \end{bmatrix}\left(I - \widehat{X}^1\right)^{-1}$, $\check{C}^1_{y^1} = \begin{bmatrix} C^1_{y^1} & C^1_{y^1} \end{bmatrix}$*, and* $\check{\Delta}$ *is the* $N \times 2N$ *matrix with entries* $\check{\Delta}_{n,n} = \Delta_n$*,* $\check{\Delta}_{n,N+n} = -1$*, and zeros elsewhere.*

Proof. This follows from reinterpreting profit income as labor supply with wedge -1 and then following the proof of Proposition 18. □

Intuitively, the planner targets "profit-wedges" in the same manner as labor supply wedges. These both reduce the social cost of government spending and provide a motive for Keynesian stimulus.

Finally, a similar network-irrelevance result holds as in the case without profits.

[52]This ensures that the final two correction terms in Equation 18 are zero for all partial equilibrium shocks.

Proposition 21. *Impose Assumptions 8 and 9. Now, suppose that all households rationed to on the margin at the optimum have no marginal labor disutility, i.e. if* $(R^1 C^1_{y^1})_{n,-} \neq \vec{0}$ *then* $\Delta_n = 0$*. Then Equation A151 holds with respect to variations in first-period transfers if and only if, for all* $n \in N$*,*

$$\gamma = \frac{\widetilde{\lambda}_n}{1 - m_n} \tag{A152}$$

Alternatively, suppose that the social gains from first-period government expenditure are equal to some $\tilde{v}$ *across goods and constraints bounding expenditures above zero do not bind. Then Equation A138 holds with respect to variations in first-period expenditures if and only if, for all* $i \in I$*,*

$$\gamma = \tilde{v} + \frac{1}{1 - \tilde{m}_i} \left(-\widetilde{\lambda \Delta}_i \right) \tag{A153}$$

where $\tilde{m}_i \equiv \left(m^T R^1 \right)_i$ *is the rationing-weighted average MPC in the production of good* i *and* $\widetilde{\lambda \Delta}_i \equiv \left(\widetilde{\lambda}^T \check{\Delta} \check{R}^1 \right)_i$ *is the rationing-and-welfare-weighted average rationing wedge in the production of good* i*, where* R^1 *is as in proposition 19 and* $\check{R}^1$ *and* $\check{\Delta}$ *are as in proposition 20.*

Proof. Again, this follows from reinterpreting profit income as labor supply with wedge -1 and then following Appendix A.5, plus imposing $\Delta_n = 0$ for marginal labor-suppliers in the transfer case. □

C. Measuring Rationing Wedges

In this Appendix, we describe how to recover rationing wedges in the data and describe how we estimate the counterfactual welfare effects of fiscal stimulus in the Great Recession. We present two microfoundations for the same, particularly simple form of the rationing wedge in terms of the demographic level percentage change in unemployment from before the Great Recession to during the Great Recession. In particular, we provide two microfoundations for the following expression for the change in welfare induced by fiscal stimulus dG^1:

$$dW = \sum_{n \in N} \underbrace{\frac{l_n^2 - l_n^1}{l_n^2}}_{\text{Gap in Labor Income}} \times \underbrace{\left(R(I - C_{y^1}^1 R)^{-1} dG^1\right)_n}_{\text{Labor Income Effect of Stimulus}} \tag{A154}$$

C.1. Intensive margin microfoundation

Our first microfoundation assumes that all households within each demographic group can be treated as having the same quantity of labor supply. This is equivalent to the assumption that all labor supply adjustment happens on the intensive margin (hours worked), and that workers within any demographic group experience the same change in hours.

By optimality of second period labor supply (and noting that $w^t = 1$):

$$v_n^{2\prime} = \kappa_n^2 \tag{A155}$$

where $\kappa_n^t = u_n^{t\prime}$. The rationing wedge is defined as the wedge in the first period intra-temporal Euler equation:

$$v_n^{1\prime} = \kappa_n^1 (1 + \Delta_n) \tag{A156}$$

See that $\Delta_n < 0$ corresponds to involuntary underemployment, $\Delta_n = 0$ is consistent with optimal labor supply and $\Delta_n > 0$ corresponds ot involuntary overemployment. The intertemporal Euler equation is given by:

$$\kappa_n^1 = \beta_n \frac{1 + r^1}{1 - \phi_n} \kappa_n^2 \tag{A157}$$

where $\phi_n \geqslant 0$ is a wedge stemming from the potentially-binding borrowing constraint. Combining these equations yields:

$$v_n^{1\prime} = \beta_n \frac{1 + r^1}{1 - \phi_n} v_n^{2\prime} (1 + \Delta_n) \tag{A158}$$

Thus, the rationing wedge is given by:

$$\Delta_n = \frac{1 - \phi_n}{\beta_n (1 + r^1)} \frac{v_n^{1\prime}}{v_n^{2\prime}} - 1 \tag{A159}$$

We now assume that (i) all households have slack borrowing constraints $\phi_n = 0$ and that (ii) $\beta_n (1 + r^1) = 1$, which is empirically justifiable with standard estimates for discount factors and the real interest rate in the US during the Great Recession. Alternatively, it follows exactly from (i) and (A157) in the special case where consumption utility is linear. Under these assumptions, the rationing wedge is given by:

$$\Delta_n = \frac{v_n^{1\prime}}{v_n^{2\prime}} - 1 \tag{A160}$$

Under the assumption that labor disutility is time invariant and isoelastic, we have that:

$$v_n^t(l) = \xi \frac{l^{1+\psi}}{1+\psi} \tag{A161}$$

The rationing wedge is then given by:

$$\Delta_n = \left(\frac{l_n^1}{l_n^2}\right)^{\psi} - 1 \tag{A162}$$

Intuitively, whenever the household is working less than that steady state value, they are underemployed. This is because wages are not changing and their preferences and interest rate are such that they apply no dollar discount to future disutility. A standard calibration allows us to set $\psi = 1$. In this case, the rationing wedge is the percentage gap in labor supply from the steady state:

$$\Delta_n = \frac{l_n^1 - l_n^2}{l_n^2} \tag{A163}$$

It follows by Proposition 3 that – in the absence of direct willingness to pay for government spending – the change in welfare induced by a change in first period government spending dG^1 is given by:

$$dW = -\sum_{n \in N} \widetilde{\lambda}_n \Delta_n \left(R^1 (I - C_{y^1}^1 R)^{-1} dG^1\right)_n \tag{A164}$$

Assuming no distributional motive $\widetilde{\lambda}_n = 1$, we then obtain that the welfare benefits of stimulus spending dG^1 are given by:

$$dW = \sum_{n \in N} \frac{l_n^2 - l_n^1}{l_n^2} \left(R^1 (I - C_{y^1}^1 R)^{-1} dG^1\right)_n \tag{A165}$$

C.2. Extensive margin microfoundation

Our second microfoundation focuses on the polar case in which, before a change in government spending, all households within a demographic group are either fully employed or fully unemployed. We assume that, within each demographic group n, a mass $1 - f_n$ of households never supplies labor. The complementary mass f_n supplies labor inelastically up to some level l_n^* after which their marginal dis-utility of labor supply sharply—but continuously—increases, so that they always supply close to l_n^* in equilibrium.

At the initial equilibrium, a total mass $\zeta_n \leqslant f_n$ are employed at the efficient level in the initial equilibrium; the remainder are unemployed. This implies that initially employed households supply $\approx l_n^*$ in both periods, whereas initially unemployed households supply 0 in the first period and $\approx l_n^*$ in the second period. Total first- and second-period labor supplies by group n are therefore approximately $l_n^1 \approx \zeta_n l_n^*$ and $l_n^2 \approx f_n l_n^*$, respectively.[53]

Now consider a change in labor demand induced by government spending. We assume that each firm rations the same expected amount of marginal labor to each member of the demographic group within the employable subpopulation f_n. However, in the case of employed workers, they do this by rationing infinitesimally more labor to a continuum of workers, whereas in the case of unemployed workers, they do this by hiring workers at their efficient level of labor supply. The former has only second-order welfare consequences. The latter increases welfare by

$$\begin{aligned}\widetilde{\Delta}_n \equiv & \max_{\substack{\vec{1}^T c^1 + \frac{\vec{1}^T c^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \leqslant l_n^1 + \frac{l_n^2}{1+r^1} \\ l_n^1 - \vec{1}^T c_n^1 - \tau_n^1 \geqslant \underline{s}_n^1}} u_n^1(c^1) - v_n^1(l_n^1) + \beta_n \left[u_n^2(c^2) + v_n^2(l_n^2)\right] \\ & - \max_{\substack{\vec{1}^T c^1 + \frac{\vec{1}^T c^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \leqslant \frac{l_n^2}{1+r^1} \\ -\vec{1}^T c_n^1 - \tau_n^1 \geqslant \underline{s}_n^1}} u_n^1(c^1) - v_n^1(0) + \beta_n \left[u_n^2(c^2) + v_n^2(l_n^2)\right]\end{aligned} \tag{A166}$$

per newly employed worker.

Finally, note that the fraction of the f_n-sized subgroup which is initially unemployed – and therefore experiences the welfare gain $\widetilde{\Delta}_n$ if employed – is equal to

$$\frac{f_n - \zeta_n}{f_n} = \frac{(f_n - \zeta_n) l_n^*}{f_n l_n^*} \approx \frac{l_n^2 - l_n^1}{l_n^2} \tag{A167}$$

The expected welfare gain per marginal dollar rationed to n in the first period is therefore

$$\approx \widetilde{\lambda}_n \widetilde{\Delta}_n \frac{l_n^2 - l_n^1}{l_n^1} \tag{A168}$$

Finally, we assume that the planner puts weight $\widetilde{\lambda}_n = \widetilde{\Delta}_n^{-1}$ on each group n, i.e. she values change in utility from employment equally across demographic groups. Combining this assumption with our earlier formula for the change in employment in each industry, we recover

$$dW \approx \sum_{n \in N} \frac{l_n^2 - l_n^1}{l_n^2} \left(R(I - C_{y^1}^1 R)^{-1} dG^1\right)_n \tag{A169}$$

C.3. Estimation

We have already estimated R and $C_{y^1}^1$, so to compute the welfare effects of stimulus in any given episode, we require only estimates of the demographic-level gap in labor income from the steady state at any point in time. For our Great Recession analysis, we compute this in the ACS by taking the percentage change in labor hours worked from 2005-06 to 2009-10 in each of our state-by-demographic bins. When there are no observations in any given bin, we assume that the change in labor hours is given by the state-level average.

[53] These approximations are exact in the limit where labor disutility is kinked at l_n^*.

For robustness, we compute a version of the state-by-demographic level rationing wedge by imposing that each demographic group's rationing wedge is the change in hours for that demographic group nationwide compared to the average multiplied by the average change in hours across demographics at the state level. The results are very similar, with the R^2 of multipliers in explaining welfare changes dropping slightly to 54%.

D. Validating the Model

The model that we develop and estimate in this paper makes stark predictions about the propagation of industry- and region-specific shocks. In this section, we attempt to empirically validate those quantitative predictions. Specifically, Proposition 1 provides an expression relating total output to spending in a state s and industry i:

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 = M \partial Q^1 \tag{A170}$$

where M is the generalized multiplier matrix. The $m_{s,r}$ entry gives the total change in output in state s when there is a one-unit partial equilibrium shock to state r distributed across industries in proportion to their share of total output in state r. Any identified partial equilibrium shock G will be some component of the many partial equilibrium shocks hitting the economy, which we can express as $\partial Q^1 = G + U$, where U is the partial equilibrium effect on demand of the unobserved shocks hitting the economy. Plugging this in, we arrive at the foundation for our estimating equation:

$$dY_t = M(G + U) = \beta M G_t + \epsilon_{i,t} \tag{A171}$$

where G is the vector of identified industry-by-region shocks and M is our estimated generalized multiplier. The strict prediction of our model is that $\beta = 1$, meaning that we have perfectly predicted the heterogeneous effects of the shocks on output growth. Note that the matrix M includes not only heterogeneity in the response to a shock in one's own market, but also how each market will respond to other markets to spillovers arising from spending network effects. Therefore, in addition to testing $\beta = 1$, we also test separately for the existence of spillovers of the nature predicted by the model. More specifically, we run the following regression:

$$dY_t = M(G + U) = \alpha_0 \left(M_{diag}\right) G_t + \alpha_1 \left(M_{offdiag}\right) G_t + \epsilon_{i,t} \tag{A172}$$

where M_{diag} is the diagonal entries of the multiplier matrix (i.e. all other entries are set to 0) and $M_{offdiag}$ are the off-diagonal entries of the multiplier matrix. α_0 captures the degree to which the multiplier accurately captures the effect of a direct shock and α_1 captures the degree to which the model accurately captures the nature of the spillovers across regions and industries.

In the following sections, we will use two different identified shocks for G – state-level military spending shocks from Nakamura and Steinsson (2014) and a growth in industry imports from Autor et al. (2013). Of course, bringing this to the data presents several identification challenges particular to the shock in question. We address the challenges particular to each shock below as we slightly modify Equation A171 to fit the particular setting.

D.1. Government Spending Shocks from Nakamura and Steinsson (2014)

The first shock that we consider is the local government spending shock developed by Nakamura and Steinsson (2014) to estimate local fiscal spending multiplier. We refer the reader to that paper for the details on the construction of the shock. We closely follow their original specification, using data on US states from 1966-2006. We restrict our attention to variation across states and our dependent variable is the 2-year change in state GDP per capita, divided by the level of state GDP lagged 2 periods. The state spending shock is the 2-year change in military spending per capita, also divided by the level of state GDP lagged 2 periods.Specifically, we run the following regression

$$\frac{y_{s,t} - y_{s,t-2}}{y_{s,t-2}} = \beta \frac{(MG)_{s,t} - (MG)_{s,t-2}}{y_{s,t-2}} + \gamma_s + \gamma_t + e_{s,t} \tag{A173}$$

where γ_s and γ_y are state and year fixed effects, respectively. The central concern is that military spending is not random and may be directed towards states based on their economic performance. Therefore, we follow Nakamura and Steinsson (2014) and instrument the state changes in spending with state dummies interacted with national changes in military spending. Table A1 shows the results. First, Column 1 shows the replication of the result in Nakamura and Steinsson (2014), which is the equivalent of imposing that M has 1 on the diagonals but is 0 elsewhere (call this M_1). Column 2 shows the estimate of Equation A173. The estimates are noisy, but two small pieces of evidence suggest that including the multiplier provides a better fit for the data than the simple specification. First, while we cannot reject that the coefficient on either M_1G or MG is 1, the coefficient on MG is closer to 1 than the coefficient on M_1G, suggesting that the heterogeneity embedded in M is getting us closer to capturing all of the variation in the data. Second, the r-squared in Column 2 is slightly higher than that in Column 1. However, the estimates are noisy and largely inconclusive.

The remaining columns of Table A1 show the estimates separating the own and spillover effects as in Equation A172. A finding that the coefficient on the spillover term were positive and close to 1 would suggest that our measure was accurately picking up the experienced spillovers. Here, the estimates are also too noisy to be conclusive.

	Baseline			Robustness		
				No State FE	post-1980	post-1990
State Spending (M_1G)	1.474***					
	(0.373)					
Model Prediction (MG)		1.189***				
		(0.299)				
Model Prediction ($M_{diag}G$)			1.251***	1.166***	1.569**	0.657
			(0.355)	(0.309)	(0.611)	(0.908)
Model Prediction ($M_{nodiag}G$)			-0.145	0.496	-7.112	-8.899
			(3.367)	(3.242)	(5.443)	(9.385)
Constant						
Observations	1989	1989	1989	1989	1377	867
R-Squared	0.316	0.319	0.316	0.309	0.305	0.308

Table A1: Reduced Form Validation: Government Spending from Nakamura and Steinsson (2014)

D.2. Chinese Import Shocks from Autor et al. (2013)

We also explore the predictions of our model using import shocks constructed as in Autor et al. (2013). While the government spending shocks were primarily at the level of the state, import shocks are primarily at the level of the industry. Thus, as in Autor et al. (2013), we construct the state-level exposure to the China shock ($\Delta IP_{s,t}$) using the industry distribution in each state as:

$$\Delta IP_{s,t} = \sum_j \frac{L_{sjt}}{L_{st}} \frac{\Delta \text{Imports}_{j,t}}{L_{i,1991}} \tag{A174}$$

where j is the industry, s is the state, and $Imports_{j,t}$ are the imports from China to the US. Variation across states in import exposure stems from differences across states in their industry distribution. We assume that there are no imports to non-manufacturing industries. Using this measure as our state-level demand shock, we estimate

$$\Delta log Y_{s,t} = \beta_1 M \ \Delta IP_{s,t} + \gamma_s + \gamma_t + \epsilon_{st} \tag{A175}$$

where $Y_{s,t}$ is state GDP and γ_s and γ_t are state and year fixed effects, respectively. We use stacked 5-year changes and utilize data from 1991-2011. The central concern is that imports grow most in areas that are already shrinking or growing, and therefore we instrument the China shock $\Delta IP_{s,t}$ with the imports from China in eight other developed countries as in Autor et al. (2013).

Table D.2 shows the results. Column 1 first shows the baseline estimate where $M = M_1$, where M_1 is a diagonal matrix of ones. As predicted given the results in Autor et al. (2013), states with a larger growth in imports experienced lower GDP growth rates. The following columns test for the ability of our estimated multiplier to predict the magnitude of the effect as well as the direction of the spillovers. The coefficient on M_{nodiag} is generally negative and similar in magnitude to the coefficient on M_{diag}. This means that we generally find that the model correctly predicts the direction of the spillovers. However, the results are too noisy to draw any firm conclusions.

	Baseline			Robustness	
				Excluding GR	Rolling Window
China Shock G)	-1.847**				
	(0.782)				
Model Prediction (MG)		-1.485**			
		(0.596)			
Model Prediction ($M_{diag}G$)			-1.322*	-1.481**	-1.993***
			(0.684)	(0.638)	(0.620)
Model Prediction ($M_{nodiag}G$)			-2.638	-1.819	-2.855
			(1.814)	(1.637)	(1.903)
Constant					
Observations	204	204	204	153	561
R-Squared	0.482	0.481	0.485	0.409	0.485

Table A2: Reduced Form Validation: China Shock from Autor et al. (2013)

E. Additional Tables and Figures

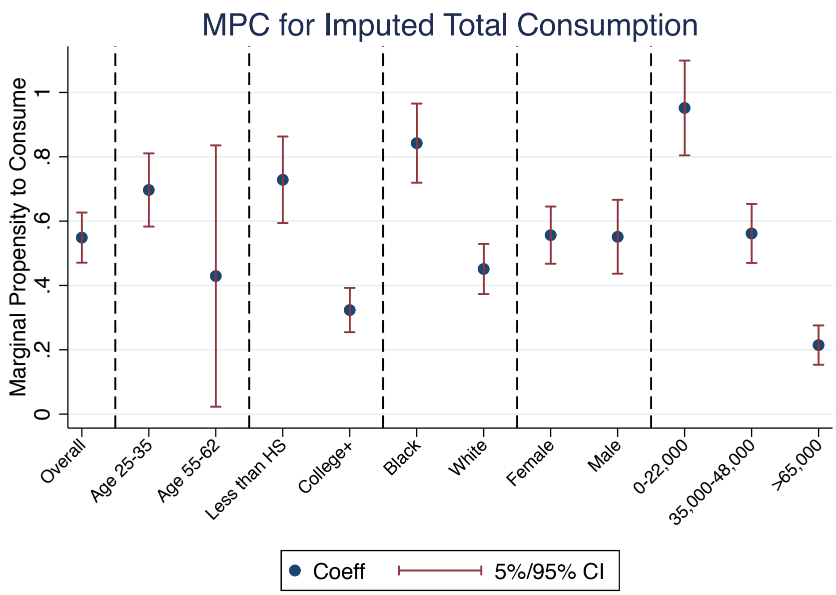

Fig. A1. Heterogeneity in estimated MPCs for total consumption across demographic groups.

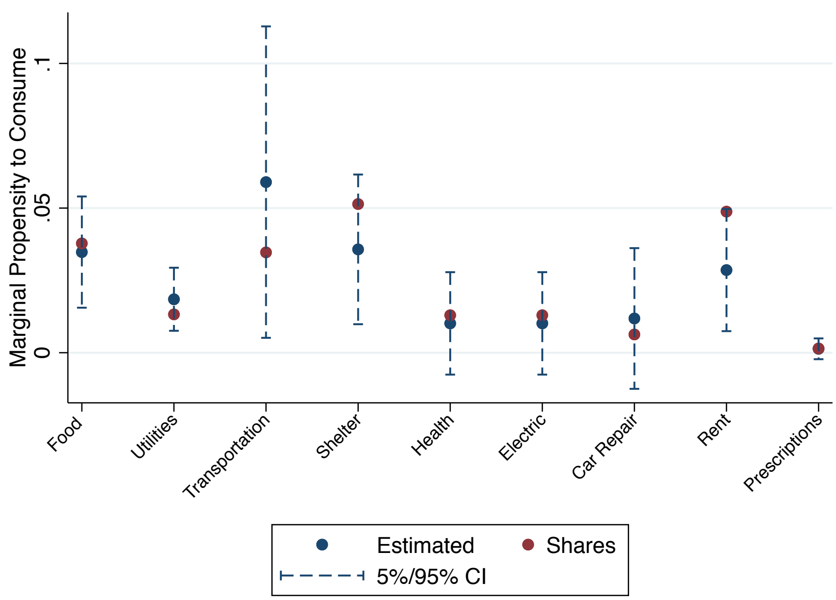

Fig. A2. Estimated Directed MPCs Vs. CEX basket-weighted MPCs

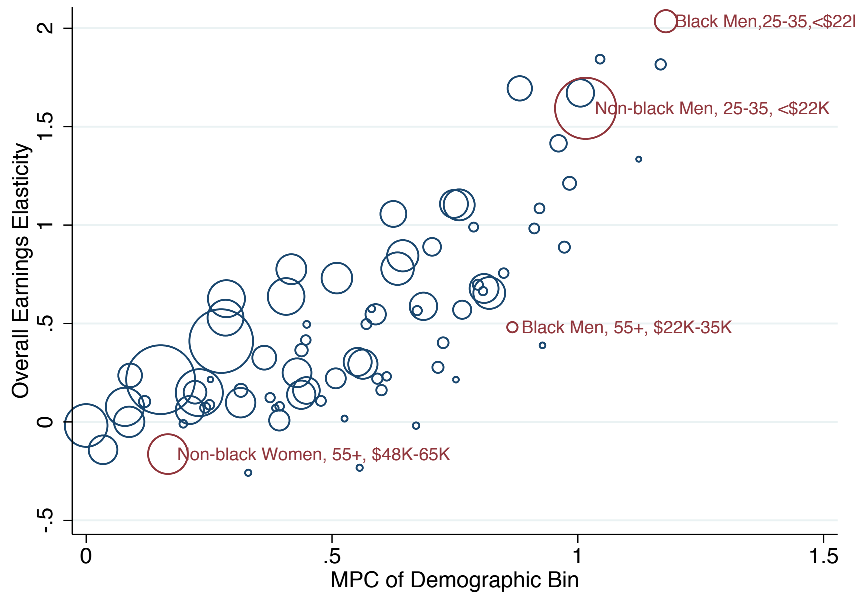

Fig. A3. Earnings elasticity to GDP shocks scattered against estimated MPC. See Patterson (2019) for more details.

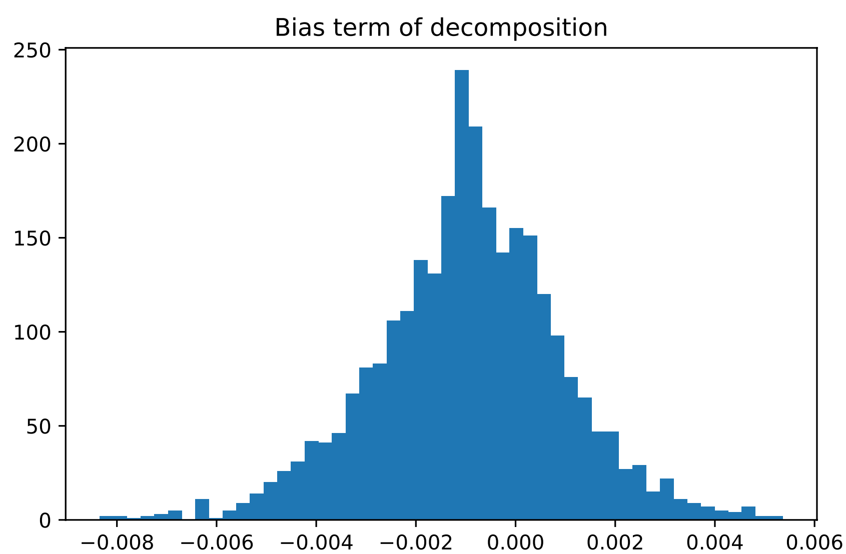

Fig. A4. Histogram of the bias terms from the decomposition in Proposition 2 for each unit demand shock to the 2805 sector-region pairs, with baseline y^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

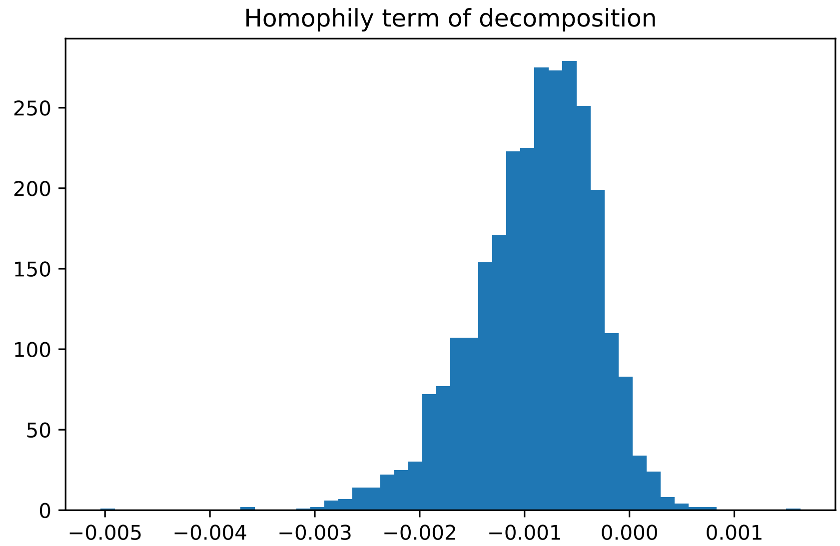

Fig. A5. Histogram of the homophily terms from the decomposition in Proposition 2 for each unit demand shock to the 2805 sector-region pairs, with baseline y^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

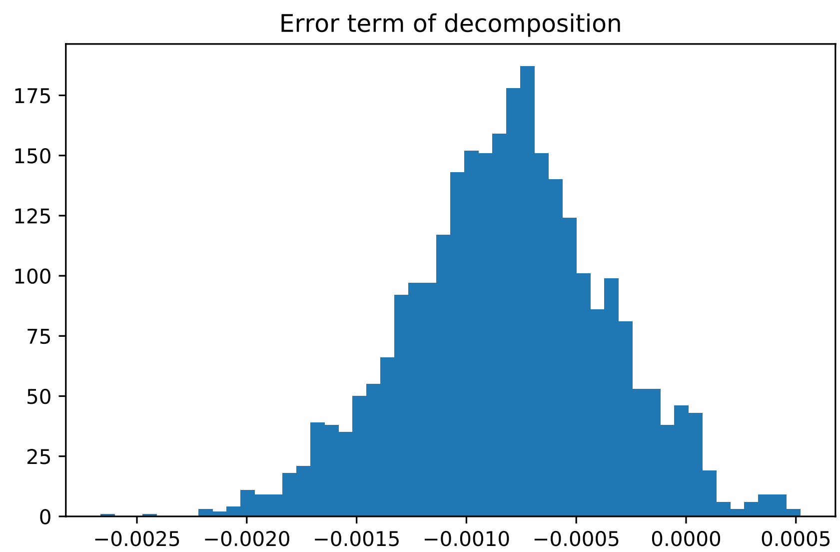

Fig. A6. Histogram of the error terms from the decomposition in Proposition 2 for each unit demand shock to the 2805 sector-region pairs, with baseline y^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

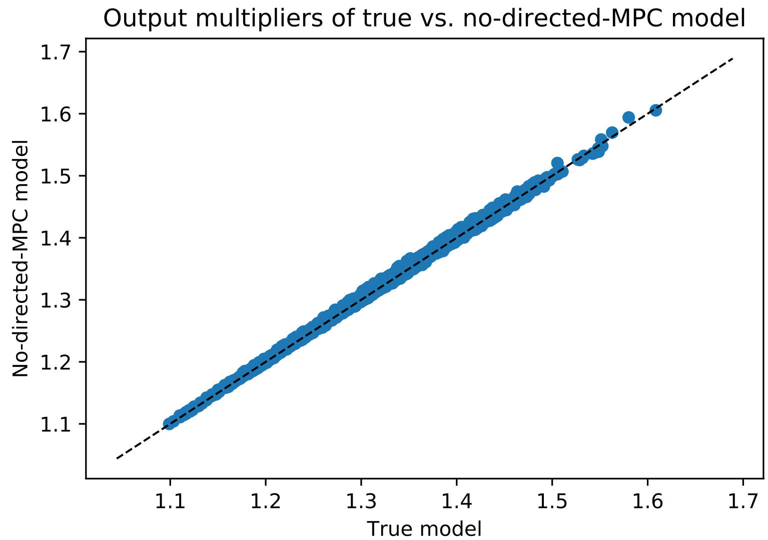

Fig. A7. Scatter plot of output multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which all households have homogeneous consumption baskets in proportion to aggregate consumption (y-axis).

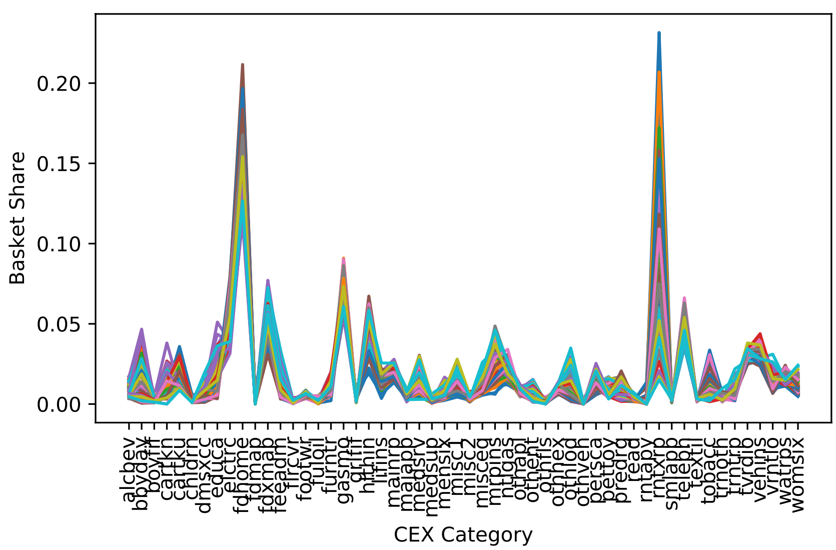

Fig. A8. Consumption basket weights for each demographic group (each line is a demographic group) across each CEX consumption category.

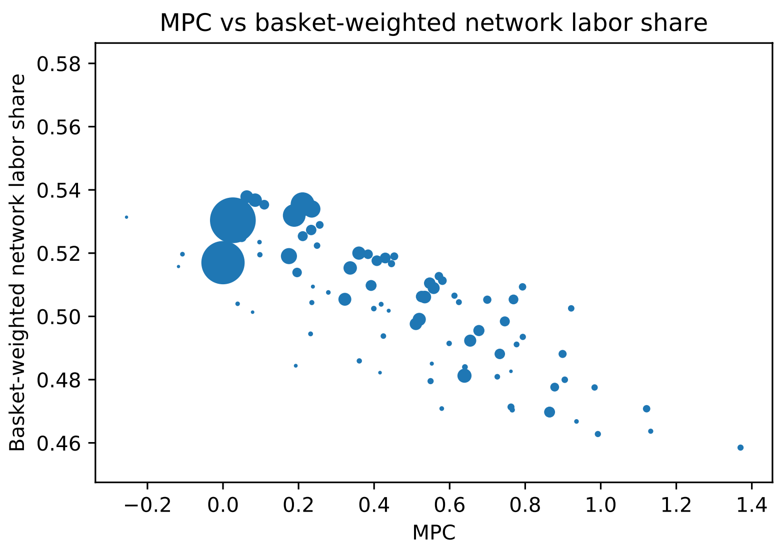

Fig. A9. Scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume.

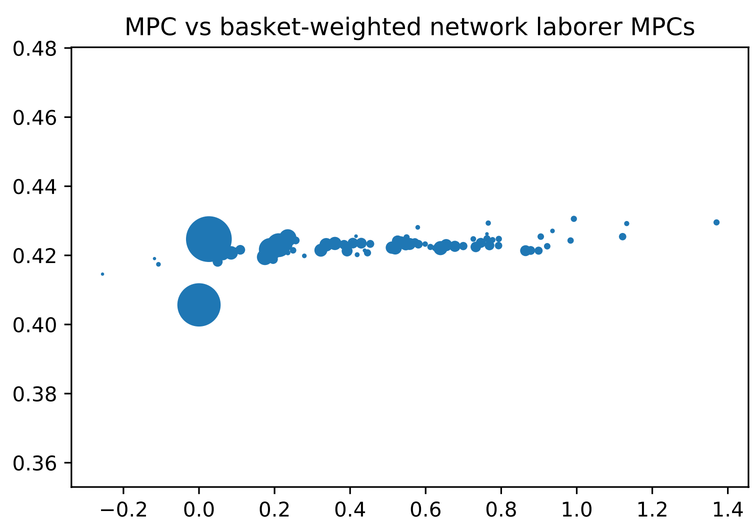

Fig. A10. Scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume.

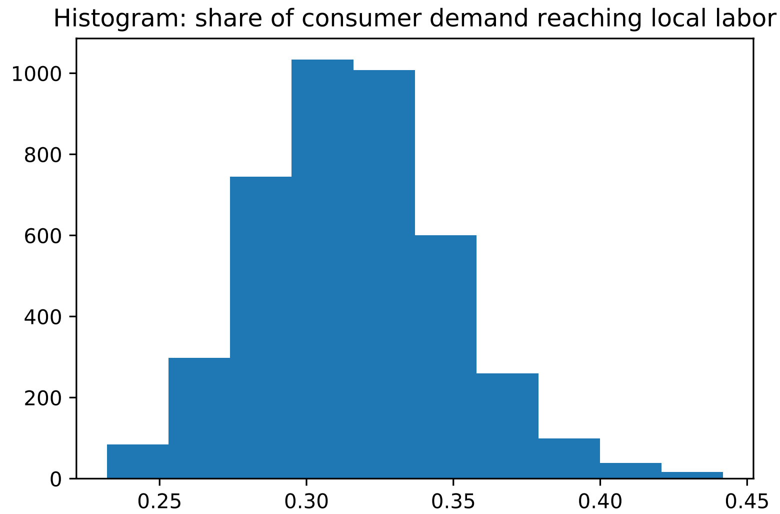

Fig. A11. Histogram of the fraction of consumer demand resulting in income for labor within the same state for each state-demographic pair.

Fig. A12. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the share of income from production that goes directly to labor (as opposed to capital, foreigners, or inputs).

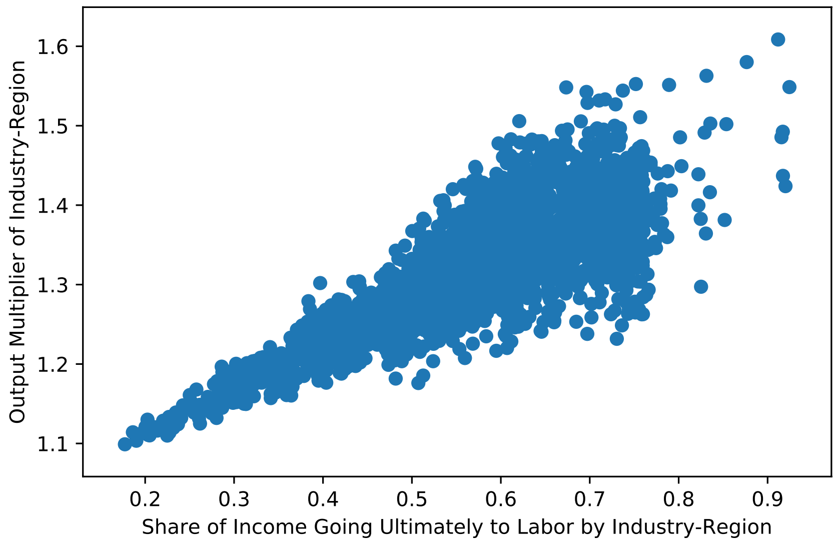

Fig. A13. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the ultimate labor share accounting for labor employed in the production of intermediates.

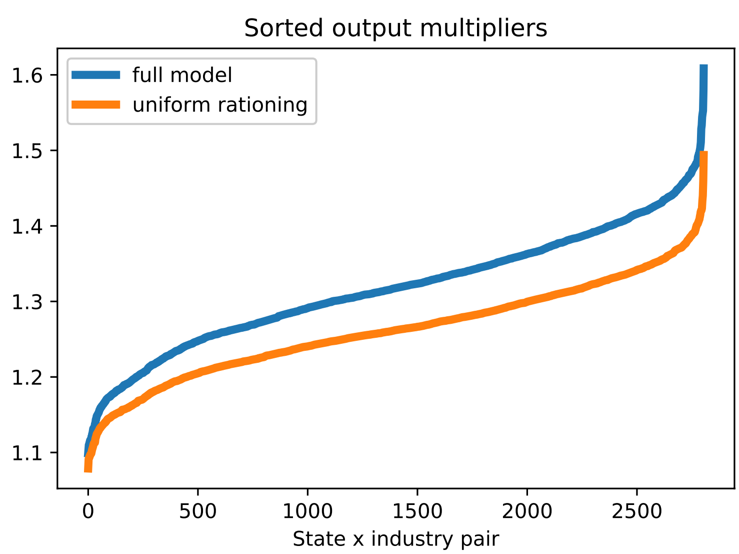

Fig. A14. Sorted change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their labor income.

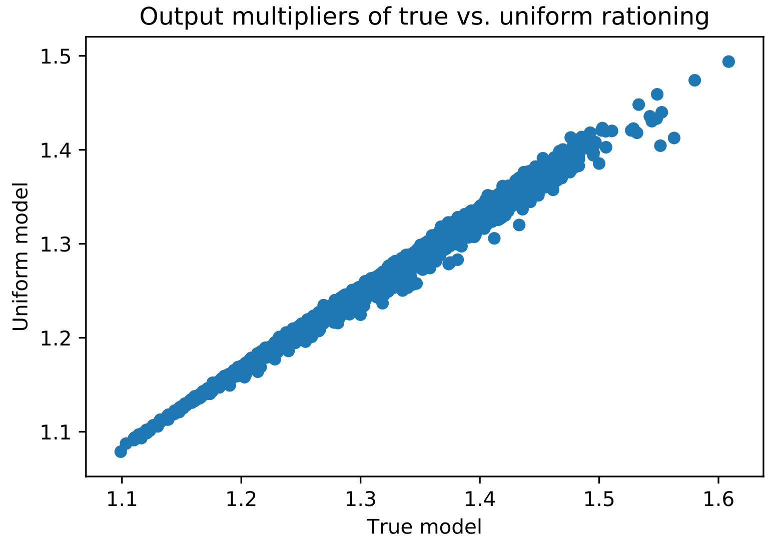

Fig. A15. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their income.

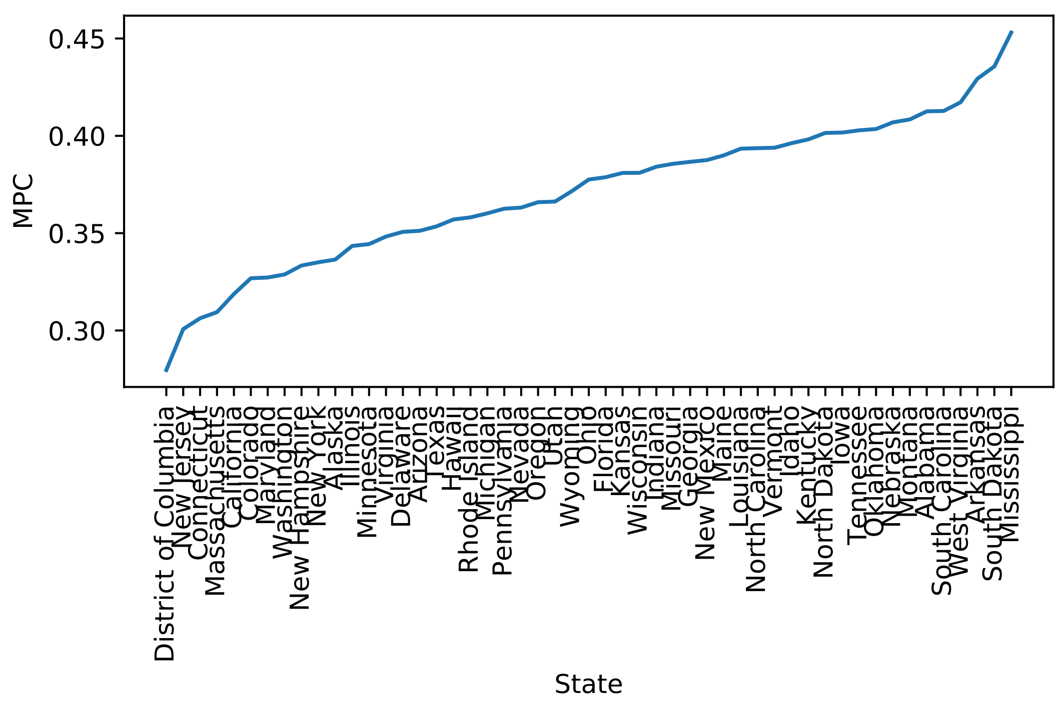

Fig. A16. Income-weighted average MPC by state.

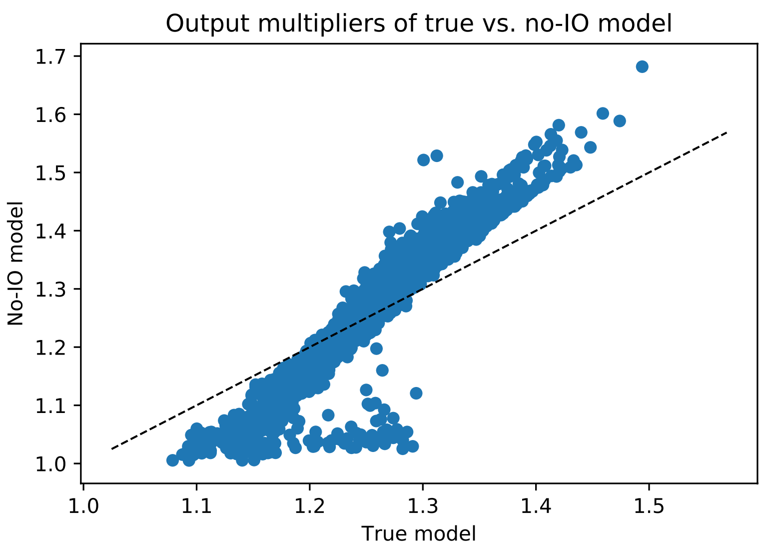

Fig. A17. Scatter plot of output multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which there is no intermediate goods use by firms (y-axis).

Component	Incidence multiplier	Bias	Homophily	Total	Error
Magnitude	1.302	-0.002	-0.001	1.298	0.000

Table A3: Homophily decomposition for a shock to demand proportional to 2012 GDP across sectors. "Incidence multiplier" includes the first and second terms in in Proposition 2. "Bias" is the bias correction and "homophily" is the homophily correction. Error is the difference between the sum of these terms and the exact multiplier.

	No Directed MPC	Directed MPC
Uniform Rationing	1.23	1.23
MPC Rationing	1.28	1.28

Table A4: Multiplier of a GDP-proportional output shock across model specifications. In this table, we eliminate regional structure and instead have 55 industries at the national level. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

	No Directed MPC	Directed MPC
Uniform Rationing	1.25	1.25
MPC Rationing	1.30	1.30

Table A5: Multiplier of a GDP-proportional output shock across model specifications. In this table, everything is as in the baseline except we eliminate regional trade and assume that all consumption and intermediate goods use is within each state. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

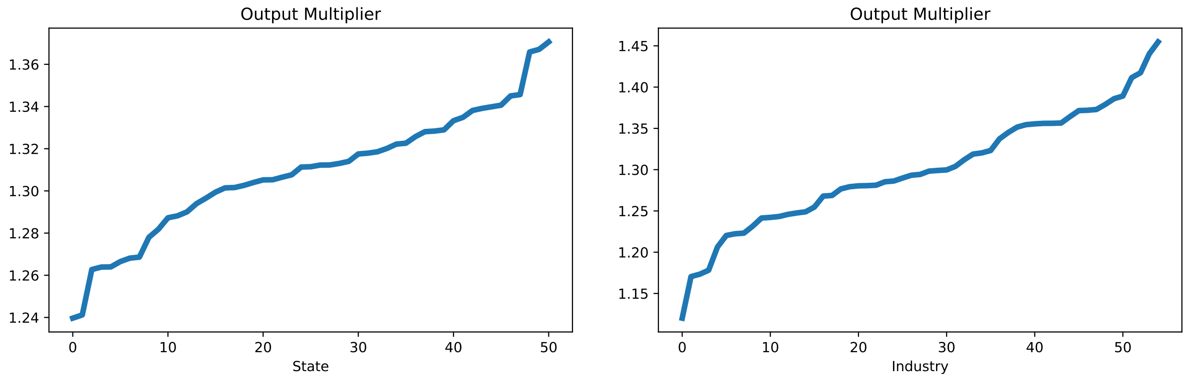

Fig. A18. Multipliers for state-level and industry level shocks. Formally, we take the shock for each state r as $\partial Q_r = \left(\mathbb{I}[s=r]\frac{y_{sj}}{\sum_k y_{rk}}\right)_{sj}$, where y_{rj} is BEA output for sector j in state r and each industry j as $\partial Q_j = \left(\mathbb{I}[k=j]\frac{y_{rk}}{\sum_s y_{sj}}\right)_{rk}$. That is, we marginalize across each dimension according to output shares.

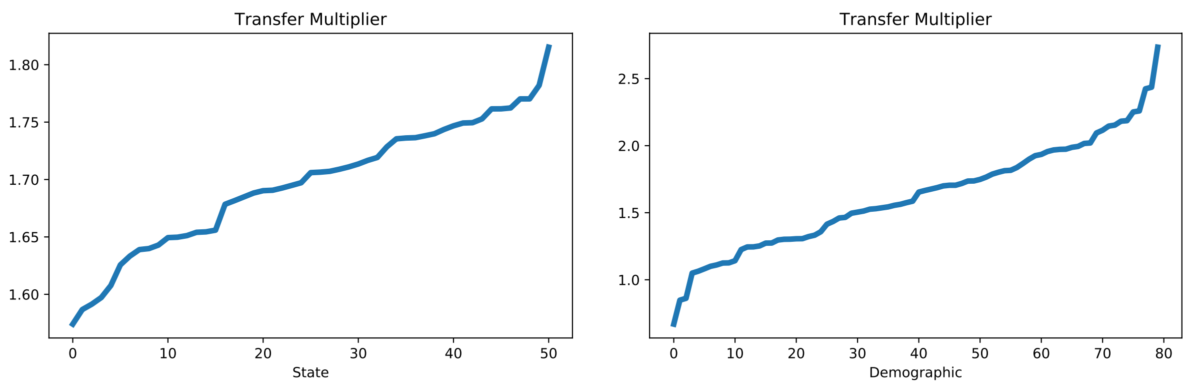

Fig. A19. Multipliers for state-level and demographic level transfer shocks. Formally, for the state-level shock, we transfer each state one dollar, in proportion to the demographic composition of that state. For the demographic-level shock, we transfer each demographic group one dollar, in proportion to the distribution of that demographic across states.

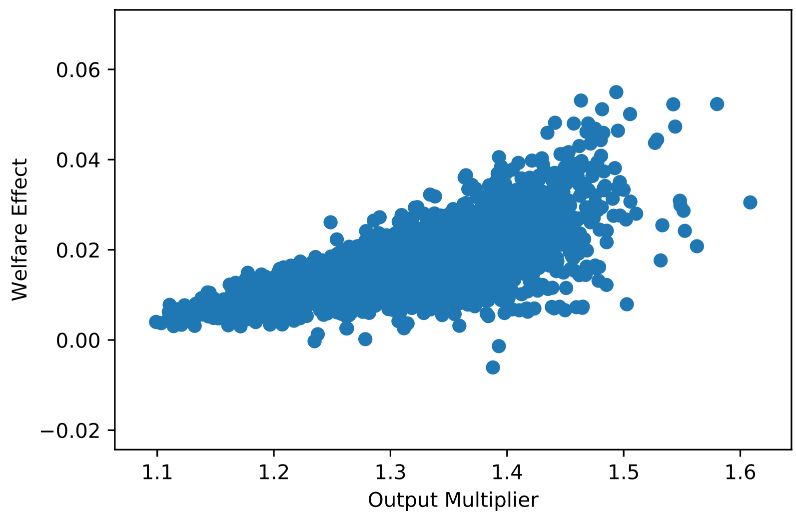

Fig. A20. Comparison of output multipliers and welfare effects of government spending. The x-axis gives the output multiplier for a dollar of government spending targeting each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government spending targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession.

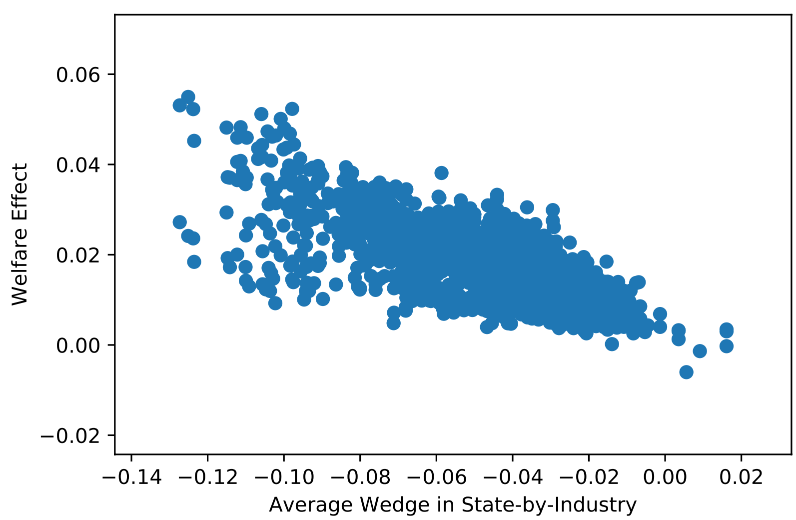

Fig. A21. Comparison of wedges in each state-industry pair and welfare effects of government spending. The x-axis gives the population-weighted Great Recession rationing wedge of employees in each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government spending targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession.

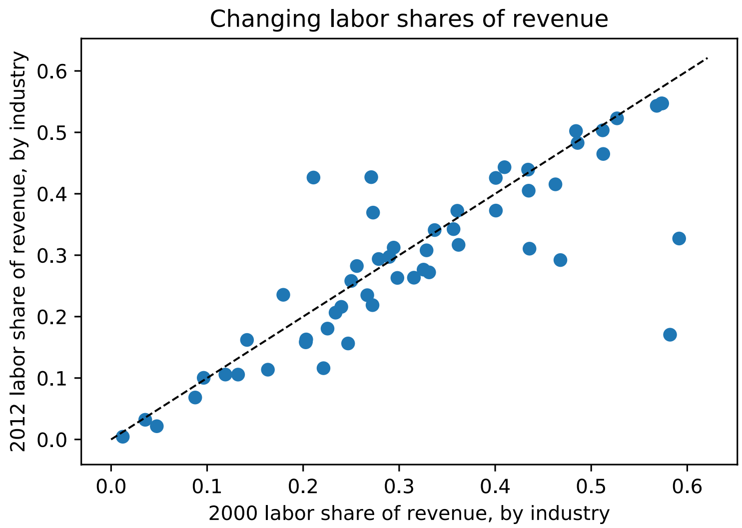

Fig. A22. Labor shares of revenue, by industry, in 2000 vs. 2012. Most industries experience a modest decline in labor share. The most dramatic decline is in the sector labelled "data processing, internet publishing, and other information services." The most dramatic increase is in the sector labelled "apparel and leather and allied products."

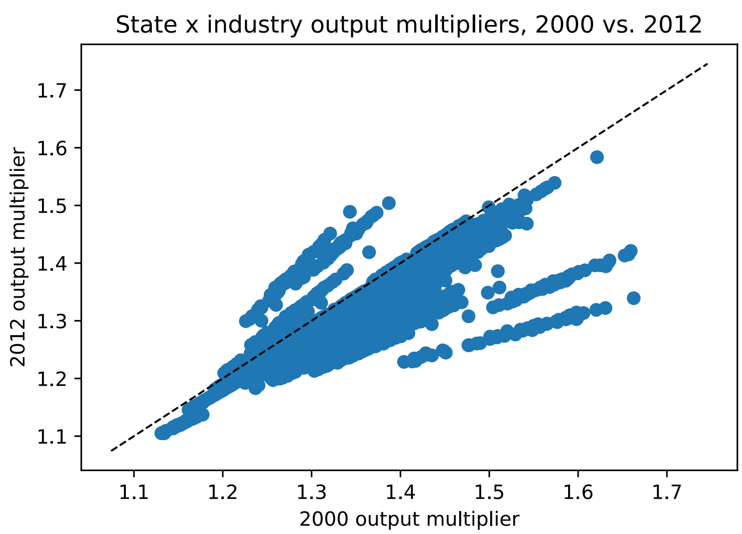

Fig. A23. Scatter plot of output multipliers in 2000 vs. 2012, by state-industry pair.