# Shock Propagation and the Fiscal Multiplier: the Role of Heterogeneity

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**Abstract**

We study how household heterogeneity and regional and industrial linkages affect shock propagation and optimal fiscal policy in a model with short-run labor rationing. In our model, households differ in their income sensitivity to shocks affecting each industry and region, in the magnitudes of their MPCs, and in the industries and regions where they spend their marginal dollars. Theoretically, we express the general equilibrium multiplier of demand and supply shocks in terms of estimable sufficient statistics and provide simple formulae for optimal fiscal policy. Empirically, we take our multiplier to the data and find that, depending on which industries and regions are shocked, amplification differs by a factor of six. This heterogeneity is driven by differences in worker MPC and labor shares, and is attenuated by input-output linkages. In contrast, the network structure of linkages between heterogeneous households is quantitatively unimportant. As a result, fiscal policy that targets households based on their MPCs is close to maximally expansionary and optimal.

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# 1. Introduction

Fiscal stimulus is a primary tool used to address weak or negative output growth. But not all fiscal stimuli are created equal. Practitioners emphasize the "three Ts": fiscal stimulus should be timely, temporary and *targeted* (Elmendorf and Furman, 2008). Just as fiscal stimuli may target particular sectors, regions, or households, so do many shocks outside of policymakers' control: price shocks to key commodities, trade shocks, monetary shocks, financial shocks, deleveraging shocks, etc. Moreover, the initial shock is also amplified through heterogeneous multiplier effects, as workers who lose their jobs cut demand, leading to further job losses, and so on. The magnitude and incidence of these knock-on effects themselves may vary across space, sectors, and time. Such observations suggest two key questions about the ways in which the micro structure of the economy affects the aggregate response to shocks. Descriptively, what shocks are amplified the most, and how does heterogeneity across industries, households and regions affect their propagation? Moreover, how should policymakers design targeted fiscal policy to account for this heterogeneity?

These questions have been the subject of large theoretical and quantitative literatures that variously stress the role of input-output structure (Long and Plosser, 1987; Gabaix, 2011; Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012; Baqaee and Farhi, 2019; Rubbo, 2019), regional linkages and trade (Farhi and Werning, 2017; Caliendo, Parro, Rossi-Hansberg, and Sarte, 2018), and heterogeneous households (Werning, 2015; Kaplan, Moll, and Violante, 2018; Baqaee and Farhi, 2018; Auclert, Rognlie, and Straub, 2018; Auclert, 2019; Bilbiie, 2019). At the same time, a more empirical literature has used both time-series and cross-regional identification strategies to estimate multipliers from fiscal and other shocks (Mian and Sufi, 2011; Ramey, 2011; Nakamura and Steinsson, 2014; Chodorow-Reich, 2014; Mian, Sufi, and Verner, 2017). So far unexplored is how these various forms of heterogeneity intersect to shape amplification and optimal policy.

This paper addresses this question with a semi-structural model that features input-output structure, regional linkages and trade, and heterogeneous households, allowing us to explore the importance of these features for amplification and optimal policy. We provide theoretical results that decompose the various channels of amplification and take theoretically-motivated sufficient statistics to the data to determine the empirically important dimensions of heterogeneity and their consequences for fiscal policy.

Overall, this paper makes five contributions. First, we build a semi-structural model that incorporates rich heterogeneity among and between households and firms. On the household side, we allow for heterogeneity in both the magnitude of households' MPCs and their direction toward different goods. On the firm side, we allow for many sectors and

and regions, linked to one another through an arbitrary input-output structure. Finally, we allow for any pattern of employment of households across the various firms, generating heterogeneous household income processes. Within this rich setting, we study a rationing equilibrium where wages are sticky and thus labor is rationed, meaning that households can lie off their labor supply curves and be involuntary un(der)employed.[1] We derive and study the *multiplier* that maps arbitrary partial equilibrium changes in output corresponding to any demand or supply shock to the resulting, general equilibrium change in output.[2] At the zero lower bound, the multiplier admits a particularly simple representation that we can take directly to the data. Formally, the change in the vector of first period output across industries and regions $dY^1$ in response to the partial equilibrium effect on product demand of any shock $\partial Q^1$ is given by the following expression:[3]

$$\underbrace{dY^1}_{\substack{\text{GE change}\\\text{in output}}} = \left(I - \underbrace{C_{y^1}}_{\substack{\text{Directed}\\\text{MPC matrix}}} \underbrace{l^1_{L^1}\widehat{L}^1}_{\substack{\text{Rationing of}\\\text{labor income}}} \underbrace{\left(I - \widehat{X}^1\right)^{-1}}_{\substack{\text{Leontief-inverse}\\\text{of IO matrix}}}\right)^{-1} \underbrace{\partial Q^1}_{\substack{\text{PE effect}\\\text{on demand}}} \tag{1}$$

where $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ is the heterogeneous analog of the aggregate $MPC$ in the classical multiplier of Keynes (1936). Intuitively, the Leontief-inverse of the input-output matrix maps any change in demand to changes in production of all firms. Next, the rationing matrix maps these changes in production to changes in household labor income. Finally, the directed MPC matrix of households maps these changes in household income to changes in their demand for all goods in the economy. This loop repeats ad infinitum, generating the outer Leontief-inverse.

Our second contribution is to theoretically parse out the mechanisms through which these interconnections amplify or dampen the response of aggregate output to partial equilibrium shocks. We decompose the effects of any shock into three distinct adjustments to the benchmark Keynesian multiplier $\frac{1}{1-MPC}$: incidence, bias, and homophily. The incidence effect accounts for the differential exposure of households' incomes to a partial equilibrium change in output. In partciular, shocks with incidence onto higher-MPC households change output by more. The bias effect accounts for the possibility that households affected by the partial equilibrium shock (through the incidence channel) may disproportionately direct their

[1]We provide general, technical results on the existence of equilibria as well as a no-substitution theorem whereby prices are determined independently of demand.

[2]We can accommodate government spending, tax and transfer, preference, and productivity shocks. At the level of abstraction of our household consumption functions, preference shocks may be interpreted to include deleveraging shocks, liquidity shocks and risk shocks.

[3]Such a formula appears in the regional economics literature on social accounting matrices dating back to Miyazawa (1976). To our knowledge, we provide the first full micro-foundation for such a representation of the multiplier.

spending, in turn, toward goods produced by high-MPC households. Finally, the homophily effect accounts for the covariance between the MPC of households directly affected by the shock and the MPCs of the households producing the goods that directly-affected households buy. The multiplier is larger when high (low) MPC households direct their spending to other high (low) MPC households—for example due to geographic concentration.

Our third contribution is to characterize optimal fiscal policy in this environment. We primarily focus on the case of a planner who seeks solely to mitigate the un(der)employment of households who supply labor inelastically.[4] In this context, the optimal fiscal policy coincides with the aggregate-income-maximizing fiscal policy. Intuitively, when all marginally employed households have no dis-utility of labor, the planner simply aims to direct as much income to them as possible, either through transfers or labor income. Motivated by the potentially immense informational requirements for a planner in a model with such rich shock propagation, we provide a number of results concerning the optimality of simple policies. Strikingly, one may design optimal transfers using only information on household level MPCs. This is always true for an economy initially at the optimum, and is true away from the optimum when the bias and homophily terms are zero.

Our fourth contribution is to rigorously take our model to the data and empirically quantify the importance of heterogeneity in amplifying shocks. To this end, we combine several public-use datasets to estimate three key empirical objects: the regional input-output matrix describing the input-use requirements of every industry-region pair; the rationing matrix describing how much each demographic-region pair's income changes in response to a one dollar change in production of each industry-region pair; and the directed MPC matrix describing how much each demographic-region pair consumes from each industry-region pair. We estimate output multipliers that differ by a factor of six depending on the sector and region that is initially shocked.

While the theoretical results demonstrate that many channels of amplification are possible, the empirical results strongly conclude that the heterogeneous incidence of the shock is the only quantitatively relevant dimension of heterogeneity. In particular, we document that for all feasible demand shocks, the spending network effects captured by the bias and homophily effects are very close to zero. As a result, aggregate amplification can be understood precisely through the incidence of shocks onto households' MPCs. Dispersion in this incidence across states and industries is increased by cross-industry differences in labor shares and average worker-MPCs, as well as heterogeneity in the exposure of different workers' incomes within firms. Conversely, it is dampened by input-output linkages, which effectively mix together heterogeneous industries. In this context, we also show that the change

[4] In appendix B.7, we show that the results below also generalize to an environment with markups.

in output owing to a GDP proportional shock, or *aggregate multiplier*, is not sensitive to most of these modelling features. Still, the heterogeneous MPCs and differential exposure of higher-MPC agents to the business cycle as studied by Patterson (2019) is quantitatively important and increases amplification by 25%.

Our calibration also allows us to quantify the extent of geographic spillovers in shock amplification. We find that 47% of shock amplification comes from spillover effects across states. Recognizing that the multiplier is an endogenous object which may change through time, we explore the implications of the fall in the labor share from 2000 to 2012. We find that it reduces the amplification of most output shocks and the fiscal multiplier from government expenditures, but hardly affects the multiplier on direct transfers to households.

Our final contribution is to connect our empirical findings with earlier results on optimal fiscal policy. Owing to the empirical unimportance of spending network effects, MPC targeting is optimal for a planner seeking to address capacity under-utilization. Our empirical estimates of multiplier heterogeneity imply large gains from targeting government expenditures according to the MPCs of affected workers; differences in worker MPCs imply even larger gains from targeting transfers, since transfers target MPC directly. Whereas MPC targeting is simple for transfers, targeting government expenditures requires knowledge of the input-output network and labor rationing process, both of which shape how income is directed from firms to workers, as in Baqaee (2015). Naive targeting according to the MPC of workers in each industry-region pair is moderately effective but leaves substantial gains on the table.

## *1.1. Relation to the Existing Literature*

This paper relates to both the theoretical and empirical literatures on shock propagation and fiscal multipliers. Theoretically, our model unifies a range of elements treated separately in the existing literature. First, we incorporate heterogeneity in household MPCs; this implies that the incidence of shocks has implications for the spending it induces, as emphasized by Kaplan et al. (2018) and Auclert (2019). Second, our model features many goods and allows for heterogeneity across households in the *direction* of MPCs toward these goods. Insofar as these goods are spatially indexed, this connects to papers on cross-regional aggregate demand externalities such as Corsetti, Kuester, and Müller (2013) and Farhi and Werning (2017). Third, we allow for an input-output network whereby each good uses others as inputs. Baqaee (2015) focuses on this channel, emphasizing, as we do, that shocks to an industry affect not only the factors employed in that industry but also those used in producing its inputs, motivating a "network adjustment" to the labor share of each industry. Fourth, we allow each firm to employ different workers and to ration labor to them disproportionately, allowing for heterogeneity in income cyclicality as studied theoretically in Werning (2015) and documented empirically in Patterson (2019) and Guvenen and Smith (2014). While each of these papers focuses on one or two dimensions of heterogeneity, we integrate them in a model rich enough to bring to the data through sufficient statistics. This more reduced-form approach is similar methodologically to Auclert et al. (2018), Wolf (2019), and Koby and Wolf (2019), who each adopt sufficient statistics approaches in their different contexts.

Our paper is not the first to incorporate spending, input-output, and employment linkages between individuals and regions. In fact, this approach dates back to the much earlier regional accounting literature which emphasized how demand may spill over across regions (Miyazawa, 1976). We micro-found this literature's focus on fixed prices in an environment with a single factor, sticky wages, and a binding zero lower bound. In recent work, Baqaee and Farhi (2018) develop rich macroeconomic models featuring these channels as well as endogenous prices and markups. At the level of generality of their approach, it is hard to disentangle the role that various modelling elements have in shaping amplification. By contrast, we abstract away from price movements but are able to precisely characterize the channels through which economic linkages affect aggregate shock propagation and how these matter for optimal stimulus policy. Relatedly, Zorzi (2020) studies the interaction of cyclicality in durable consumption and investment with sector-specific employment in a parametric environment. This paper and ours are related insofar as they involve the interaction of directed demand and heterogeneous labor rationing. However, we abstract away from the specific microfoundation of directed demand and take a more reduced form approach that emphasizes richer connections between households and firms.

The theoretical results in this paper also relate to the literature on optimal fiscal policy at a zero lower bound. As in Werning (2011), we decompose the planner's motive for government spending into "opportunistic spending" and "stimulus spending." In our model, fiscal stimulus propagates through rich economic networks, nesting Baqaee (2015). From a theoretical point of view, our main contribution is to provide a set of assumptions under which a planner need not know the network linking each household's spending to other households' income in order to evaluate the optimality of a stimulus package. Moreover, our empirical findings imply that a planner motivated purely by factor under-utilization need not know this network, even away from the optimal policy. The marriage of our theoretical results and empirical findings suggests both that fiscal stimulus targeting MPCs is optimal and that there are large gains from such targeting. This echoes the empirical finding that MPCs are heterogeneous in the population and the accompanying argument that stimulus packages directing money to high-MPC households are therefore more effective (Parker, Souleles, Johnson, and McClelland, 2013; Broda and Parker, 2014; Kaplan and Violante, 2014; Jappelli and Pistaferri, 2014; Carroll, Slacalek, Tokuoka, and White, 2017).

Lastly, this paper also connects to a large empirical literature estimating multipliers from fiscal shocks. On one end of the literature, our estimates complement empirical estimates of open-economy multipliers. Our aggregate multiplier calibration of 1.30 is somewhat smaller, but within the established confidence intervals of those in Ramey (2011), Nakamura and Steinsson (2014), Chodorow-Reich (2019) and Corbi, Papaioannou, and Surico (2019). A major challenge in this literature is that SVAR strategies are methodologically unsuited for capturing heterogeneity and regional identification strategies are generally underpowered to capture heterogeneity in multipliers and isolate the importance of several distinct channels. Our semi-structural empirical strategy is therefore complementary to this literature, using minimal theoretical structure to quantify the role of heterogeneity, while remaining consistent with the valuable reduced-form evidence provided by this literature. The more recent empirical literature on fiscal multipliers estimates spillovers from demand shocks directly using finer geographical and sectoral data. For example, Feyrer, Mansur, and Sacerdote (2017) document geographical spillovers in demand from counties with increased fracking production onto nearby regions. Auerbach, Gorodnichenko, and Murphy (2020) leverage rich data on Department of Defense contracts, finding reduced form evidence for both back-propagation of demand through supply chains and increased demand in other industries through income multipliers. Theoretically, we provide a framework consistent with the evidence presented in these papers and provide more structural estimates of the importance of spillovers in determining the aggregate response to sectoral shocks.[5]

The rest of the paper proceeds as follows. Section 2 introduces the model and studies properties of the rationing equilibrium. Section 3 derives the multiplier and provides our main decomposition results characterizing the role of heterogeneity. Section 4 studies optimal fiscal policy, providing conditions for MPC targeting to be optimal. Section 5 introduces the data and methodology we use to estimate the multiplier. Section 6 provides our empirical results regarding the role of heterogeneity in shock propagation and targeting fiscal stimulus. Section 7 concludes.

# 2. The Model and Rationing Equilibrium

To explore shock propagation and optimal policy, we build a semi-structural model. Our goal is to develop a setting that is rich enough to capture many dimensions of household,

[5] Cox, Müller, Pasten, Schoenle, and Weber (2019) use the same procurement data and account for heterogeneity in price stickiness across sectors subject to fiscal shocks, which lies outside of our framework.

industrial, and regional heterogeneity, but general enough to deliver equations that we can bring directly to the data. In the model, a continuum of heterogeneous households interact in a competitive multi-sector, multi-region economy over two periods. We consider a rich class of household-level consumption and labor supply functions that accommodate arbitrary preference heterogeneity, household borrowing constraints, and most behavioral frictions, as well as a rich, constant returns to scale input-output structure. We consider a rationing equilibrium, where first period wages are fixed and first period labor supply is determined by exogenous rationing functions rather than by household optimization. This allows households to lie off their labor supply curves and thus enables the model to capture classical involuntary unemployment. In Appendix B, we extend this analysis to an arbitrary number of time periods and imperfect competition with fixed markups, and we compare our rationing equilibrium to a flexible-wage equilibrium.

## *2.1. Model Primitives*

Time is indexed by $t \in \{1,2\}$. There is a finite set of goods $\mathcal{I}^t$ in each period, each of which is produced by one representative firm $j$ using intermediates $X_j^t = (X_{j1}^t, ..., X_{j|\mathcal{I}^t|}^t) \geqslant 0$, labor $L_j^t \geqslant 0$ and a production technology $F(X_j^t, L_j^t, z_j^t)$ that is CRS in inputs and labor, where $z_j^t$ is a vector of parameters that determine the production function. In each period, firms take prices $p^t$ and wages $w^t$ as given and maximize profits. We normalize the wage to one within every period, i.e. $w^t = 1$; intertemporal price comparisons are possible via the real interest rate $r^1$. Firms choose labor and intermediate inputs to maximize profits in each period:

$$\begin{aligned} p_i^t F_L(X_i^t, L_i^t, z_i^t) &= 1 \\ p_i^t F_{X_j}(X_i^t, L_i^t, z_i^t) &= p_j^t \end{aligned} \tag{2}$$

There is a continuum of households on the interval $[0,1]$ indexed by $i$, and a finite set of types $N$, where each type $n \in N$ has mass $\mu_n > 0$ such that $\sum_{n \in N} \mu_n = 1$. Households consume a vector of goods $c_n^t = \{c_{ni}^t\}_{i \in \mathcal{I}^t}$ in each period $t$, and they save an amount $s_n^1$ between periods at a real rate $1 + r^1$; households have no initial savings or debt.[6] Each household $n$ supplies labor $l_{ni}^t$ to each firm $i$ in period $t$, totalling $l_n^t = \sum_i l_{ni}^t$. Household $n$ therefore has labor income $y_n^t = l_n^t$ in period $t$. Rather than explicitly microfounding households' decision problems, we simply assume there exist exogenous functions that describe their consumption and labor supply as a function of variables outside their control (see Section

[6] This is without loss, as we can replace initial debt between agents with heterogeneous lump-sum taxes and transfers.

2.2). This allows us to nest non-homothetic preferences, behavioural frictions and borrowing constraints. Households always satisfy their lifetime budget constraint:

$$l_n^1 + \frac{l_n^2}{1+r^1} = p^1 c_n^1 + \frac{p^2 c_n^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \tag{3}$$

The government levies (possibly negative) lump-sum taxes $\tau_n^t$ on households, and it buys $G_i^t$ units of good $i \in \mathcal{I}^t$ subject to running a balanced budget over the two periods. To finance its fiscal spending and tax/transfer programs, the government issues bonds at a real interest rate of $r^1$ in the first period. The real lifetime government budget constraint is therefore

$$\sum_{n \in N} \mu_n \left( \tau_n^1 + \frac{1}{1+r^1} \tau_n^2 \right) = p^1 G_i^1 + \frac{1}{1+r^1} p^2 G_i^2 \tag{4}$$

So that the government budget constraint continues to hold when prices or the interest rate changes, we assume expenditures are given by an exogenously specified function of real prices, taxes, and a government spending preference parameter $\theta_G$. In particular, $G^t = G^t\left(\varrho, (\tau_n)_{n \in N}, \theta_G\right)$, where $\varrho$ is the price vector $(p^1, p^2, r^1)$ and we assume $G^t(\cdot)$ is such that Equation 4 always holds. Finally we assume $G^t(\cdot)$ is continuous in $r^1$.

## *2.2. Rationing Equilibrium*

We consider a sticky-wage rationing equilibrium. In this equilibrium concept, first-period wages are exogenously fixed and, consequently, first-period labor is rationed, rather than determined by household optimization. Such an equilibrium notion corresponds well to an environment with wage rigidity of the kind commonly observed in the data (Solon, Barsky, and Parker, 1994; Grigsby, Hurst, and Yildirmaz, 2019; Hazell and Taska, 2019).[7] This concept allows us to capture classical involuntary unemployment, *i.e.* in economic downturns there are households who would like to work but cannot because firms are unwilling to hire them. Firms, in turn, are unwilling to hire because they cannot sell more without reducing prices, which they cannot do since wages are fixed. We assume that the same, fixed wage also applies to new hires, so that firms cannot simply fire existing workers and hire under-employed households at lower wages. Fundamentally, this narrative is about households lying off their labor supply curves, which we capture by assuming that in the short run, households do not choose their labor supply but rather have it rationed to them.

Formally, first period labor is determined by a differentiable rationing function that maps the vector of labor demands $(L_i^1)_{i \in \mathcal{I}^1}$ to a vector of total labor supplied by each household

[7] To the extent that wages are rigid across them, our model can accommodate an arbitrary number of different types of labor.

type $l^1\big((L_i^1)_{i\in\mathcal{I}^1}\big)$. The rationing function treats all households within each type identically, and is such that the labor market clears:

$$\vec{1}\cdot l^1\big((L_i^1)_{i\in\mathcal{I}^1}\big) = \sum_{i\in\mathcal{I}^1} L_i^1 \tag{5}$$

At this level of generality, our model accommodates arbitrary rationing across household types. For example, it allows for heterogeneous migration by households across regions or sectors. In the second period, households choose their second-period labor supply and the prices of all goods and wages are set so that all markets clear. We model this behavior by assuming that households take not only prices but also first period labor income as given, while allowing consumption $c_n^t(\varrho, y_n^1, \tau_n, \theta_n)$ and second period labor supply $l_n^2(\varrho, y_n^1, \tau_n, \theta_n)$ to be given by arbitrary functions of prices, first-period income, taxes, and a preferences parameter $\theta_n$.

All other markets clear in the usual fashion:

$$F(X_i^t, L_i^t, z_i^t) = \sum_{j\in\mathcal{I}^t} X_{ji}^t + \sum_{n\in N}\mu_n c_{ni}^t + G_i^t, \qquad \sum_{i\in\mathcal{I}^2} L_i^2 = \sum_{n\in N}\mu_n l_n^2 \tag{6}$$

We assume that the nominal interest rate set by the central bank directly pins down the real interest rate that enters into both the government and household budget constraints. We therefore suppose that the central bank sets real interest rates directly, potentially as a function of output of any good in any period:[8]

$$r^1 = r^1(Q) \tag{7}$$

We assume that interest rates have an upper and lower bound, i.e. $r^1(Q)\in[\underline{r},\bar{r}]$ for some $\underline{r},\bar{r}\in\mathbb{R}$, and that $r$ is differentiable.

A rationing equilibrium is therefore defined as follows:

**Definition 1.** *A rationing equilibrium is a set of first and second period, agent- and market-level variables* $\{s_n^1, \{c_{ni}^t, l_n^t\}_{t\in\{1,2\},i\in\mathcal{I}^t}\}_{n\in N}$ *and* $\{r_i^1, \{p_i^t, \{X_{ij}^t\}_{j\in\mathcal{I}^t}, L_i^t, C_i^t, G_i^t\}_{t\in\{1,2\}}\}_{i\in\mathcal{I}^t}$ *that satisfy conditions (2) – (7) given initial conditions.*

In appendix B.3, we compare the rationing-equilibrium to a benchmark model with flexible price. In that setting, the interest rate moves in the first period to clear the labor market while workers remain on their labor supply curves. Household MPCs play no direct part in determining the response of output to a demand shock.

[8]This specification nests Taylor rules that condition on both sector-level output and inflation as well as money supply targeting via a quantity theory.

### 2.3. Equilibrium Properties

Before proceeding with the analysis of the multiplier, we ensure that the problem is well-posed and eliminate any nuisance terms that unnecessarily complicate the analysis. To this end, we first provide a no-substitution theorem that ensures prices are technologically determined – and thus independent of demand – and, second, prove the existence of a rationing equilibrium.

The following technical conditions on production technologies and household preferences are sufficient for the no-substitution theorem. Assumption 1 provides basic technical conditions on production and Assumption 2 imposes a simple positivity condition on demand such that there is demand for all goods.

**Assumption 1.** *For all $i$ and $z_i$, production $F(X_i, L_i, z_i)$ is continuous, weakly increasing, strictly quasi-concave, and homogeneous of degree one in $(X_i, L_i)$. Further, labor is essential in production, i.e. $F(X_i, 0, z_i) = 0$, and production is strictly increasing in labor. Finally, there exists some $\overline{p} \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i, L_i\}_{i \in \mathcal{I}^t}$ such that for all $i$, $F(X_i, L_i, z_i) \geqslant 1$ and $\overline{p}X_i + L_i \leqslant \overline{p}_i$.*[9]

**Assumption 2.** *For any $\varrho, y^1, \tau, \theta$: for each good $i$ there is a household type $n$ for which $c_{ni}^t > 0$.*

Under these two rather weak assumptions, we can show that:

**Proposition 1.** *Under Assumptions 1 and 2, for a given $z^t$, there exists a unique $p^t$ consistent with rationing equilibrium, independent of demand.*

*Proof.* See Appendix A.1. □

Proposition 1 establishes that, under Assumptions 1 and 2, a no-substitution theorem holds: given $(z^1, z^2)$, there exist unique, positive prices $p^1(z^1), p^2(z^2) \in \mathbb{R}_+^{\mathcal{I}^t}$ consistent with equilibrium. This result allows us to reduce the number of endogenous price variables in considering comparative statics that keep $z^1$ and $z^2$ fixed, allowing us to keep track of just the real interest rate. Implicit in this no-substitution economy is the assumption that good prices respond instantaneously to changes in technology, which is irrelevant in the case of demand shocks.

Moreover, Proposition 1 also implies a simple form for aggregate input demands $X^t(p^t, Q^t)$ in equilibrium. In particular, for any technology $z$, we define the equilibrium unit input demands as:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i, L_i) \text{ s.t. } F(X_i, L_i, z_i) \geqslant 1} p(z)X_i + L_i \tag{8}$$

[9] A sufficient but not necessary condition is that every good can be produced using only labor.

Constant returns to scale imply that aggregate input and labor demands are simply a scaling of these unit input demands. Formally:

**Corollary 1.** *The aggregate input demand $X^t(p^t, Q^t)$ and labor demand $L^t(p^t, Q^t)$ vectors are given by:*

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{9}$$

*where $\widehat{X}(z^t)$ is the matrix with $i^{th}$ column $\widehat{X}_i(z^t)$ and $\widehat{L}(z^t)$ is the diagonal matrix with $i^{th}$ entry $\widehat{L}_i(z^t)$.*

*Proof.* See Appendix A.2. □

Proposition 1 implies two additional, useful results. First, the Leontief-inverse matrix always exists. Second, one can use the Leontief-inverse to obtain a useful closed-form expression for the demand-independent prices. This is stated formally in the following corollary:

**Corollary 2.** *The Leontief-inverse matrix $(I - \hat{X}(z))^{-1}$ exists. Moreover, prices are given uniquely by the following expression:*

$$p(z) = (I - \hat{X}(z)^T)^{-1}\hat{L}(z)\vec{1} \tag{10}$$

*Proof.* See Appendix A.3. □

Given the above simplifications, throughout the paper we will write $\widehat{X}^t, \widehat{L}^t$ for $\widehat{X}(z^t), \widehat{L}(z^t)$ when $z^t$ is fixed. We write $\widehat{X}$ and $\widehat{L}$ for the block-diagonal matrices composed of $\widehat{X}^1$ and $\widehat{X}^2$, and $\widehat{L}^1$ and $\widehat{L}^2$ respectively.

Having simplified the structure of the model, we proceed to establish that the analysis of equilibrium is well posed by providing regularity conditions under which equilibria exist. To this end, we assume basic continuity properties of demand and that household consumption in the first period is bounded away from fully consuming first period income as income grows large.

**Assumption 3.** *The primitives satisfy the following properties:*

1. *The consumption and labor functions $c_n^t$ and $l_n^1$ are continuous in $r^1$ and $y^1$.*
2. *For all $n, \varrho, \tau_n, \theta_n$, we have that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in $y_n^1$.*
3. *For any $p, \tau, \theta$: there exists $\overline{y} \in \mathbb{R}_+$ and $\overline{c} < 1$ such that for all $n \in N$, $r^1 \in [\underline{r}, \overline{r}]$, and $y_n^1 > \overline{y}$, we have that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$.*

This assumption is extremely mild and satisfied by virtually all standard household problems of which we are aware.[10] With this additional structure we are now able to prove the existence of rationing equilibria for the economy under consideration.

**Proposition 2.** *Under assumptions 1, 2, and 3, there exists a rationing equilibrium.*

*Proof.* See Appendix A.4. □

As we have established conditions under which an equilibrium exists, our analysis of equilibria going forward will be well-posed. While the fixed-point theorems we use are familiar, we employ a somewhat different strategy to usual existence proofs in (i) leveraging the structure of no-substitution and (ii) clearing markets intertemporally and then constructing intratemporal market clearing from the resulting fixed point interest rate. This provides a common structure to both rationing equilibrium and flexible-wage equilibrium (see Appendix B.3) existence and may be useful to other authors proving equilibrium existence in economies with labor rationing.

# 3. The Multiplier

Within the setting outlined in Section 2, we explore the general equilibrium impact of supply and demand shocks. Our goal is to derive an expression for the generalized multiplier that we can take to the data to quantify the role of heterogeneity in shock propagation and the optimal fiscal policy.

## *3.1. The Partial Equilibrium Response to Shocks*

Our main results will express the economy's general equilibrium responses to shocks to exogenous parameters as a function of their partial equilibrium effect on goods demand. The partial equilibrium effect on good demand $\partial Q$ is the change in output in response to a shock before prices or incomes have been allowed to adjust. The demand and supply shocks that we consider in our setting are changes in government spending, taxes and transfers, preferences, and technology.

We begin by parameterizing aggregate demand. Recognizing that each household's decisions depend only on real quantities, we can represent type $n \in N$'s Marshallian demand

[10] It is easy to see how Assumption 3 holds if households are utility maximizers whose utility functions satisfy various standard assumptions. Existence and continuity of the consumption and labor functions follow from continuity and quasiconcavity of utility, and from Berge's theorem. Satisfying the lifetime budget constraint follows from non-satiation. Consumption being asymptotically bounded away from first-period income follows from sufficiently decreasing marginal utility.

for good $j \in \mathcal{I}^t$ at time $t \in \{1,2\}$ as $c^t_{nj}(y^1_n, \varrho, \tau_n, \theta_n)$, where $\varrho = (p^1, p^2, r)$, and $\tau_n = (\tau^1_n, \tau^2_n)$. Aggregate consumption demand $C^t_j$ is then given by:

$$C^t_j(\varrho, \tau, \theta) = \sum_{n \in N} \mu_n \; c^t_{nj}(y^1_n, \varrho, \tau_n, \theta_n) \tag{11}$$

where $\theta = (\theta_1, ..., \theta_N)$ and so forth.

To find the partial equilibrium effect of each type of shock, we totally differentiate the goods market clearing condition, given by

$$Q^t = \widehat{X}^t Q^t + C^t + G^t \tag{12}$$

We then collect the terms corresponding to changes in demand for goods before accounting for the endogenous response of interest rates and income and for the higher-order effects those responses generate. Doing so yields the following partial equilibrium effect of each shock:[11]

**Proposition 3.** *The following shocks have partial equilibrium effects on aggregate demand given by:*

1. *A change in government preferences $\theta_G$ by $d\theta_G$:*

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G) d\theta_G \tag{13}$$

2. *A change in household preferences $\theta$ by $d\theta$:*

$$\partial Q = C_\theta(\varrho, \tau, \theta) d\theta \tag{14}$$

3. *A change in taxes or transfers by $d\tau$:*

$$\partial Q = C_\tau(\varrho, \tau, \theta) d\tau + G_\tau(\varrho, \tau, \theta_G) d\tau \tag{15}$$

4. *A change in productivity $z$ by $dz$:*

$$\partial Q = (C_p + G_p) p_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1 \tag{16}$$

*Proof.* See Appendix A.5. □

The form of the pure demand shocks (preferences and lump-sum taxes) is very simple: to compute the partial equilibrium effect, we simply need to track how much private demand

[11]Below, and for the rest of the text, we assume that derivatives exist as needed.

changes. To understand the partial equilibrium effect of a productivity shock, there are three forces to consider. First, changes in productivity lead to changes in prices, which induce changes in consumption and government purchases. Second, changes in productivity alter the input mix of current production. Third, changes in productivity affect the labor share, which influences labor income and thereby demand via households' directed MPCs.

## *3.2. Deriving The Multiplier*

Having shown how primitive shocks map into partial equilibrium changes in demand, we next explore how these shocks map into changes in output in general equilibrium. We study the impact of a small shocks or, equivalently, the impact of large shocks to first order. Proposition 4 presents the general equilibrium mapping, an object we refer to as the generalized Keynesian multiplier.

**Proposition 4.** *For any small shock to parameters, there exist a pair of rationing equilibria production $Q$ and $Q + dQ$ before and after the shock. Then if the shock induces a partial equilibrium change in production $\partial Q$, the general equilibrium change $dQ$ is given to first order by:*

$$dQ = \begin{bmatrix} \widehat{X}^1 & 0 \\ 0 & \widehat{X}^2 \end{bmatrix} dQ + \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^1} & \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^1} & \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^2} \end{bmatrix} dQ + \partial Q \quad (17)$$

*where all quantities above are evaluated at the initial equilibrium. Moreover, defining the matrix $D$ by*

$$dQ = (D + \widehat{X})dQ + \partial Q, \quad (18)$$

*we have that—so long as the modulus of $D(1 - \hat{X})^{-1}$ is not equal to one—the general equilibrium effect on output is given by:*

$$dY = \left(I - D\left(I - \widehat{X}\right)^{-1}\right)^{-1} \partial Q \equiv M \partial Q \quad (19)$$

*Proof.* See Appendix A.6. □

The matrix $M$ is the generalized Keynesian multiplier and is central to our analysis. To understand the form of $M$, see that $D$ is the mapping from changes in production to changes in both government and private consumption demand. Moreover, see that these changes in demand stem from two sources: direct changes in labor income affecting consumption and changes in interest rates affecting both consumption and government expenditure. In the absence of input-output structure, the multiplier would simply be given by infinite iteration of $D$ in analogy to the canonical Keynesian multiplier: output goes up by $\partial Q$, this induces a change in demand of $D\partial Q$, which induces a change in demand of $D^2\partial Q$ and so on, yielding the form $(I-D)^{-1}\partial Q$. The presence of input-output linkages changes this in two ways. First, whenever output increases there is a direct effect on intermediate goods demand of $\widehat{X}dQ$, which we have to account for in computing the total change in production from any shock. Second, as we are ultimately interested in changes in final output, not merely production, we have to remove total intermediate goods production. The combination of these two effects results in the Leontief inverse matrix $(I-\widehat{X})^{-1}$ post-multiplying $D$ in the multiplier expression.

### *3.3. The Multiplier at the Zero Lower Bound*

At the zero lower bound, Proposition 4 simplifies substantially and provides the key sufficient statistics expression for the multiplier that we directly take to the data. Concretely, we explore a special case of the multiplier when either (i) the agents are unresponsive to the real interest rate or (ii) the real interest rate does not move in response to shocks. This special case is relevant since many developed economies hit the zero lower bound in the Great Recession and will likely continue to hit it in future downturns. Moreover, empirical evidence using both macro and micro data suggests that the consumption response to interest rates is small, suggesting the relevance of this special case even away from the zero lower bound (Campbell and Mankiw, 1989; Kaplan, Violante, and Weidner, 2014; Vissing-Jorgensen, 2002). Formally, we make the following assumption:

**Assumption 4.** *At least one of the following statements is true:*

1. *The consumption and government responses to real interest rates sum to zero:*

$$C^1_{r^1} + G^1_{r^1} = 0 \tag{20}$$

2. *The central bank response of real interest rates to production is zero:*

$$r^1_Q = 0 \tag{21}$$

Under this assumption, which we maintain for the rest of the paper, the earlier expression for the output multiplier simplifies dramatically into a particularly interpretable form:

**Corollary 3.** *Under the conditions of Proposition 4 and Assumption 4, the impact of a* *demand shock $\partial Q$ on first period output $dY^1$ is given by:*

$$dY^1 = \left( I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \right)^{-1} \partial Q^1 \tag{22}$$

*Proof.* See Appendix A.7. □

This is the key formula of the paper and can be understood as a generalization of the traditional Keynesian multiplier $(1 - MPC)^{-1}$ to the case of input-output networks and heterogeneous households. The term

$$C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \tag{23}$$

is the analog of the $MPC$ in the traditional multiplier formula. In this economy, following a demand shock to firms, the term $\left( I - \widehat{X}^1 \right)^{-1}$ maps changes in final demand to changes in production via the input-output network. Having pinned down the change in required production, $\widehat{L}^1$ maps these to changes in firms' demand for labor. Next, the rationing function $l^1_{L^1}$ maps those to changes in each household's income. Finally, the directed MPCs of households $C^1_{y^1}$ map those changes to changes in aggregate consumption of each good. The final multiplier is the Leontief inverse of this object as this loop repeats *ad infinitum*.

The crucial difference relative to the traditional Keynesian multiplier is that the structure of production, employment and consumption matters. First, it is important whether shocks load onto low or high MPC households, as studied by Patterson (2019). Moreover, the *interaction* between the input-output network and the directed consumption network matters: the multiplier is largest when it is not only partial equilibrium shocks but also higher order responses that load onto high MPC households, due to those households spending their marginal dollars at firms that hire high MPC workers or at firms that buy inputs from firms hiring high MPC workers, and so forth.[12]

Throughout the rest of the paper, we assume the moduli of $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1}$ and $l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} C^1_{y^1}$ are less than one, which guarantees that the output multiplier—as well as a similar multiplier in the income space—are well defined.

[12] This same multiplier expression appears in the regional economics literature on social accounting matrices, dating back to Miyazawa (1976). Our result provides the first fully-microfounded justification of this formula, which receives widespread use in the regional economics literature and applied work to compute expenditure multipliers (such as the BEA's RIMS II system). The connection to the social accounting literature motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households. See Appendix B.4 for a formal description of this interpretation.

## *3.4. Decomposing the Role of Heterogeneity*

Several dimensions of heterogeneity interact to produce the multiplier $M$ in Corollary 3. There are three effects induced by the network that lead to amplification relative the basic Keynesian case: an incidence effect whereby the average MPCs of households who are affected by different shocks differs; a biased spending direction effect whereby households direct their marginal consumption toward those with higher MPCs; and a homophily effect where high MPC households largely consume goods produced by other high MPC households and low MPC household largely consume goods produced by other low MPC households. Finally, we discuss knife-edge special cases conditions under which these adjustments to the baseline Keynesian multiplier are zero.

We first simplify notation by renormalizing the units of all goods in each period so that all pre-shock, intra-temporal prices are equal to one, i.e. $p_n^t = 1$. For demand shocks, which do not affect prices, this is without loss. We therefore only consider only the response of GDP to demand shocks in this section. Analyzing supply shocks requires accounting for how prices move in response to these shocks. This factor takes the form of an adjustment that depends only on initial GDP and the primitive production functions; we derive and present that extension in Appendix B.5.

Toward decomposing the role of heterogeneity, we now define the aggregate spending network in terms of primitives. First, let $C_{y^1}^1$ be written as the product $\overline{C}_{y^1}^1 \hat{m}$ of a diagonal matrix $\hat{m}$ of MPCs (the column sums of $C_{y^1}^1$) and spending direction $\overline{C}_{y^1}^1$; and define $m$ as the vector of MPCs. Second, define

$$\mathcal{G} \equiv l_{L^1}^1 \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \overline{C}_{y^1}^1 \tag{24}$$

as the map from an additional dollar of spending by one household to the vector of income changes in generates for each other household. Since every dollar spent eventually becomes income, every column of $\mathcal{G}$ sums to one. Lastly, define

$$\partial y^1 \equiv l_{L^1}^1 \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^1 \tag{25}$$

as the partial equilibrium incidence of a shock on labor income. Proposition 5 first rewrites the generalized multiplier from Proposition 3 in terms of these newly defined terms – $\partial y^1$ and $\mathcal{G}$. Intuitively, this separates the first loop in the multiplier (the effect of the partial equilibrium demand shock on labor incomes) from all other iterations of the loop (the effect of changing incomes on demand, the effect of those demand changes on income, and so forth).

**Proposition 5.** *The total change in first-period output due to a partial equilibrium demand*

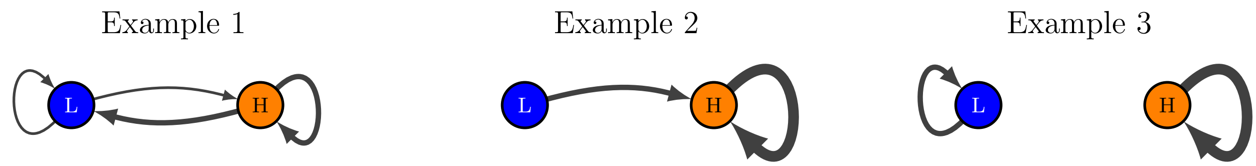


Fig. 1. Example 1: "Neutral" spending-to-income network. Example 2: Typical HH's marginal spending directed toward HHs with higher than own MPC ("bias"). Example 3: Each HH directs marginal spending toward HHs with same MPC ("homophily").

*shock with labor income incidence* $\partial y^1$ *can be expressed as*

$$\vec{1}^T dY^1 = \vec{1}^T \partial y^1 + m^T \left( \sum_{k=0}^{\infty} (\mathcal{G}\hat{m})^k \right) \partial y^1 \tag{26}$$

*Proof.* See Appendix A.8. □

Proposition 5 clarifies that any shock inducing a partial equilibrium change in labor incomes has two components: a direct effect of increasing GDP and an indirect, or multiplier, effect. The multiplier effect exactly maps changes in incomes through MPCs and the network of spending to compute all higher-order effects.

The best way to understand the effects of the spending-to-income network $\mathcal{G}$ on amplification is through three examples. In each example below, there are two households: one with low MPC $m_L$ and one with high MPC $m_H > m_L$. We consider a shock $\partial y^1$ that has incidence $\frac{1}{2}$ on each household's income, so that the incidence-weighted aggregate MPC is $\bar{m} \equiv \frac{m_L + m_H}{2}$. The difference between each example is in the structure of the spending-to-income network $\mathcal{G}$.

Our first example illustrates a neutral case in which network structure is irrelevant. In particular, each household divides its marginal spending equally between the two sectors (see the left panel of Figure 1). In this case, the incidence of spending induced by the income earned in meeting the partial equilibrium demand shock is exactly $m$ times the shock's incidence for each household; similarly for spending induced by income earned in meeting this secondary demand, and so on. Thus, the total change in output is given by the standard $\frac{1}{1-\text{MPC}}$ formula using the incidence MPC, $\bar{m}$.

In the second example, each household instead directs all of its marginal spending to the sector employing the high-MPC household (see the middle panel of Figure 1). Unsurprisingly, this generates higher amplification: the original shock has magnitude 1 and the consumption response of households employed to meet the partial equilibrium demand shock increases output by $\bar{m}$. Then, this spending propagates according to the $\frac{1}{1-\text{MPC}}$ multiplier *at the* *high MPC,* $m_H$. The total change in output is then given by $1 + \frac{\bar{m}}{1-m_H}$, which exceeds $\frac{1}{1-\bar{m}}$. Intuitively, the bias of consumption baskets toward higher-MPC households increases amplification.

In the final example, each household directs all of its marginal spending toward itself (see the right panel of Figure 1). In this case, each household's share of the shock incidence propagates separately, at $\frac{1}{1-\text{MPC}}$ with that household's MPC. The total change in output is then:

$$\frac{1}{2}\left(\frac{1}{1-m_L} + \frac{1}{1-m_H}\right) > \frac{1}{1-\bar{m}} \tag{27}$$

where the inequality comes from the fact that $\frac{1}{1-\text{MPC}}$ is *convex* is MPC. Intuitively, since the high-MPC household spends more of its increase in income, it increase output more by directing spending toward the high-MPC household than by directing spending the the low-MPC household. This network homophily increases amplification.

These examples illustrate the three channels by which network structure matters for amplification. First, one must account for the incidence of a shock onto households of higher or lower $MPC$. Second, the multiplier is higher when households' marginal spending is biased toward households with higher MPCs than their own. Third, homophily in the spending network in the form of correlation between household MPCs and MPCs of households they spend on also generates amplification. Proposition 6 establishes that these three channels exactly capture the effects of the spending-to-income network $\mathcal{G}$, to second order in MPCs. Appendix A.9 provides an exact decomposition in terms of Bonacich centralities of $\mathcal{G}$.

**Proposition 6.** *The total change in first-period output due to a demand shock with unit-magnitude labor income incidence $\partial y^1$ can be approximated as:*

$$\begin{aligned} 1^T dY^1 = &\frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\textit{Incidence effect}} \\ &+ \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{next}] - \mathbb{E}_{y*}[m_n]\right)}_{\textit{Biased spending direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}}\Bigg) + O^3(|m|) \end{aligned} \tag{28}$$

*where $y^*$ is any reference income weighting of unit-magnitude and $m^i_{next}$ is the average MPC of households who receive as income $i$'s marginal dollar of spending.*

*Proof.* See Appendix A.9. □

To gain further insight, we now discuss important benchmarks in which the various alterations to the Keynesian multiplier are zero.

**Proposition 7.** *The following statements are true:*

1. *(No incidence or bias effects) Suppose that consumption preferences and labor rationing are homothetic, that no households are net borrowers in period* 1*, and that there is no government spending.*[13] *Then, for a GDP-proportional, unit-magnitude demand shock, the incidence and bias effects are zero, so that we have:*

$$\vec{1}^T dY^1 = \frac{1}{1 - \mathbb{E}_{y^1}[m_n]} \left( 1 \ + \ \underbrace{\mathbb{C}ov_{y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}} \right) + \ O^3(|m|) \tag{29}$$

   *where $y^1$ is the vector of first-period incomes.*

2. *(No incidence, bias, or homophily effects) Suppose that all industries have a common rationing-weighted average MPC, m.*[14] *Then the incidence, bias, and homophily effects are zero, so that for any reference weighting $y^*$ that can be induced by a demand shock, the change in output corresponding to any unit-magnitude demand shock is:*[15]

$$\vec{1}^T dY^1 = \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} = \frac{1}{1 - m} \tag{31}$$

*Proof.* See Appendix A.10 □

The first part of the proposition shows how, even in a "homothetic economy," heterogeneity in household consumption baskets and sectoral employment can generate network effects through homophily. This happens even at the same time as homotheticity eliminates the bias effect by ensuring that each household's marginal consumption is proportional to its initial consumption, so that the income-weighted average of marginal consumption is proportional to output. Still, when households with different MPCs direct their spending toward different goods, the households employed to produce the goods consumed by higher-MPC households experience a greater change in income – not from the initial, uniform shock, but from the economy's response to it. Insofar as these households have different MPCs from the average, homophily is still possible. This mechanism generate non-neutrality for the multiplier, even if the economy and the shock considered are "neutral" in all other aspects. Aggregate neutrality requires (to second order in MPCs) that the economy feature exactly

[13] By homothetic labor rationing, we mean that marginal and average rationing of income are equal. Formally, if we let $\mathcal{L}^1 \equiv \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} Y^1$ be the vector of first-period firm-level labor bills, then we require that $y^1 = l^1_{L^1} \mathcal{L}^1$.

[14] Formally, $\sum_{n \in N} (l^1_{L^1})_{ni} m_n = m$ for all $i \in \mathcal{I}^1$.

[15] Formally, saying that $y^*$ can be induced by a demand shock says that there exists a $\partial Q^*$ such that:

$$y^* = l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^* \tag{30}$$

zero correlation between households' MPCs and the MPCs of the households they spend on.

The second part of the proposition imposes that each firm's marginal employees have the same average MPC as one another. This eliminates the incidence, bias, and homophily effects, leaving only the classical Keynesian multiplier. That is, wherever in the economy a shock strikes, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $l^1_{L^1} = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm's marginal employees have the population average MPC.

The conditions required to eliminate the network adjustments are knife-edge. In all other cases, the distribution of shocks does affect aggregate responses, and the IO and directed consumption networks affect both the size and direction of these responses.

# 4. Optimal Fiscal Policy

So far, we have studied how shocks propagate to affect output and income in general equilibrium. Guided by the observation that this analysis applies not only to exogenous shocks but also to changes in taxes and government expenditures, we now bring the results of Section 3 to the policy problem of a planner who designs government expenditure and transfer policy to maximize welfare. We draw particular attention to important special cases in which it is optimal o simply target household MPCs.

## *4.1. Welfare and the Planner's Problem*

In previous sections, we have not specified household utility functions, instead simply working with Marshallian demands. In order to analyze welfare, we assume each household $n$ has an additively-separable utility function over consumption, labor supply, and government purchases.[16] At time $t = 1$, households of type $n$ choose consumption but not labor supply, and face a borrowing constraint in the form of a minimum level $\underline{s}_n$ of savings. At time $t = 2$,

[16]Separability between consumption and labor supply ensure that MPCs out of income and transfers are the same. Separability of consumption and labor from government purchases ensures that household decisions do not respond to government purchases directly.

households are unconstrained. The household's problem is therefore:

$$
\begin{aligned}
\max_{\tilde{c}^t, \tilde{l}^t} \; & \sum_{t=1,2} \beta_n^{t-1} \left[ u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t) \right] \\
\text{s.t. } \; & \vec{1}^T \tilde{c}^1 + \frac{\vec{1}^T \tilde{c}^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \leqslant \tilde{l}^1 + \frac{\tilde{l}^2}{1+r^1} \\
& \tilde{l}^1 - \vec{1}^T \tilde{c}^1 - \tau_n^1 \geqslant \underline{s}_n^1 \\
& \tilde{l}^1 = l_n^1
\end{aligned} \tag{32}
$$

We assume that the planner is utilitarian, placing some welfare weight $\lambda_n$ on households of type $n$. The planner maximizes this objective subject to household optimization, market clearing, labor rationing, supply-determined prices, a budget constraint, and a zero lower bound. We assume the zero lower bound is binding throughout so that the planner simply takes $r^1$ as given. The planner's problem is:[17]

$$
\begin{aligned}
\max_{\{c_{ni}^t, l_n^t, Q_i^t, G_i^t, \tau_n^t\}_{t \in \{1,2\}, n \in N, i \in \mathcal{I}^t}} \; W \equiv \sum_{n \in N} \mu_n \lambda_n \sum_{t=1,2} \beta_n^{t-1} \left[ u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t) \right] \\
\text{s.t. } (c_n^1, c_n^2, l_n^2) \text{ solves Equation 32 given } l_n^1 \\
\text{and all equilibrium conditions hold}
\end{aligned} \tag{33}
$$

Below, we will denote the Lagrange multiplier on the government budget constraint by $\gamma$ and refer to it as the "marginal value of public funds" or MVPF.

## 4.2. *Optimal Targeting of Fiscal Stimulus*

Our main goal is to answer the question of where the planner should spend the marginal dollars so as to have the greatest effect on welfare. To this end, we first decompose the change in welfare due to a small change in either transfers or government expenditure.

**Proposition 8.** *The change in welfare $dW$ due to a small change in taxes and government expenditure—at a constant interest rate—can be expressed as:*

$$
\begin{aligned}
dW = \sum_{n \in N} \mu_n \widetilde{\lambda}_n \Bigg[ & \underbrace{-\Delta_n dl_n^1}_{\text{Address under-emp.}} - \underbrace{\left( d\tau_n^1 + (1-\phi_n) \frac{d\tau_n^2}{1+r^1} \right)}_{\text{Make transfers}} \\
& + \underbrace{\left( WTP_n^1 dG^1 + (1-\phi_n) \frac{WTP_n^2}{1+r^1} dG^2 \right)}_{\text{Make expenditures}} \Bigg]
\end{aligned} \tag{34}
$$

[17] For a full statement, see Appendix A.11.

*where $\widetilde{\lambda}_n$ is the value the planner places on the marginal transfer of first-period wealth to a household of type $n$, $\Delta_n$ and $\phi_n$ are $n$'s implicit first-period labor wedge and borrowing wedge, and $WTP_n^t$ is the vector of $n$'s marginal willingness to pay for period $t$ government expenditures on each good, in period $t$ dollars. The change in $n$'s first-period employment, in turn, is given by*

$$\widehat{\mu} dl^1 = R^1 \left(I - C^1_{y^1} R^1\right)^{-1} \left(dG^1 - C^1_{y^1} \left(\widehat{\mu} d\tau^1 + \frac{1_{\phi_n=0}\widehat{\mu} d\tau^2}{1+r^1}\right)\right) \tag{35}$$

*where $R^1 \equiv l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ is the partial-equilibrium map between output and total employment of each type (across individuals) in the first period.*

*Proof.* See Appendix A.11. □

Equation 34 clarifies three different motives of the planner. First, she seeks to alleviate involuntary un(der)employment by changing the labor allocation so as to provide more employment to households with large negative labor wedges (the underemployed). Second, she may make transfers between households, both in the name of pure redistribution and to help borrowing-constrained households substitute intertemporally. Third, she makes government purchases; here the borrowing wedge enters, as borrowing-constrained households undervalue future purchases due to an artificially high value of wealth in the first period.[18]

Our next results consider the special case of a planner whose only motive is to reduce involuntary un(der)employment. This planner is indifferent to redistribution, borrowing constraints, and the direct benefits of government expenditures. More formally, we make the following assumption:

**Assumption 5.** *The planner's objective satisfies the following conditions:*

1. *The planner is indifferent between households, i.e.* $\widetilde{\lambda}_n = 1$
2. *Government purchases have no intrinsic value, i.e.* $WTP_n^t = 0$
3. *Borrowing constraints do not bind, i.e.* $\phi_n = 0$
4. *All un(der)employed households have no marginal disuility of labor, i.e. if* $\Delta_n < 0$ *then* $\Delta_n = -1$

*Moreover, all households $n$ to which labor is rationed are un(der)employed.*

Our next result shows that under these assumptions, the planner's motives simplify considerably, so that she simply maximizes aggregate income. This makes the analysis of

[18] In Appendix B.6, we provide a further decomposition of these terms for small variations in policy starting at the global optimum, similarly to Werning (2011).

optimal fiscal policy policy tractable as maxmimally expansionary fiscal policy and optimal fiscal policy coincide.

**Proposition 9.** *Under assumption 5, the welfare change from a change in expenditures is proportional to the resulting change in output, whereas the welfare change from a change in transfers is proportional to the resulting change in income. Formally,*

$$dW = \vec{1}^T \frac{dY^1}{dG} dG + \vec{1}^T \frac{dl^1}{dy^1} \left( -\hat{\mu} d\tau^1 - \frac{\hat{\mu} d\tau^2}{1 + r^1} \right) \tag{36}$$

*where* $\frac{dY^1}{dG^1} = (1 - C^1_{y^1} R^1)^{-1}$ *and* $\frac{dY^1}{dG^2} = 0$ *are first-period output multipliers and* $\frac{dl^1}{dy^1} = (1 - R^1 C^1_{y^1})^{-1}$ *is the first-period income multiplier.*

*Proof.* See appendix A.12 □

Key to Proposition 9 is the observation that its assumptions imply there is zero social cost of production; all marginal production is done by underemployed households, who are indifferent to working more. For these households, earned income is as good as a pure transfer. In the case of government expenditure shocks, the total change in income is equal to the total change in output. In the case of lump-sum transfer shocks, income changes both directly and through earnings generated by the change in the output; this generates the difference in multipliers.

In Appendix B.7, we show that this result carries over directly to environments with non-zero markups in the first period.[19] Intuitively, profit owners can be thought of as providing capital services with completely elastic supply. Conveniently, this allows us to treat capital owners "as if" they simply supply labor and are rationed to in proportion to their firm's markup. The only modification required to accommodate this broader interpretation is that the output and income multipliers must be extended to include capital income. This contrasts sharply with Baqaee (2015), who proposes that a labor-wedge-reducing planner should target the industry with the highest network-adjusted labor share. The difference comes from that Baqaee's model features competitive firms (hence no markups) and fully-utilized capital (no capital wedge).

## *4.3. When is MPC Targeting Optimal?*

In the context of rich interconnections between households and firms, policymakers must consider not only what is the optimal fiscal stimulus in principle, but also whether they

[19] We allow for non-zero markups in the second period as well, provided (a) the government encourages second-period profit creation with consumption and input subsidies proportional to markups and (b) the MPC out of future capital income is zero.

have enough information to implement the optimal policy in practice. This section provides conditions under which a simple policy – targeting a stimulus toward households with the highest MPC – is optimal, offering some hope that planners with imperfect knowledge of the economy can still design policy effectively.

Our first result builds on Proposition 9, which showed that – for a planner only concerned with the under-utilization of labor – the best small change in policy is that which maximizes output. Below, we provide a further assumption, under which the planner's output-maximizing policy is simply to target households based on their MPCs. This is true exactly when consumption network effects through bias and homophily are both zero, so that only the first correction term of Equation 28 is non-zero.

**Corollary 4.** *Suppose that all households' marginal spending is directed to households whose average MPC is equal to the incidence-weighted average MPC corresponding to a uniform output shock.*[20] *Formally,* $m_n^{next} = \mathbb{E}_{y*}[m_{n'}]$ *for all* $n$*, where* $y^*$ *is the income incidence of a uniform output shock. Then, under assumption 5, the welfare change from a policy is given by:*

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1 + r^1}\right) \tag{37}$$

*Dollar-for-dollar, the best policy is the one most effectively targeting household MPC.*

*Proof.* See appendix A.13 □

This result holds for two reasons. First, all households direct their consumption in the same way for the purposes of amplification in the absence of network effects. Second, the planner simply wants to maximize output. As a result, to generate maximal amplification and thereby construct optimal policy, the best thing a planner can do is target households with the highest MPCs. A further implication of this result is that, for the same amount of spending, transfers weakly dominate government expenditures for stimulus purposes. This is because transfers more directly target MPCs, a household-level variable. At the same time, if it is possible to target MPCs close to as well with expenditures as with transfers,[21] then expenditures are likely to dominate transfers so long as government spending has some direct value.

Corollary 4 always holds on the margin under the assumption of no spending network effects – whether the current policy is optimal or not. Our final policy result flips these conditions, applying only *at* the optimal policy but without restrictive empirical assumptions on the spending network. It provides a test that a planner may use to evaluate whether the

[20] This ensures that the final two correction terms in 28 are zero for all partial equilibrium shocks.

[21] This is likely if, for example, it is politically untenable to make transfers to only high-MPC groups.

current policy is optimal, without information on the network connecting one household's spending to another's income.

**Proposition 10.** *Suppose that the marginal social dis-utility of labor supply is constant across all households rationed to on the margin at the optimum, i.e. if $(R^1 C^1_{y^1})_{n,-} \neq \vec{0}$ then $\widetilde{\lambda}_n(1+\Delta_n) = const$. Then $dW = 0$ with respect to changes in first-period transfers if and only if, for all $n \in N$,*

$$\gamma = \widetilde{\lambda}_n \left(1 + \frac{m_n}{1-m_n}(-\Delta_n)\right) \tag{38}$$

*where $\gamma$ is the marginal value of public funds. Alternatively, suppose that the social gains from first-period government expenditure are equal to some $\tilde{v}$ across goods and constraints bounding expenditures above zero do not bind. Then $dW = 0$ with respect to changes in first-period expenditures if and only if, for all $i \in \mathcal{I}^1$,*

$$\gamma = \tilde{v} + \frac{1}{1-\widetilde{m}_i}\left(-\widetilde{\lambda\Delta}_i\right) \tag{39}$$

*where $\tilde{m}_i \equiv \left(m^T R^1\right)_i$ is the rationing-weighted average MPC in the production of good $i$ and $\widetilde{\lambda\Delta}_i \equiv \left(\widetilde{\lambda}^T \widehat{\Delta} R^1\right)_i$ is the rationing-and-welfare-weighted average rationing wedge in the production of good $i$.*

*Proof.* See appendix A.14. □

When government purchases and redistribution have no direct value and marginally employed workers have zero disutility of labor – as in Proposition 9, where output-maximization was optimal – the first-order conditions simplify further, to

$$\gamma = \frac{1}{1-m_n}, \qquad \gamma = \frac{1}{1-\widetilde{m}_i}$$

more closely echoing Corollary 4. Intuitively, these conditions express that in order to be at the optimal policy, all households / industries must have the same MPC – otherwise output could be increased by shifting money between them.

Away from these cases – e.g. when government purchases or redistribution have value – Proposition 10 says that the planner continues to require only very limited information in order to verify whether the current policy is optimal. In the transfer case, the planner only needs information on household level welfare weights, rationing wedges, and MPCs – not the network of marginal spending flows between households. In the expenditures case, the planner needs to know the average MPC and welfare-weighted rationing wedge by industry; these require knowledge of the rationing function linking output to incomes, but not the directed consumption matrix.

The main idea underlying Proposition 10 is that—at an optimum—the social value of additional spending by any household is independent of how that spending is directed. This is clearest in the case of transfers: For any household employed in order to produce marginally-demanded goods, the social value of their employment is equal to the value of a transfer to that household, less the dis-utility of labor. Since (by assumption) the dis-utility of labor is constant across households, and since—at an optimum—the value of transfers must also be constant across households, it follows that the social value of additional employment is constant across households. Since the planner is indifferent over the direction of household spending, she targets solely based on the magnitude of that spending—i.e. household MPCs—as well as household welfare weights. A similar argument applies in the case of government expenditures.

# 5. Data and Estimation Methodology

Using our framework, we have so far derived a simple sufficient statistics expression for the generalized multiplier. We also demonstrated theoretically how rich household, industry and regional heterogeneity can interact to potentially amplify shocks and shape optimal policy. We now take our multiplier to the data to empirically evaluate the importance of these dimensions of heterogeneity. To do this, we directly estimate the sufficient statistics that comprise the multiplier using a variety of datasets. In this section, we describe both the datasets we use to estimate these sufficient statistics and the methodology we employ to calculate the components of the multiplier.

First, recall from Proposition 3 that in the case of zero interest rate responsiveness the response of output to a shock $\partial Q_1$ is given by:

$$dY^1 = \Big(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 (I - \widehat{X}^1)^{-1}\Big)^{-1} \partial Q_1 \tag{40}$$

To estimate the multiplier, we therefore need estimates of three key objects: the regional input-output matrix $\widehat{X}^1$ describing the input use requirements of every region-industry pair, the rationing matrix $l^1_{L^1}\widehat{L}^1$ describing how much each demographic-region pair's income changes in response to a one dollar change in revenue of each region-industry pair, and the directed MPC matrix $C^1_{y^1}$ describing how much each demographic-region pair consumes from each region-industry pair when they receive a one dollar income shock.

In going to the data, we must also account for three empirically-relevant factors that were absent from our baseline model – capital, profit, and foreign income. At a high level, our strategy is to (1) model capital as an input, (2) model profits by assuming constant

| Dataset | Input Output | Rationing | Directed MPC |
|---|---|---|---|
| American Community Survey (ACS) | | X | |
| BEA Make and Use Tables (IO) | X | X | |
| BEA Regional Accounts (RA) | | X | |
| Consumer Expenditure Survey (CEX) | | X | X |
| Commodity Flow Survey (CFS) | X | | X |
| Consumer Price Index (CPI) | | X | X |
| Internal Revenue Service Statistics of Income (IRS SOI) | | X | |
| Panel Study of Income Dynamics (PSID) | | X | X |

Table 1: Summary of Datasets Used in the Estimation of Our Sufficient Statistics.

markups, as in appendix B.2, and (3) model foreign factors as a type of "labor" with zero MPC, reflecting that payments leaving the economy do not re-enter through income effects.

The following subsections describe in detail how we estimate each of the three components of the generalized multiplier: the input-output, rationing, and directed consumption matrices. Table 1 shows which datasets are used in the estimation of each object. We restrict our attention to the United States in 2012, which is the most recent year for which we have several of the key datasets.

## *5.1. The Regional Input-Output Matrix*

The regional input-output matrix $\widehat{X}^1$ is an $(R \times I) \times (R \times I)$ matrix where $I$ is the number of industries and $R$ is the number of regions. The $(ri, sj)$ component of this matrix corresponds to the amount of sector $i$ in region $r$'s good required to produce a single unit of sector $j$ in region $s$'s good. To estimate this object, we must first take a stand on the level of granularity at which to model sectors and regions. Guided by the level at which input-output data are available, we largely follow the BEA's collapsed input-output sector classification, leaving us with 55 sectors which loosely correspond to the 3-digit NAICS classification.[22] Similarly, to take full advantage of the CFS microdata on interstate trade, we set regions at the level of the state (including Washington D.C.), leaving us with 51 regions. This leaves us with 2805 sector-regions.

We construct the regional input-output matrix in three steps. First, following others in the literature, we use data from the 2012 BEA make, use, and imports tables to construct the domestic, national input-output matrix, which measures the dollar value of products from industry $j$ that are used by industry $i$. In constructing this table, we assume that conditional on sourcing a commodity, the commodity is provided by industries in proportion to the

[22]For full details on the definition of these sectors and for similar details, see the replication files.

amount of that commodity produced by those industries. We also make an adjustment to account for linkages across industries in capital investment. This is necessary as the standard use table accounts only for changes in intermediate goods usage. To impute each industry's expenditure on investment goods, we assume that all industries invest the same fraction of their gross operating surplus (available in the use table) in capital. To compute the direction of this investment toward different industries, we assume that each firm demands the same investment good and compute its industrial composition with the same procedure – using the use, make, and import tables – as we use for inputs. We then add this investment correction to the previously constructed input-output matrix.

Second, we use the 2012 public-use microdata from the Commodity Flows Survey (CFS) to construct a matrix describing how much each state imports from all other states. The CFS is a survey conducted by the US Census Bureau and includes data on 4,547,661 shipments from approximately 60,000 establishments. The data records the location of the shipping establishment, the commodity being shipped, the value of the shipped commodity, and the location to which the commodity is being shipped. The public use microdata file modifies this underlying data by introducing noise and top-coding extremely large shipments. Using this information, we calculate the total value of shipments between each pair of states for each tradable industry using the mapping between commodities and industries outlined in the BEA's make table.[23] For all nontradable industries, we assume that the commodity is sourced entirely within the state.

Finally, we construct the regional input-output matrix by combining the national industry-level input-output with state-by-state trade flows. Specifically, the amount of industry $i$ in state $r$ used by industry $j$ in state $s$ is the product of the share of industry $j$'s inputs that come from industry $i$ and the fraction of sector $i$ goods flowing to $s$ from $r$ (out of all origin states). This yields a matrix describing, for each industry-region pair, how much of each other industry-region pair's production is used to produce a single unit of output.

## *5.2. The Directed MPC Matrix*

The directed MPC matrix $C^1_{y^1}$ corresponds to an $(R \times I) \times (R \times N)$ matrix where $N$ is the number of demographic groups. The $(ri, sn)$ component of this matrix maps how a one dollar change in demographic $n$ living in region $s$'s income changes that household's consumption of good $i$ in region $r$. Again, this first requires us to take a stand on the level of granularity at which to model demographic groups. Guided by the level at which precise estimation of MPCs is possible in the PSID, we set the number of demographic groups at

[23]Caliendo et al. (2018) use a similar methodology to construct their regional input-output matrix.

82, comprising 80 baseline groups (five income groups, four age groups, two gender groups, two race groups) and two dummy groups for the owners of capital and foreigners.[24]

We construct the directed MPC matrix in three steps. First, we construct MPCs for total consumption expenditure for each of our 80 demographic groups using the PSID, CPI and CEX following the methodology in Patterson (2019). Specifically, we follow the procedure of Gruber (1997), usinging the panel structure of the PSID to estimate the equation:

$$\Delta C_{ht} = \sum_x \left(\beta_x \Delta E_{ht} \times x_{ht} + \alpha_x \times x_{ht}\right) + \delta_{s(h)t} + \varepsilon_{ht} \tag{41}$$

where $C_{ht}$ is household $h$'s consumption at time $t$, $E_{ht}$ is household $h$'s labor earnings at time $t$, $x_{ht}$ is a demographic characteristic of the individual, and $\delta_{s(i)t}$ is a state by time fixed effect. Estimating Equation 41 we then obtain the following estimate of the MPC for household $h$ at time $t$:

$$\widehat{MPC}_{ht} = \sum_x \hat{\beta}_x x_{ht} \tag{42}$$

However, there are two challenges in performing this estimation. The first issues arises as there are a wide range of factors that could simultaneously move income and consumption. To address this, we instrument for changes in labor market earning using transitions into unemployment. This is desirable as such shocks are both large and persistent. Unemployment shocks therefore capture that variation most important to understanding recessions. Indeed, if recessions can be seen as shocks of the same persistence as unemployment, then this MPC is exactly the right object to capture shock propagation in the manner suggested by the model.[25]

The second issue stems from measurement in the PSID: for most of the PSID sample, only expenditure on food consumption is measured. Using only this measure is problematic as food is a necessity and expenditure on food is likely to be distorted by the provision of food stamps (Hastings and Shapiro, 2018). To overcome this issue, we use overlapping information in the PSID and CEX to impute a measure of total consumption expenditure, following the methodology of Blundell, Pistaferri, and Preston (2008) and Guvenen and Smith (2014). Concretely, we use the CEX to estimate demand for food expenditure as

[24] Our five income groups correspond to: less than \$22,000, \$22,000-\$35,000, \$35,000-\$48,000, \$48,000-\$65,000 and more than \$65,000. Our four age groups correspond to those 25-35, 36-45, 46-55 and 56-62. Our race groups are black and non-black. Our gender groups are men and women.

[25] While the MPC out of an unemployment shock is relevant for the general equilibrium amplification of shocks, it is potentially not the right MPC for determining the response of consumption to targeted transfers, which is the focus of the optimal policy analysis. We return to this in Section 6, but note here that the MPCs estimated here are close in magnitude and have similar cross-demographic patterns as those estimated using tax rebates or lottery winnings (Parker et al., 2013; Fagereng, Holm, and Natvik, 2019).

a function of durable consumption, non-durable consumption, demographic variables and relative prices from the CPI. Under the assumption of monotone food expenditure, this function can be inverted to predict total consumption as a function of food expenditure and demographics in the PSID. This procedure generates substantial heterogeneity across households in estimated MPCs (see Figure A1 in Appendix D).

Next, we estimate the consumption basket shares in each of our 55 industries for each of our 80 demographic groups using the CEX and CPI. We first deflate consumption over the 54 measured categories using the CPI and then compute the average consumption basket share of each demographic group. Using a concordance between NIPA goods and our industry classifications, we then map consumption at the household level in each category to the 55 industries used in our analysis.

We use these consumption basket shares and our estimated MPCs to construct an estimate of the directed MPC for each of the 80 demographic groups out of each of the 55 industries. We do this by assuming linear Engel curves of households for each category of consumption. Formally, we estimate the directed MPC of household $h$ at time $t$ as:

$$\widehat{MPC}_{n(ht)i} = \alpha_{n(ht)i}\widehat{MPC}_{n(ht)} \tag{43}$$

where $n(ht)$ is the demographic group of household $h$ at time $t$ – which we from now on suppress when clear from context – and $\alpha_{n(ht)i}$ is the demographic-specific consumption basket weight of good $i$. Naturally, the imposition of linear Engel curves may be overly restrictive. However, our estimates always lie in the 95% confidence interval of estimates of good-specific MPCs from the PSID in the years in which this is possible (see Figure A2 in Appendix D), suggesting that we are capturing reasonable dimensions of heterogeneity with this assumption.

Finally, we use our estimated state-state gross flows in goods to arrive at the regionally-directed MPCs. Formally, for tradable goods, we assume that all households in a state consume from all other states in proportion to the fractions of imports of that good that originate from those states:

$$\widehat{MPC}_{risn} = \lambda_{irs}\widehat{MPC}_{ni} \tag{44}$$

where $\lambda_{rs}$ is the fraction of shipments of good $i$ from state $s$ to state $r$ as a function of the total shipments of good $i$ to state $r$, as we earlier computed to construct the regional input-output matrix.[26] We assume all nontradable goods are consumed within the state.

[26]This potentially sources too much consumption from outside the state given that the CFS comprises both consumption goods and intermediate goods flows. In section 6, we explore the robustness of this modelling assumption for how consumption is sourced by considering a model with total consumption autarky where all consumption is sourced within the state. This has a very small impact on the results.

The procedure above provides the directed MPC entries for the 80 demographic groups. It remains to estimate the directed MPCs for capitalists and foreigners. For foreigners, we simply set all entries to zero. This coincides with the assumption that, of all foreign recipients of income that leaves the US, none spend this income in the US or indirectly cause other spending in the US. For capitalists, we take the MPC out of stock market wealth as estimated by Chodorow-Reich, Nenov, and Simsek (2019) at 0.028. We then allocate this in the direction of the aggregate consumption basket as reported in the BEA use table.

## *5.3. The Rationing Matrix*

The rationing matrix $l^1_{L^1}\hat{L}^1$ corresponds to an $(R \times N) \times (R \times I)$ matrix where $N$ is the number of demographic groups. The $(rn, si)$ component of this matrix maps how a one dollar change in the production of good $i$ in region $s$ translates to a change in labor income for demographic $n$ in region $r$.

We construct the rationing matrix in three steps. We first use the ACS to compute, within each state-industry pair, the total labor earnings by each demographic group in 2012. We also use state-level data from the BEA on compensation and output by industry to compute labor shares of value added for each state-industry pair.

Second, we use these two components, along with the estimated demographic group MPCs, to construct the the labor rationing entries for workers. Concretely, we employ the following formula:

$$\left(l^1_{L^1}\hat{L}^1\right)_{rnsi} = \mathbb{I}[r = s]\frac{y_{inr}}{\sum_n y_{inr}}\alpha_{ir}\beta_i\left(1 + \gamma\left(MPC_n - \overline{MPC}_{ir}\right)\right) \tag{45}$$

where $y_{inr}$ is total earnings of demographic $n$ in industry $i$ in region $r$, $Y_{ir}$ it total output in industry $i$ in region $r$, $\alpha_{ir}$ is state x industry labor share of value added, $\beta_i$ is the national value added to output ratio in industry $i$, $\gamma$ is the correlation between MPCs and earnings elasticities, and $\overline{MPC}_{ir}$ is the earnings-weighted MPC of all workers in industry $i$ in region $r$. The indicator function imposes the condition that all labor earnings are received within the state where production occurs. This is the unique functional form that both preserves a constant correlation between MPC and earnings elasticity, of which there is strong evidence from Patterson (2019) and preserves total income received across all demographic groups in each industry-region pair. We set $\gamma = 1.332$, the correlation of MPC with earnings elasticity to aggregate shocks measured in Patterson (2019).[27]

Finally, it remains to allocate those factor payments that are not received by labor. These

[27] See Patterson (2019) for more details and discussion.

take two forms: payments made to the domestic owners of capital and payments made to foreign factors. We compute directly payments made to domestic owners of capital via the following procedure. We first compute profits in each region-industry pair. To do this, we compute the domestic profit share of production from the BEA use table and add this to the residual value added in each state-industry pair that is not paid to labor. We then allocate these profits to the capitalist demographic group in each state according to that state's share of dividend income in the IRS SOI data. Finally, we compute payments made to foreigners as the residual of payments made to intermediate producers, payments made to labor and payments made to capitalists.[28]

# 6. The Role of Heterogeneity for Shock Propagation and Optimal Fiscal Policy

In this section, we quantify the role of heterogeneity in shock propagation and the design of optimal fiscal policy. In both cases, we use our estimated sufficient statistics and employ earlier theoretical results to marshal the discussion. Finally, we explore the implications of our estimates for the size of geographic spillovers and investigate the effect of changes in the labor share on shock propagation.

## *6.1. Heterogeneity and Shock Propagation*

We begin our empirical analysis by quantifying the degree of heterogeneity in the multiplier across industries and regions. Since the multiplier is linear in the partial-equilibrium effect of any shock, the aggregate effect of any shock can be constructed as a linear combination of the effects of unit magnitude shocks to each industry-region pair. Therefore, it is informative to understand the degree to which the effect differs depending on the origin of the shock. The left panel of Figure 2 plots the change in GDP induced by a one dollar shock to each industry-region pair. One sees immediately the striking fact that across industry and space, multipliers differ by a factor of six. This reveals that accounting for heterogeneity is critical for understanding shock propagation. Our goal is now to understand what channels in the data generate this wide dispersion in multipliers.

[28]In a small fraction of cases, this leads to a *negative* foreign share of revenues, which is unrealistic. To avoid this, we could alternatively reduce the profit share of revenue in region-industry pairs with high labor shares. Insofar as we use similarly small MPCs for foreigners and capitalists, this alternative calibration would generate similar quantitative results.

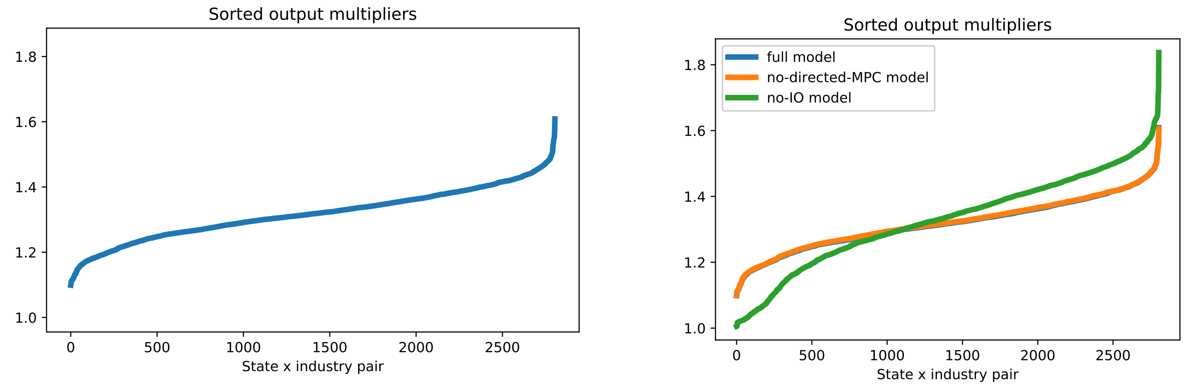


Fig. 2. Sorted change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline and plotted in the left pane. No directed MPC assumes that all households direct their consumption in proportion to aggregate consumption. No IO assumes that there is no use of intermediate goods.

### *6.1.1. An Empirical Decomposition*

Recall from Proposition 6 in Section 3.4 that for any shock causing a unit magnitude partial equilibrium change in labor incomes $\partial y^1$, the total change in GDP is:

$$\vec{1}^T dY^1 \approx \frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1+\underbrace{\mathbb{E}_{\partial y^1}[m_n]-\mathbb{E}_{y*}[m_n]}_{\text{Incidence correction}} + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{\text{next}}]-\mathbb{E}_{y*}[m_n]\right)}_{\text{Biased spending direction correction}} + \underbrace{\mathbb{C}\text{ov}_{\partial y^1}[m_n, m_n^{\text{next}}]}_{\text{Homophily correction}}\Bigg) \tag{46}$$

Therefore, the dispersion in multipliers from Figure 2 could be coming from differences in 1) the incidence of the shock, meaning that shocks to some markets load more heavily on agents with higher MPCs, 2) the bias in the spending, meaning that some markets have marginal spending that is more directed towards high MPC households, or 3) the homophily, meaning that the spending networks in some markets are more segmented by MPCs. We find that all of the heterogeneity across state-by-industry pairs in Figure 2 is driven by the differential direct incidence of those shocks onto agents with different MPCs.

Proposition 6 makes clear that necessary conditions for the existence of sizeable bias and homophily terms are that there exists heterogeneity across households in their basket-weighted MPCs $m_n^{next}$ and that average basket weighted MPCs differ from the benchmark $\mathbb{E}_{y*}[m_n]$. To see this, observe that if $m_n^{next}$ is homogeneous and $\mathbb{E}_{\partial y^1}[m_n^{\text{next}}] = \mathbb{E}_{y*}[m_n]$, then both the bias and homophily terms are zero as all households effectively direct their consumption homogeneously. Figure 3 documents that in the data, we uncover minimal

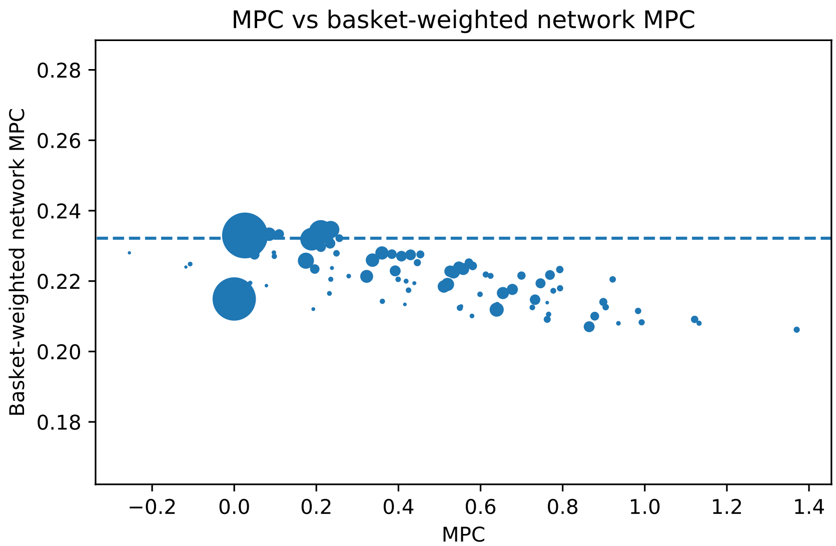


Fig. 3. Scatter of MPCs $m_n$ against basket-weighted MPCs $m_n^{next}$. The dashed line gives the average MPC $\mathbb{E}_{y*}[m_n]$ for $y^*$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

heterogeneity in basket-weighted MPCs, shown by the very shallow slope between basket-weighted MPCs and household MPCs. As a result, the homophily effects are very close to zero. Moreover, Figure 3 demonstrates that basket-weighted MPCs all lie very close to $\mathbb{E}_{y*}[m_n]$. Consequently, bias effects are also very close to zero.[29] To drive this point home, the orange line in the right panel of Figure 2 shows multipliers corresponding to a counterfactual model without heterogeneous consumption and make the bias and homophily effects identically zero. As one can see, there is effectively no difference in the full distribution of multipliers when we impose this condition, demonstrating that it plays no role in the baseline estimates.[30]

The lack of consumption network affects appears to be a real feature of the data, rather than a failure of our estimation approach to capture them. Critically, our estimates of consumption basket shares in the CEX do display substantial variation across households (see Figure A8 in Appendix D), allowing the possibililty of large network effects. The lack of

[29]To show this more formally, we construct any feasible $\partial y^1$ via a linear combination of demand shocks to each sector-region pair. We then compute the bias and homophily effects from each of these shocks and plot the full distribution of bias and homophily terms (see Figures A4 and A5 in Appendix D, respectively). Across the full distribution of shocks, the contributions of the bias and homophily terms range between zero to four tenths of a percent increase in the multiplier – they are empirically negligible for all feasible demand shocks. We also compute the full distribution of error terms arising from the approximation in our decomposition result (Figure A6 in Appendix D) and find that they are uniformly an order of magnitude smaller than the bias and homophily terms. Our approximation is therefore very tight for any feasible shock.

[30]In Figure A7 of Appendix D we compare the multipliers from these two models without sorting. The correlation in multipliers across the two models is nearly perfect.

estimated consumption network effects then stems from two opposing forces. On one hand, high MPC households disproportionately consume goods produced by low-labor-share industries (see Figure A9 in Appendix D), directing more spending toward capital, the owners of which have low MPCs.[31] On the other hand, our estimates feature substantial within-region non-tradeables demand, with around a third of total labor demand remaining within the state from which consumption originates (see Figure A11 in Appendix D). Moreover, there is spatial heterogeneity in MPCs, with income-weighted MPCs differing by a factor of 1.5 across states (See Figure A16 in Appendix D). Together, these regional forces generate a modest positive homophily effect whereby higher (lower) MPC workers direct their consumption more toward local labor which similarly features high (low) MPC. When combined, however, these labor share and local demand effects – each modest on its own – partially cancel, so that all types spend on goods baskets produced by households of very close to the average MPC.

#### *6.1.2. Sources of Heterogeneous Incidence*

Since the heterogeneity in amplification in Figure 2 does not stem from higher order network effects, it must instead come from differences in the incidence of different shocks onto the MPCs of households. Three distinct factors contribute positively to these differences: First, differences in the demographic composition of the workforce across sectors and regions causes large differences in the average MPCs of workers across firms and regions. The right panel of Figure 4 shows both this heterogeneity in average MPCs and that sectors with higher worker MPCs are naturally associated with a larger output multiplier. Second, differences in the share of labor that each sector directly employs cause large differences in the MPC of the ultimate recipients of factor income. In particular, agents employing lots of capital but little labor pass most factor payments on to the owners of capital who have very low MPC and therefore feature small output multipliers. This is shown in Figure A12 in Appendix D that plots the labor share of each industry-state pair against its output multiplier: there is substantial heterogeneity in labor use and low labor use is associated with a small output multiplier. Third, differences across firms in the covariance of worker MPC and exposure to changes in firm revenue generate additional widening of the distribution of multipliers. This is shown in Figures A14 and A15 in Appendix D where we compare the baseline model with rationing more to agents with higher MPC to a model with rationing to agents uniformly

[31] Conditional on reaching labor, the average MPC of workers producing consumption baskets is homogeneous across the MPC distribution (see Figure A10 in Appendix D), so labor share differences account for the bulk of differences in basket-weighted MPCs stemming from heterogeneous consumption baskets. This finding is also consistent with the empirical patterns in Hubmer (2019).

by income, where we observe both an upward shift in the distribution of output multipliers as well as an increase in range.

Conversely, input-output linkages serve an important role in *attenuating* the heterogeneity induced by these differences. This can be seen in the right panel of Figure 2, where the green line corresponds to the model without input-output linkages, which features a much more dispersed distribution of multipliers.[32] The role of input-output linkages in attenuating dispersion is intuitive. In the absence of inputs, when the firm directly employing the highest-MPC factors gets an additional dollar of revenue, it spends it all on those high-MPC factors. With inputs, this same firm spends a fraction of its revenue on goods produced by other firms, who in turn direct that money to their (by construction) less-than-highest-MPC factors – effectively diluting the MPC of the initial firm. This dilution effect attenuates the heterogeneity in industry multipliers.

#### *6.1.3. The Aggregate Multiplier*

Having understood the ways in which shocks to certain sectors propagate in isolation, we can also aggregate across sectors to quantify how different dimensions of heterogeneity contribute to the size of the *aggregate multiplier*, defined as the response of GDP to a GDP-proportional shock across industries and regions. Table 2 shows that in the baseline calibration, our model generates an aggregate multiplier of 1.30, a number consistent with the large literature on fiscal multipliers (Ramey, 2011; Chodorow-Reich, 2019). Our earlier finding that consumption network effects are unimportant for local shocks carries through and implies that the aggregate multiplier is almost unaffected by the direction of consumption; if one were to assume that all households consumed the same good – one sourced from each household in proportion to its income – then the bias and homophily terms would be exactly zero. Column 2 of Table 2 shows the results under this more restricted setting and reveals that they are almost identical to the baseline estimates.[33]

More surprisingly, Column 3 in Table 2 shows that accounting for IO linkages is also unimportant for the magnitude of the aggregate multiplier. This is despite the fact that – as we have shown – accounting for IO linkages is important for understanding the cross-section of multipliers. Intuitively, IO linkages reduce the effective MPC of industries with high-MPC workers and increase the effective MPC of industries with low-MPC workers, but have roughly zero effect in the aggregate as these two forces cancel out.

[32] See Figure A17 in Appendix D for a scatter plot of the multipliers across both the full model and that without input-output linkages.

[33] Table A3 in Appendix D confirms that the bias, homophily, and error terms are small in the case of a GDP-proportional demand shock.

| | No IO | No Directed MPC | IO & Directed MPC |
|---|---|---|---|
| Uniform Rationing | 1.24 | 1.25 | 1.24 |
| MPC Rationing | 1.30 | 1.30 | **1.30** |

Table 2: Multiplier of a GDP-proportional output shock across model specifications. IO and directed MPC with MPC rationing is the baseline and in bold. No IO assumes that all industries consume no intermediate goods. No Direct MPC assumes that all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

Finally, a comparison of the two rows of Table 2 shows the importance of accounting for the fact that high-MPC households are more exposed to business cycle shocks. The first row shows the multiplier in the scenario with income-proportional rationing while the second row shows the case with the empirical incidence of shocks. We find that the income-proportional rationing dampens the output response by approximately 20 percent. This echoes the finding of Patterson (2019), but in a richer model. In the appendix, we show that accounting for regional vs. national structure (see Table A4 in Appendix D) as well as inter-regional trade (see Table A5 in Appendix D) also has a limited impact on the aggregate multiplier.

In summary, the results in Table 2 demonstrate that the aggregate multiplier of a uniform GDP shock would have been the same if we had assumed a one-good model with MPC-proportional income rationing, as long as we had calibrated the MPC to reflect an average out of the labor, profits and foreign shares of income. While many-good and many-region heterogeneity is empirically relevant for determining the response to some shocks, it does not affect the amplification of an aggregate shock proportional to GDP.

## *6.2. Heterogeneity and Optimal Fiscal Policy*

The substantial heterogeneity in the total response to an additional unit of demand across states and regions suggests a large scope for targeted fiscal policy. There are two important targeting problems that we aim to answer: to which industries and in which regions should the government target expenditure? To which demographic groups and in which regions should the government provide cash transfers?

Recall from Proposition 9 that – for a planner whose sole goal is to reduce the total extent of factor underutilization – it is optimal to maximize aggregate income.[34] Therefore, we will evaluate optimal policy by the increase in income it induces. Figure 3 showed that

[34]Proposition 9 is robust to not only the presence of profits (see Appendix Proposition 20) but also to the presence of foreign income, provided the planner is indifferent to foreign factors' disutility of factor supply. To the extent profits and foreign income are small, income-maximization can alternatively be justified through the lens of its effects solely on involuntary unemployment, one of the most often mentioned concerns of policymakers (Elmendorf and Furman, 2008).

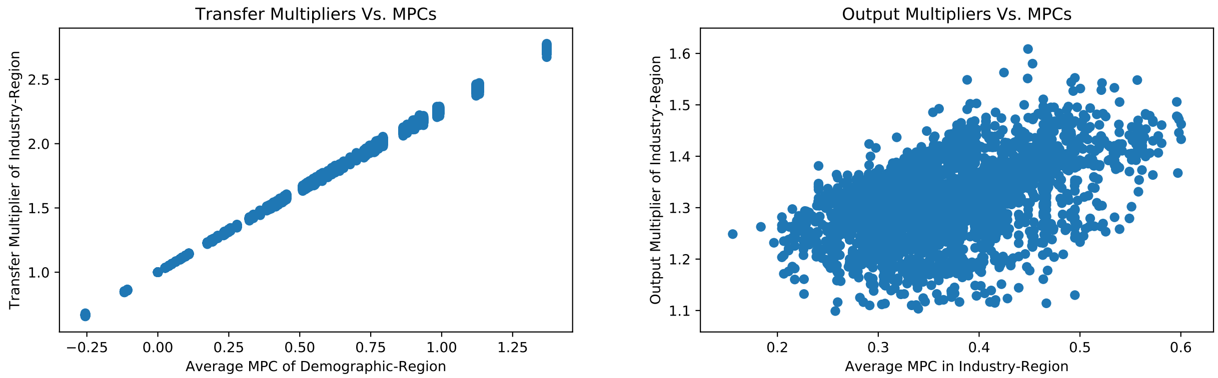


Fig. 4. Left: the effectiveness of targeting transfer stimulus by household MPCs. Right: the effectiveness of targeting expenditure stimulus by the average MPC of workers in each industry-region pair.

the average MPC of households who receive each household's marginal dollar is invariant to the household that spends that marginal dollar – and close to the average MPC weighted by the incidence of a uniform GDP shock.[35] This is precisely the condition required for Corollary 4, implying that both optimal expenditure policy and transfer policy should be designed to target agents with the highest MPCs.

Concretely, optimal transfer policy gives cash to households with the highest MPCs. Even the IO network and industry labor shares are irrelevant to the planner; a policymaker need know only household MPC. The left panel of Figure 4 shows this directly by scattering household MPCs and the resulting transfer multiplier from giving them a dollar, with an effectively perfect relationship between the two.[36]

By contrast, output-maximizing (optimal) expenditure policy targets those sectors such that when their production expands, accounting for the intermediates goods they use and the intermediates used by the producers of those intermediates and so on, the resulting change in labor income ends up in the hands of the highest MPC agents. While this requires no knowledge of the direction of household spending, it does rely on an understanding of the structure of production—through the input-output network and labor rationing. Critically, it is not sufficient for the government to target the sectors employing the highest MPC workers. Instead, they should work out the final labor income consequences of their spending

[35]In particular, Figure 3 shows us that $m_n^{next} \approx \mathbb{E}_{y*}[m_{n'}]$ for all household types $n$.

[36]Note that the MPC that we use in Figure 4 is estimated using unemployment as the identifying shock, and therefore captures the consumption response to a potentially persistent shock. The MPC that is better suited for the analysis of fiscal policy would be the MPC out of a transitory shock. If the MPC out of these two shocks are highly correlated across demographic groups, this difference should be irrelevant for the question of which demographic groups to target. While it is hard to test this explicitly, the cross-demographic patterns in MPCs that we utilize here have a correlation of 0.9 with self-reported MPCs from survey data (Jappelli and Pistaferri, 2014) and have similar patterns as those in response to tax rebates (Parker et al., 2013).

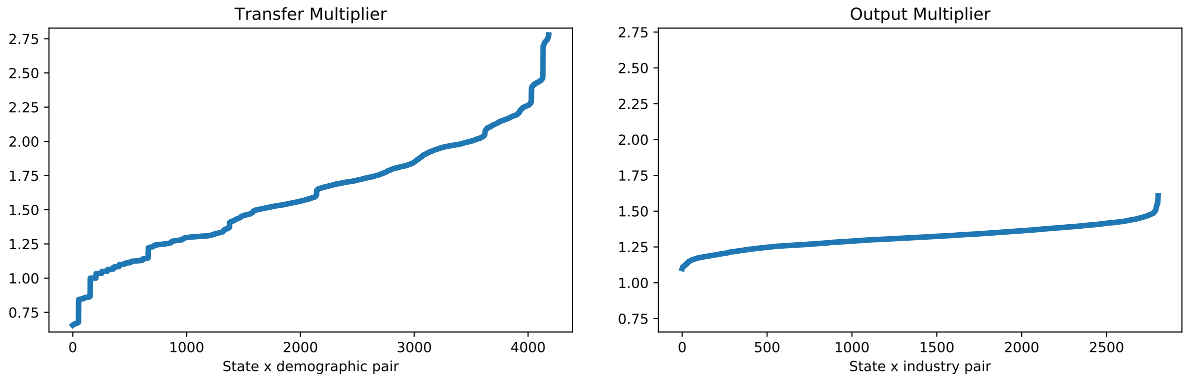


Fig. 5. Left: distribution of transfer multipliers, giving the change in aggregate income from a one dollar transfer to each state-by-demographic group. Right: distribution of output multipliers, giving the change in aggregate income (also, GDP) from one dollar of expenditure on each state-by-industry pair.

and target according to the MPC of the workers receiving that terminal labor income. This difference is quantitatively important; the right panel of Figure 4 shows how naively targeting sectors employing the highest MPC workers is effective but leaves a lot of the gains from targeting on the table.

Figure 5 shows the heterogeneity in transfer and output multipliers. Concretely, the left panel of Figure 5 shows how depending on the household to which a dollar of transfer stimulus is given, the effect on aggregate income ranges from slightly negative (some types have negative MPCs) to nearly three dollars. The right panel shows the corresponding distribution of output multipliers. Both figures emphasize that – beyond just being optimal – targeting fiscal policy based on MPC generates large gains relative to untargeted policy. The heterogeneity in multipliers – and so the gains from targeting – remain when targeting is forced to be more granular: output multipliers differ by a factor of more than three across industries and a factor of 1.5 across states (see Figure A18 in Appendix D); transfer multipliers differ by a factor of 1.3 across states and display the same heterogeneity across demographic groups (See Figure A19 in Appendix D).

Finally, observe that there is much greater heterogeneity in transfer multipliers than output multipliers and therefore that targeted transfer stimulus is likely to dominate targeted expenditures from the perspective of maximizing welfare. This is natural as transfers more effectively target households with the highest MPCs than expenditures. The clear caveat is that fiscal expenditure may have direct value. If this is the case, our analysis shows how much stimulus would have to be sacrificed to obtain that direct value, enabling a policymaker with knowledge of the value of direct government purchases to determine which policy to optimally pursue.

### 6.3. *Geographic Spillovers*

One focus of the recent empirical literature on fiscal multipliers has been the strength of fiscal spillovers across states. Quasi-random cross-regional variation in fiscal spending has allowed researchers to estimate local fiscal multipliers (Nakamura and Steinsson, 2014; Chodorow-Reich, 2019). The relationship between these local estimates and the national multiplier is complicated by the presence of potentially large local spillovers – research designs using cross-sectional estimates usually recover only the relative effect of spending more in state $i$ than in state $j$ and are unable to directly measure the potential effect that spending in state $i$ has on output in state $j$.

The regional interlinkages embedded in our model allow us to provide an estimate for the magnitude of these cross-state spillovers. We quantify these spillovers within our model by considering a unit of government spending in each state, which we assume is distributed across industries within the state in proportion those industries' shares of GDP within the state. Averaging across states, we find that total output in the economy increases by 1.3 units in response to 1 unit of additional spending. Of this 30 percent amplification, about 16 percentage points come within the state that received the additional government spending, while 14 percentage points come from from spillovers to other states – firms and households in the shocked state demand more goods and some of those are sourced from other states.[37] The spillover to any given state is small and only about 2 percent as large as the effects within the shocked state. However, each state contributes to the total effect, and overall, the spillovers contribute meaningfully to the overall effect of the shock.

These estimates are in line with some recent empirical evidence estimating the magnitude of these spillovers directly. Specifically, Auerbach et al. (2020) use detailed geographic information on local defense spending and find that large positive spillovers across geographies, suggesting the importance of positive demand spillovers through input-output networks and directed MPCs. They also find that the spillovers are decreasing in the distance between cities. Our results are consistent with this, as our estimated spillovers are largest for the geographically closer states.[38] These estimates suggest that demand spillovers across states are empirically important when evaluating the total effect of localized fiscal spending.

[37] Of course, the shock itself all remains in the shocked state, so that the total change in output within the shocked state is 1.16, on average.

[38] In Appendix Section C, we more formally explore the extent to which our model predicts the cross-state spillovers in response to several identified demand shocks. While the estimates are under-powered, we find evidence suggesting that our structural estimates are qualitatively consistent with cross-state spillovers in response Chinese-import shocks as in Autor, Dorn, and Hanson (2013).

## *6.4. Changes in the Labor Share and Amplification*

The analysis throughout this paper highlights that the multiplier is not a deep structural parameter of the economy, but rather depends critically on the incidence of the shock in consideration. Similarly, these estimates could change substantially over time as the underlying structure of the economy changes – but we have estimated our multiplier in a single year, 2012. One potentially relevant change in the economy over the past several years is the well-documented decline in the labor share in the US (Karabarbounis and Neiman, 2014; Dorn, Katz, Patterson, and Van Reenen, 2017). More recently, Hazell (2019) provides empirical evidence that this reduction in the labor share has dampened unemployment fluctuations. In this section, we perform a similar exercise in our model, comparing the output and transfer multipliers as industry-specific labor shares change from their 2000 to 2012 levels. Intuitively, if spending is directed away from high-MPC workers and toward low-MPC capitalists, aggregate amplification should fall.

Our methodology is as follows. We assume that, within each year and each industry, the shares of employee compensation in revenue is constant across states. We obtain these shares from the BEA use tables in 2000 and 2012. The aggregate labor share of value added fell from 59.2% in 2000 to 54.9% in 2012; the aggregate labor share of revenue fell from 32.1% to 30.0%. Figure A20 shows the distribution of labor shares of revenue by industry in each year. We maintain our earlier, 2012-based, estimates of demographic-specific consumption baskets and MPCs, demographic employment by region, and input-output network. We allocate the difference in labor income between 2000 and 2012 to a factor with MPC zero; this can be understood as a foreign factor or as profits accruing to MPC-zero shareholders.

Unsurprisingly, the reduction in the labor share leads to a smaller multiplier, as revenues are directed to lower-MPC households. We estimate an aggregate multiplier – i.e. the output response to a shock proportional to the 2012 distribution of output across states and industries – of 1.338 in 2000 and 1.300 in 2012. Figure 6 shows the sorted distributions of output and transfer multipliers across all shocks, for 2000 and 2012. Predictably, the distribution of output multipliers shifts down, as less of the income from a given change in demand flows to workers and more flows to low-MPC factors. Still, the multiplier does not fall for every state-industry pair. Figure A21 shows that a few industries – namely those with sufficiently increased labor shares, such as "apparel and leather and allied products" – have higher multipliers in 2012 than in 2000.

For transfer multipliers, the response to changing labor shares is almost zero. This is because transfers target households of each MPC directly, so that differences in the labor share only affect the incidence of higher-order spending.

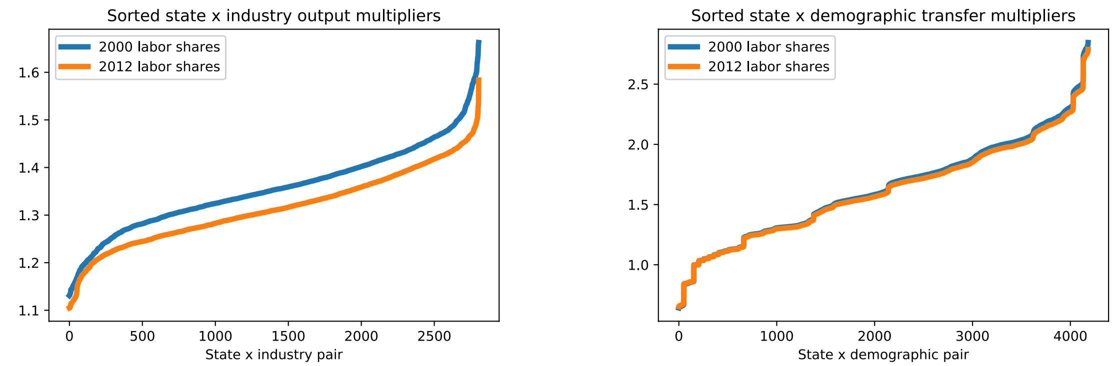


Fig. 6. Multipliers for state-industry-level output shocks and state-demographic-level transfer shocks. Differences in labor shares are more relevant for output shocks.

# 7. Conclusion

This paper has incorporated rich household heterogeneity in MPC magnitudes and directions, industrial and spatial linkages, and differential employment sensitivity into a simple, Keynesian model. All of these elements can be unified into a single reduced-form network that maps the marginal spending of any given household to the marginal income of factor owners producing the goods the household consumes.

Our output decomposition result provides a simple, novel way to understand the importance of these rich interconnections by providing three corrections to the standard Keynesian multiplier. Taking this decomposition to the data, we find that linkages through the direction of household spending are empirically unimportant, so that the effect of a demand or supply shock on aggregate output only depends on the shock's incidence onto the incomes of households of different MPCs. This incidence is shaped by supply chains—which diffuse concentrated shocks over a wider geographic area and set of industries—by differences in labor shares and worker MPCs in different industries, and by the differential employment sensitivity of high-MPC workers within firms.

Our findings have important implications for the design of fiscal policy. First, the effectiveness of government spending in different industries and regions, as well as of transfers to different demographic groups in different regions, differs by an order of magnitude. Second, we emphasize that—despite the interconnections between households in our model—a planner seeking to minimize factor under-utilization can set optimal transfer policy using only information on household MPCs. For small policy changes away from an optimum, this is an empirical fact, owing to the irrelevance of the direction of household consumption; at the optimal policy, it holds independent of our empirical findings.

# References


Acemoglu, D., Azar, P. D., 2020. Endogenous production networks. Econometrica 88(1), 33–82.

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., Tahbaz-Salehi, A., 2012. The network origins of aggregate fluctuations. Econometrica 80, 1977–2016.

Auclert, A., 2019. Monetary policy and the redistribution channel. American Economic Review 109, 2333–67.

Auclert, A., Rognlie, M., Straub, L., 2018. The intertemporal keynesian cross. Tech. rep., National Bureau of Economic Research.

Auerbach, A., Gorodnichenko, Y., Murphy, D., 2020. Local fiscal multipliers and fiscal spillovers in the usa. IMF Economic Review 68, 195–229.

Autor, D. H., Dorn, D., Hanson, G. H., 2013. The china syndrome: Local labor market effects of import competition in the united states. American Economic Review 103, 2121–68.

Baqaee, D. R., 2015. Targeted fiscal policy. Tech. rep.

Baqaee, D. R., Farhi, E., 2018. Macroeconomics with heterogeneous agents and input-output networks. Tech. rep., National Bureau of Economic Research.

Baqaee, D. R., Farhi, E., 2019. The macroeconomic impact of microeconomic shocks: beyond hulten's theorem. Econometrica 87, 1155–1203.

Bilbiie, F. O., 2019. The new keynesian cross. Journal of Monetary Economics .

Blundell, R., Pistaferri, L., Preston, I., 2008. Consumption inequality and partial insurance. American Economic Review 98, 1887–1921.

Broda, C., Parker, J. A., 2014. The economic stimulus payments of 2008 and the aggregate demand for consumption. Journal of Monetary Economics 68, S20–S36.

Caliendo, L., Parro, F., Rossi-Hansberg, E., Sarte, P.-D., 2018. The impact of regional and sectoral productivity changes on the us economy. The Review of economic studies 85, 2042–2096.

Campbell, J. Y., Mankiw, G., 1989. Consumption, income and interest rates: Reinterpreting the time series evidence. NBER Macroeconomics Annual 4, 185–246.

Carroll, C., Slacalek, J., Tokuoka, K., White, M. N., 2017. The distribution of wealth and the marginal propensity to consume. Quantitative Economics 8, 977–1020.

Chodorow-Reich, G., 2014. The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis. The Quarterly Journal of Economics 129, 1–59.

Chodorow-Reich, G., 2019. Geographic cross-sectional fiscal spending multipliers: What have we learned? American Economic Journal: Economic Policy 11, 1–34.

Chodorow-Reich, G., Nenov, P. T., Simsek, A., 2019. Stock market wealth and the real economy: A local labor market approach. Tech. rep., National Bureau of Economic Research.

Corbi, R., Papaioannou, E., Surico, P., 2019. Regional transfer multipliers. The Review of Economic Studies 86, 1901–1934.

Corsetti, G., Kuester, K., Müller, G. J., 2013. Floats pegs and the transmission of fiscal policy. Series on Central Banking Analysis and Economic Policies no. 17 .

Cox, L., Müller, G. J., Pasten, E., Schoenle, R., Weber, M., 2019. Big g. Tech. rep.

Dorn, D., Katz, L. F., Patterson, C., Van Reenen, J., 2017. Concentrating on the fall of the labor share. American Economic Review 107, 180–85.

Elmendorf, D. W., Furman, J., 2008. Three keys to effective fiscal stimulus. Washington Post, January 26.

Fagereng, A., Holm, M. B., Natvik, G. J. J., 2019. Mpc heterogeneity and household balance sheets. Tech. rep.

Farhi, E., Werning, I., 2017. Fiscal unions. American Economic Review 107, 3788–3834.

Feyrer, J., Mansur, E. T., Sacerdote, B., 2017. Geographic dispersion of economic shocks: Evidence from the fracking revolution. American Economic Review 107, 1313–34.

Gabaix, X., 2011. The granular origins of aggregate fluctuations. Econometrica 79, 733–772.

Grigsby, J., Hurst, E., Yildirmaz, A., 2019. Aggregate nominal wage adjustments: New evidence from administrative payroll data. Tech. rep., National Bureau of Economic Research.

Gruber, J., 1997. The consumption smoothing benefits of unemployment insurance. The American Economic Review 87, 192.

Guvenen, F., Smith, A. A., 2014. Inferring labor income risk and partial insurance from economic choices. Econometrica 82, 2085–2129.

Hastings, J., Shapiro, J. M., 2018. How are snap benefits spent? evidence from a retail panel. American Economic Review 108, 3493–3540.

Hazell, J., 2019. The falling labor share dampens unemployment fluctuations. Tech. rep.

Hazell, J., Taska, B., 2019. Downward rigidity in the wage for new hires. Tech. rep.

Hubmer, J., 2019. The race between preferences and technology. Tech. rep., University of Pennsylvania.

Jappelli, T., Pistaferri, L., 2014. Fiscal policy and mpc heterogeneity. American Economic Journal: Macroeconomics 6, 107–36.

Kaplan, G., Moll, B., Violante, G. L., 2018. Monetary policy according to hank. American Economic Review 108, 697–743.

Kaplan, G., Violante, G. L., 2014. A model of the consumption response to fiscal stimulus payments. Econometrica 82, 1199–1239.

Kaplan, G., Violante, G. L., Weidner, J., 2014. The Wealthy Hand-to-Mouth. Brookings Papers on Economic Activity 45, 77–153.

Karabarbounis, L., Neiman, B., 2014. The global decline of the labor share. The Quarterly journal of economics 129, 61–103.

Keynes, J. M., 1936. The general theory of employment, interest and money (1936). Kessinger Publishing.

Koby, Y., Wolf, C. K., 2019. Aggregation in heterogeneous-firm models: A sufficient statistics approach. Tech. rep., Princeton University.

Long, J. B., Plosser, C. I., 1987. Sectoral vs. aggregate shocks in the business cycle. The American Economic Review 77, 333–336.

Mian, A., Sufi, A., 2011. House prices, home equity-based borrowing, and the us household leverage crisis. American Economic Review 101, 2132–56.

Mian, A., Sufi, A., Verner, E., 2017. How do credit supply shocks affect the real economy? evidence from the united states in the 1980s. Tech. rep., National Bureau of Economic Research.

Miyazawa, K., 1976. Input-output analysis and interrelational income multiplier as a matrix. In: *Input-Output Analysis and the Structure of Income Distribution*, Springer, pp. 22–42.

Nakamura, E., Steinsson, J., 2014. Fiscal stimulus in a monetary union: Evidence from us regions. American Economic Review 104, 753–92.

Parker, J. A., Souleles, N. S., Johnson, D. S., McClelland, R., 2013. Consumer spending and the economic stimulus payments of 2008. American Economic Review 103, 2530–53.

Patterson, C., 2019. The matching multiplier and the amplification of recessions. In: *2019 Meeting Papers*, Society for Economic Dynamics, no. 95.

Ramey, V. A., 2011. Can government purchases stimulate the economy? Journal of Economic Literature 49, 673–85.

Rubbo, E., 2019. Networks, phillips curves, and monetary policy. Tech. rep., Harvard University.

Solon, G., Barsky, R., Parker, J. A., 1994. Measuring the cyclicality of real wages: how important is composition bias? The quarterly journal of economics 109, 1–25.

Vissing-Jorgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. Journal of Political Economy 110, 825–853.

Werning, I., 2011. Managing a liquidity trap: Monetary and fiscal policy. Tech. rep., National Bureau of Economic Research.

Werning, I., 2015. Incomplete markets and aggregate demand. Tech. rep., National Bureau of Economic Research.

Wolf, C., 2019. The missing intercept: A demand equivalence approach. Tech. rep., Princeton University.

Zorzi, N., 2020. Investment dynamics and cyclical redistribution. Tech. rep.

# Appendices

## A. Omitted Proofs

### *A.1. Proof of Proposition 1*

*Proof.* We follow closely the proof technique used in Acemoglu and Azar (2020). We will prove the result for an economy with arbitrary time horizon for maximum applicability. Fix a time period $t$ vector of productivity parameters $z$. For each $i$, define the unit cost function:

$$\kappa_i(p) = \underset{F(X_i, L_i, z_i) \geqslant 1,\ X_i, L_i \geqslant 0}{min} pX_i + L_i \tag{A1}$$

The minimum is well defined owing to Assumption 1, which states that $F$ is strictly increasing in labor, CRS, and strictly quasiconcave.

We now establish properties of the unit cost function on the domain $p \in \mathbb{R}_+^{\mathcal{I}}$. First, since labor is necessary for production, $\kappa_i(0) > 0$ for all $i$. Second, by the last part of Assumption 1, there exists $\overline{p}$ such that $\kappa_i(\overline{p}) \leqslant \overline{p}_i$ for all $i$. Finally, $\kappa_i(p)$ is weakly increasing in $p$ by inspection. These three properties establish that $\kappa(p) \equiv (\kappa_1(p), ..., \kappa_{\mathcal{I}}(p))$ maps $\mathbb{O} \equiv \times_{i=1}^{\mathcal{I}} [0, \overline{p}_i] \to \mathbb{O}$ and is weakly increasing. Moreover, $\mathbb{O}$ is a complete lattice with respect to the following operators:

$$\begin{aligned} p \wedge q &= \big(min(p_1, q_1), ..., min(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \\ p \vee q &= \big(max(p_1, q_1), ..., max(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \end{aligned} \tag{A2}$$

By Tarksi's fixed point theorem, the set of fixed points $\{p \in \mathbb{R}_+^{\mathcal{I}} \mid \kappa(p) = p\}$ is therefore a complete lattice.

In order for $p$ to be consistent with either our flexible-wage or rationing equilibrium, all operating firms must make zero profits. Assumption 2 implies that all firms operate in equilibrium, so $p = \kappa(p)$ is a necessary condition for any equilibrium. It therefore remains to show that $\kappa$ has a unique fixed point. To this end, we first show that each $\kappa_i$ is concave. For price vectors $p$ and $q$ and $\lambda \in (0, 1)$, we construct the price vector:

$$p^{\lambda} = \lambda p + (1 - \lambda) q \tag{A3}$$

By cost minimization,

$$\begin{aligned}\kappa_i(p) &\leqslant pX_i(p^\lambda) + L_i(p^\lambda)\\ \kappa_i(q) &\leqslant qX_i(p^\lambda) + L_i(p^\lambda)\end{aligned} \tag{A4}$$

It follows that:

$$\kappa_i(p^\lambda) = p^\lambda X_i(p^\lambda) + L_i(p^\lambda) \geqslant \lambda\kappa_i(p) + (1-\lambda)\kappa_i(q) \tag{A5}$$

establishing that each $\kappa_i$ is a concave function.

Toward a contradiction, suppose $\kappa$ has more than one fixed point. Then since the set of fixed points is a complete lattice, there must exist distinct fixed points $p^*, p^{**}$ with $p_i^* \leqslant p_i^{**}$ for all i. Now take $\lambda$ to be given by the following:

$$\lambda = \underset{i\in\mathcal{I}}{min}\ \frac{p_i^*}{p_i^{**}} \tag{A6}$$

Note that $\lambda \in (0,1)$ since $p >> 0$ for all fixed points $p$, since $\kappa_i(0) > 0$ for all $i$ and $\kappa$ is weakly increasing. We have that $p_i^* \geqslant \lambda p_i^{**}$ for all $i \in \mathcal{I}$ with equality for at least one $j$ by construction. For this $j$ such that $p_j^* = \lambda p_j^{**}$, we then have

$$\begin{aligned} 0 &= \kappa_j(p^*) - p_j^*\\ &\geqslant \kappa_j(\lambda p^{**}) - \lambda p_j^{**}\\ &\geqslant (1-\lambda)\kappa_j(0) + \lambda\kappa_j(p^{**}) - \lambda p_j^{**}\\ &= (1-\lambda)\kappa_j(0)\\ &> 0\end{aligned} \tag{A7}$$

where the first line follows from the zero profit condition, the second line follows from the fact that $\kappa_i$ is weakly increasing and $\lambda \in (0,1)$, the third line follows from concavity of $\kappa_i$, the fourth line follows again from the zero profit condition, and the final line follows from positivity of costs. This is a contradiction. Hence, there must be a unique fixed point at all times $t$. This implies the stated result and also makes the no-substitution theorem applicable to Appendix B.1 where we extend the baseline model to allow for multiple time periods. □

## A.2. *Proof of Corollary 1*

*Proof.* Fixing $z$, by Proposition 1, there exists a unique price vector $p(z)$ consistent with equilibrium. The unit input demands for any firm $i$ at this price solve the following program:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i,L_i) \text{ s.t. } F(X_i,L_i,z_i)\geqslant 1} p(z)X_i + L_i \tag{A8}$$

CRS then implies that for a firm producing $Q_i$ units in equilibrium,

$$X_i = Q_i\widehat{X}_i(z) \quad L_i = Q_i\widehat{L}_i(z) \tag{A9}$$

Stacking these equations over $\mathcal{I}^t$ gives

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{A10}$$

□

## A.3. *Proof of Corollary 2*

*Proof.* We first prove that the matrix $(I - \widehat{X}(z))$ is invertible. The zero-profit condition for all $i$ implies that:

$$p(z)X_i + L_i = p_i(z)Q_i \tag{A11}$$

Normalizing by the quantity yields:

$$p(z)\widehat{X}_i(z) + \widehat{L}_i(z) = p_i(z) \tag{A12}$$

Stacking this equation yields the matrix equation:

$$\widehat{L}(z)\vec{1} + \widehat{X}^T(z)p(z) = p(z) \tag{A13}$$

This allows us to solve for the unit labor demands as the unique diagonal matrix such that:

$$\widehat{L}(z)\vec{1} = \left(I - \widehat{X}(z)^T\right)p(z) \tag{A14}$$

Iterating this equation $k \in \mathbb{N}$ times yields:

$$p(z) = \left(1 + \widehat{X}(z)^T + ... + \left(\widehat{X}(z)^T\right)^k\right) \widehat{L}(z)\vec{1} + \left(\widehat{X}(z)^T\right)^{k+1}p(z) \tag{A15}$$

Recall that $\widehat{X}(z)$ is non-negative, $\widehat{L}(z)\vec{1}$ is strictly positive because labor is essential, and $p(z)$ is positive. A necessary condition for $p(z)$ to exist is therefore that $\left(\widehat{X}(z)^T\right)^k \to 0$ as $k \to \infty$. This implies that $\widehat{X}(z)^T$ (and therefore also $\widehat{X}(z)$) has modulus strictly less than unity. It is immediate that the inverse $\left(I - \widehat{X}(z)\right)^{-1}$ exists. From this it follows that:

$$p(z) = \left(1 - \widehat{X}(z)^T\right)^{-1}\widehat{L}(z)\vec{1} \tag{A16}$$

completing the proof. □

## A.4. Proof of Proposition 2

*Proof.* Fix all exogenous parameters. Note that by Proposition 1, prices $p^1$ and $p^2$ are pinned down by technology and so can be taken as given as well.

The outline of the proof is as follows. First, for any interest rate $r^1$, we will construct a function $\Psi_{r^1}$ that maps vectors of first-period income to vectors of first-period income and show that any fixed point of this map corresponds to an equilibrium with constant $r^1$. Second, we extend this map to construct a second function $\Psi$ that takes as inputs both a vector of incomes and an interest rate, and we show that any fixed point of this extended map corresponds to an equilibrium of the model. We then apply Brouwer's fixed point theorem to $\Psi$ to show that such a fixed point exists.

First, by Assumption 3 we have the following two facts:

1. For any $p^1, p^2, \tau, \theta$: $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in $y_n^1$ for any $n$, $r^1 \in [\underline{r}, \overline{r}]$
2. For any $p^1, p^2, \tau, \theta$: there exists some $\overline{y} \in \mathbb{R}_+$ and some $\overline{c} < 1$ such that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$ for all $n$, $y_n^1 > \overline{y}$, $r^1 \in [\underline{r}, \overline{r}]$

Thus, given any vector of incomes $y^1$, total first period consumption spending $C^1$ is bounded above:

$$C^1 \leqslant \overline{cy} + \overline{c}\vec{1}'y^1 \tag{A17}$$

Thus, aggregate spending is bounded above by:

$$C^1 + G^1 \leqslant \overline{c}(\overline{y} + \vec{1}'y^1) + \max_{r \in [\underline{r}, \overline{r}]} p^1 G^1(p^1, p^2, r^1, \tau, \theta_G) \tag{A18}$$

where this maximum exists by continuity of $G^1(\cdot)$ in $r^1$ and compactness of $[\underline{r}, \overline{r}]$. Since $\overline{c} < 1$, it follows that there exists $\overline{Y}$ such that if $y^1 \in Y^1 \equiv \{y^1 \in \mathbb{R}_+^N \mid \vec{1}'y^1 \leqslant \overline{Y}\}$, then aggregate spending—and so, as all spending flows to wages, also the resulting aggregate income—is weakly less than $\overline{Y}$. Formally:

$$\forall r^1 \in [\underline{r}, \overline{r}], y^1 \in Y^1 : l^1 \left(\widehat{L}^1(1-\widehat{X}^1)^{-1}\left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G)\right)\right) \in Y^1 \tag{A19}$$

This observation allows us to define, for any $r^1 \in [\underline{r}, \overline{r}]$, a function $\Psi_{r^1} : Y^1 \to Y^1$ given by:

$$\Psi_{r^1}(y^1) = l^1 \left(\widehat{L}^1(1-\widehat{X}^1)^{-1}\left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G)\right)\right) \tag{A20}$$

where recall $\varrho$ denotes $(p^1, p^2, r^1)$ and where the previous argument establishes that $\Psi_{r^1}(y^1)$ is indeed contained in $Y^1$. Moreover, continuity of $l^1(\cdot), C^1(\cdot)$ and $G^1(\cdot)$ establishes that $\Psi_{r^1}$ is a continuous function.

Second, we define an extended function $\Psi : Y^1 \times [\underline{r}, \overline{r}] \to Y^1 \times [\underline{r}, \overline{r}]$ by setting:

$$\Psi(y^1, r^1) = \left(\Psi_{r^1}(y^1), r^1(Q)\right) \tag{A21}$$

where $Q = (Q^1, Q^2)$ is given by:

$$Q^t = (1-\widehat{X}^1)^{-1}\left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G)\right) \tag{A22}$$

and where $r^1(\cdot)$ is the monetary policy function, which recall selects an interest rate in $[\underline{r}, \bar{r}]$.

Third, we now claim that $\Psi$ has a fixed point $(y^1, r^1)$. This follows from Brouwer's theorem: $Y^1 \times [\underline{r}, \overline{r}]$ is a compact, convex domain, and $\Psi$ is continuous because $l^1(\cdot)$ and $r^1(\cdot)$ are continuous, $c_n^t(\varrho, y_n^1, \tau_n, \theta_n)$ is continuous in $y_n^1$ and $r^1$, and $G^t(\varrho, \tau, \theta_G)$ is continuous in $r^1$.

Finally, given a fixed point $(y^1, r^1)$ of $\Psi$, we can construct a rationing equilibrium as follows: Let $p^t$ be the no-substitution-theorem prices implied by $z^t$. Let $c_n^t$, $l_n^2$, and $G^t$ be given by the relevant functions taking in prices $p^t$, real rate $r^1$, and incomes $y^1$. Let production in each period be:

$$Q^t = (I-\widehat{X}^t)^{-1}(G^t + C^t) \tag{A23}$$

The definition of the consumption, labor supply, and government spending function ensure that household and government budget constraints hold. The construction of $Q^t$ ensures that each goods market clears. Because $(y^1, r^1)$ is a fixed point, first period income is consistent with the rationing function and the first period labor market clears; also because $(y^1, r^1)$ is a fixed point, the interest rate $r^1 = r^1(Q)$ is consistent with central bank policy. Finally, the second period labor market clears by Walras' law. □

## A.5. *Proof of Proposition 3*

*Proof.* We consider each shock case by case. For each, we totally differentiate the goods market clearing condition and group all terms that have no dependence on resulting changes in equilibrium output. To this end, recall that the goods market clearing condition is given by:

$$Q = \widehat{X}Q + G + C \tag{A24}$$

Total differentiation yields:

$$\begin{aligned} dQ \;&= \widehat{X}dQ + \widehat{X}_z dzQ + C_{\hat{p}}\hat{p}_z dz + C_{r^1}dr^1 + C_{y^1}dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}}\hat{p}_z dz + G_{r^1}dr^1 + G_\tau d\tau + G_{\theta_G}d\theta_G \end{aligned} \tag{A25}$$

where $dy^1 = l^1_{L^1}\widehat{L}^1 dQ^1 + l^1_{L^1}d\widehat{L}^1 Q^1$

We now isolate the partial equilibrium effect of each type of shock by both zeroing all general equilibrium effects that operate through changes in gross production $Q$ and zeroing all other shocks:

1. A change in government preferences $\theta_G$ by $d\theta_G$:

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G)d\theta_G \tag{A26}$$

2. A change in household preferences $\theta$ by $d\theta$:

$$\partial Q = C_\theta(\varrho, \tau, \theta)d\theta \tag{A27}$$

3. A change in taxes or transfers by $d\tau$:

$$\partial Q = C_\tau(\varrho, \tau, \theta)d\tau + G_\tau(\varrho, \tau, \theta_G)d\tau \tag{A28}$$

4. A change in productivity $z$ by $dz$:

$$\partial Q = (C_p + G_p)p_z dz + \widehat{X}_z dzQ + C_{y^1}l^1_{L^1}\widehat{L}^1_z dzQ^1 \tag{A29}$$

□

## A.6. *Proof of Proposition 4*

*Proof.* The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that

$\omega_n \to \omega$. By Proposition 2, we know that for each $\omega_n$ there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit $\omega$. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at $e$: household budget balance, government budget balance or market clearing. But by Assumption 3, continuity of the fiscal rule, continuity of the interest rate rule and continuity of the rationing function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{A30}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions. Our differentiability assumptions allow us to express

$$\begin{aligned} dQ &= \widehat{X} dQ + \widehat{X}_z dz Q + C_{\hat{p}} \hat{p}_z dz + C_{r^1} dr^1 + C_{y^1} dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}} \hat{p}_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G \end{aligned} \tag{A31}$$

Plugging in for $dr^1$ and $dy^1 = l^1_{L^1} \widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X} dQ + C_{y^1} l^1_{L^1} \widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1}) r^1_Q dQ + \partial Q \tag{A32}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}}) \hat{p}_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1 + (C_\tau + G_\tau) d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. Recognizing that $dY = (I - \widehat{X}) dQ$ and subsituting completes the proof. □

## A.7. *Proof of Corollary 3*

*Proof.* Recall from Proposition 4 that:

$$dY = \left( I - D \left( I - \widehat{X} \right)^{-1} \right)^{-1} \partial Q \equiv M \partial Q \tag{A33}$$

where:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left( C^1_{r^1} + G^1_{r^1} \right) r^1_{Q^1} & \left( C^1_{r^1} + G^1_{r^1} \right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left( C^2_{r^1} + G^2_{r^1} \right) r^1_{Q^1} & \left( C^2_{r^1} + G^2_{r^1} \right) r^1_{Q^2} \end{bmatrix} \tag{A34}$$

Under Assumption 4, we have that this reduces to:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \end{bmatrix} \tag{A35}$$

Simple matrix manipulations show that one may extract just the first $\mathcal{I}^1$ rows:

$$dY^1 = \left( I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \right)^{-1} \partial Q^1 \tag{A36}$$

□

## A.8. Proof of Proposition 5

*Proof.* Starting from Corollary 3 and using that the modulus of $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1}$ is less than 1, we can express:

$$\begin{aligned} dY^1 &= \sum_{k=0}^{\infty} \left[ C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \right]^k \partial Q^1 \\ &= \partial Q + \overline{C}^1_{y^1} \hat{m} \sum_{k=0}^{\infty} \left[ l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \overline{C}^1_{y^1} \hat{m} \right]^k l^1_{L^1} \widehat{L}^1 \left( I - \widehat{X}^1 \right)^{-1} \partial Q^1 \qquad \text{(A37)} \\ \vec{1}' dY^1 &= \vec{1}^T \partial Q + m^T \left( \sum_{k=0}^{\infty} (\mathcal{G} \hat{m})^k \right) \partial y^1 \end{aligned}$$

where the last line uses the definitions of $\mathcal{G}$ and $\partial y^1$, and the fact that $\vec{1}^T \overline{C}^1_{y^1} = \vec{1}^T$ (by construction).

Finally, $\vec{1}^T \partial Q^1 = \vec{1}^T \partial y^1$ because $\vec{1}^T \cdot l^1_{L_1} \widehat{L}_1 (I - \hat{X}^1)^{-1} = \vec{1}^T$, since firms earn zero profits.

□

## A.9. Proof of Proposition 6

*Proof.* Let $b \equiv \vec{1}^T (I - \mathcal{G}\hat{m})^{-1}$ be the vector of Bonacich centralities of households in the income-to-spending network; these are well defined as we have assumed the modulus of $\mathcal{G}\hat{m}$ is less than one. Let $(b^{next})^T = b^T \mathcal{G}$ be the row vector with $n^{th}$ entry equal to the average Bonacich centrality of the household to whom $n$'s marginal spending flows.

We begin by providing a lemma that exactly decomposes the general equilibrium change in output, in terms of Bonacich centralities.

**Lemma 1.** *For any $x \in \mathbb{R}$, the total change in first-period output due to a partial equilibrium* *demand shock with unit-magnitude labor income incidence $\partial y^1$ is equal to*

$$\vec{1}^T dY^1 = (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - x\right) + \mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}] \tag{A38}$$

*Setting $x$ equal to the $\frac{1}{1-MPC}$ multiplier with the MPC weighted by income $y^*$, we obtain an exact decomposition in the spirit of Proposition 6.*

$$\begin{aligned}\vec{1}^T dY^1 = & \underbrace{\frac{1}{1-\mathbb{E}_{y*}[m_n]}}_{\text{Keynesian multiplier}} + \underbrace{\frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1-\mathbb{E}_{y*}[m_n]}}_{\text{Incidence effect}} \\ & + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - \frac{1}{1-\mathbb{E}_{y*}[m_n]}\right)}_{\text{Biased MPC direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}]}_{\text{Homophily effect}}\end{aligned} \tag{A39}$$

*Proof.* Note that Proposition 5 implies that the change in output resulting from some shock with unit incidence is given by

$$\vec{1}^T dY^1 = b^T \partial y^1 = \vec{1}^T \partial y^1 + b^T \mathcal{G} \hat{m} \partial y^1 \tag{A40}$$

Letting $b^{nextT} = b^T \mathcal{G}$ be the row vector with $i^{th}$ entry equal to the average Bonacich centrality of the household who $i$'s marginal spending flows to. We then have, for any $x \in \mathbb{R}$:

$$\begin{aligned}\vec{1}^T dY^1 &= 1 + \mathbb{E}_{\partial y^1}[m_n b_n^{\text{next}}] = 1 + \mathbb{E}_{\partial y^1}[m_n] \cdot \mathbb{E}_{\partial y^1}[b_n^{\text{next}}] + \mathrm{Cov}_{\partial y^1}[m_n, b_n^{\text{next}}] \\ &= (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{\text{next}}] - x\right) + \mathrm{Cov}_{\partial y^1}[m_n, b_n^{\text{next}}]\end{aligned} \tag{A41}$$

□

We can now prove Proposition 6. First, note that:

$$b_n = 1 + m_n + O(|m|^2) = 1 + \frac{m_n}{1 - \mathbb{E}_{y^*}[m_{n'}]} + O(|m|^2) \tag{A42}$$

Plugging this into Equation A39, we have

$$\begin{aligned}
\vec{1}^T dY^1 &= \frac{1}{1-\mathbb{E}_{y*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1-\mathbb{E}_{y*}[m_n]} \\
&+ \mathbb{E}_{\partial y^1}[m_n]\left(1 + \frac{\mathbb{E}_{\partial y^1}[m_n^{\text{next}}]}{1-\mathbb{E}_{y*}[m_{n'}]} + O(|m|^2) - 1 - \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y*}[m_n]}\right) \\
&+ \mathbb{C}\text{ov}_{\partial y^1}\left[m_n, 1 + \frac{m_n^{\text{next}}}{1-\mathbb{E}_{y*}[m_{n'}]} + O(|m|^2)\right] \\
&= \frac{1}{1-\mathbb{E}_{y*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}{1-\mathbb{E}_{y*}[m_n]} \\
&+ \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y*}[m_n]}\mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] \\
&+ \left(\frac{1}{1-\mathbb{E}_{y*}[m_n]}\right)\mathbb{C}\text{ov}_{\partial y^1}\left[m_n, m_n^{\text{next}}\right] + O(|m|^3)
\end{aligned} \tag{A43}$$

Rearranging, we have Equation 28. □

## A.10. Proof of Proposition 7

*Proof.* We prove the two claims separately:

1. Recalling that $(m^{\text{next}})^T \equiv m^T\mathcal{G}$, and the shock satisfies $\partial y^1 \propto y^1$, the following are equivalent:
$$m^T\mathcal{G}y^1 - m^T y^1 = 0 \iff \mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] = 0 \tag{A44}$$
It therefore suffices to show that $\mathcal{G}y^1 = y^1$.
Plugging in the definition of $\mathcal{G}$, we have $\mathcal{G}y^1 = l_{L^1}^1\hat{L}^1\left(I - \hat{X}^1\right)^{-1}\overline{C}_{y^1}^1 y^1$. Since each household saves zero on net, $y^1$ is equal to total spending. Homotheticity of consumption implies that $\overline{C}_{y^1}^1 y^1$, then, is the vector of total consumption of goods; since there is no government spending, this equals total output, $Y^1$. Finally, homotheticity of rationing implies that $l_{L^1}^1\hat{L}^1\left(I - \hat{X}^1\right)^{-1}Y^1 = y^1$.
2. Recall by Corollary 3 that when either $C_{r^1} + G_{r^1} = 0$ or $r_{Q^1}^1 = 0$, the general equilibrium effect on income of a partial equilibrium shock is given by:
$$dY^1 = \left(I - C_y^1 l_{L^1}^1\hat{L}^1(1-\hat{X}^1)^{-1}\right)^{-1}\partial Q^1 \tag{A45}$$
We wish to investigate whether there exists some $m \in (0,1)$ such that the following holds for all $\partial Q$:
$$\vec{1}^T dY^1 = \frac{1}{1-m}\vec{1}^T\partial Q^1 \tag{A46}$$

First, we note a simple fact of linear algebra. Suppose an invertible matrix $M$ has columns summing to some constant $m$. This is equivalent to:

$$\vec{1}^T M v = m\vec{1}^T v, \quad \forall v \tag{A47}$$

It is then true that for any $v$:

$$m\vec{1}^T(M^{-1}v) = \vec{1}^T M(M^{-1}v) = \vec{1}^T v \tag{A48}$$

Thus, $M^{-1}$ has columns summing to $\frac{1}{m}$.
Second, note that the desired result (A46) holds if and only if

$$\left(I - C_y^1 l_{L^1}^1 \widehat{L}^1(1-\widehat{X}^1)^{-1}\right)^{-1} \tag{A49}$$

sums to $\frac{1}{1-m}$. This is equivalent, by the first observation, to the claim that each column of:

$$C_y^1 l_{L^1}^1 \widehat{L}^1(1-\widehat{X}^1)^{-1} \tag{A50}$$

sums to $m$.
It remains to show that this claim is equivalent to the condition provided in the statement of the Proposition. Namely, we must show that

$$\vec{1}^T C_y^1 l_{L^1}^1 \widehat{L}^1(1-\widehat{X}^1)^{-1} = m\vec{1}^T \iff \vec{1}^T C_y^1 l_{L^1}^1 = m\vec{1}^T \tag{A51}$$

Multiplying each side by $(I - \widehat{X}^1)(\widehat{L}^1)^{-1}$—which exists since labor is essential in production—reveals that (A51) holds if $(I-\widehat{X}^1)(\widehat{L}^1)^{-1}$ has columns summing to one. By our earlier linear algebra observation, this holds if and only if $\widehat{L}^1(1-\widehat{X}^1)^{-1}$ has columns summing to one. This can be seen by recalling the no-profit condition

$$p^1 = (I - (\widehat{X}^1)^T)^{-1}\widehat{L}^1\vec{1}, \tag{A52}$$

using our normalization $p = \vec{1}$, and taking the transpose of both sides.

□

## A.11. Proof of Proposition 8

The full version of the planner's problem, Equation 33, is

$$\begin{aligned}
\max_{\{c^t_{ni}, l^t_n, Q^t_i, G^t_i, \tau^t_n\}_{t\in\{1,2\}, n\in N, i\in\mathcal{I}^t}} \quad & W \equiv \sum_{n\in N} \mu_n \lambda_n \sum_{t=1,2} \beta_n^{t-1}\Big[u^t_n(\tilde{c}^1) - v^t_n(\tilde{l}^t) + w^t_n(G^t)\Big] \\
\text{s.t. } & (c^1_n, c^2_n, l^2_n) \text{ solves Equation 32 given } l^1_n \\
& Q^t = \mu^T c^t + \widehat{X}^t(z^t)Q^t + G^t \\
& \widehat{\mu} l^1 = l^1(\widehat{L}^1 Q^1),\ \ \mu^T l^2 = \vec{1}^T \widehat{L}^2(z^t)Q^2 \\
& \vec{1} \equiv p^t = \Big(I - \widehat{X}^t(z^t)\Big)^{-1} \widehat{L}^t(z^t)\vec{1} \\
& \vec{1}^T G^1 + \frac{\vec{1}^T G^2}{1+r^1} + \mu^T\tau^1 + \frac{\mu^T\tau^2}{1+r^1} = 0
\end{aligned} \tag{A53}$$

*Proof.* To begin, we define $\kappa^t_n$ to be $n$'s marginal value of additional expenditure in period $t$, i.e. for all $i$, $u^t_{nc_i} = \kappa^t_n$ (recall prices are normalized to one). Therefore,

$$\begin{aligned}
dW &= \sum_{n\in N} \lambda_n \mu_n \sum_{t=1,2} \beta_n^{t-1}\Big(u^t_{nc}dc^t_n - v^{t\,\prime}_n dl^t_n + w^t_{nG}dG^t\Big) \\
&= \sum_{n\in N} \lambda_n \mu_n \sum_{t=1,2} \beta_n^{t-1}\left[\kappa^t_n\left(\vec{1}^T dc^t_n - \frac{v^{t\,\prime}_n}{\kappa^t_n} dl^t_n\right) + w^t_{nG}dG^t\right]
\end{aligned} \tag{A54}$$

Next note that in the second period, free labor supply implies $v^{2\prime}_n = \kappa^2_n$. In the first, there may be some wedge $\Delta_n$ such that $v^{2\prime}_n = \kappa^2_n(1+\Delta_n)$; a positive wedge indicates that $n$ works as if the wage was higher than it is, i.e. oversupplies labor; a negative wedge represents involuntary un(der)employment. In these terms, we have

$$\begin{aligned}
dW = \sum_{n\in N} \lambda_n \kappa^1_n \mu_n \Bigg[ &- \Delta_n dl^1_n + \sum_{t=1,2} \frac{\kappa^t_n}{\kappa^1_n}\beta_n^{t-1}\left(\vec{1}^T dc^t_n - dl^t_n\right) \\
&+ \left(\frac{w^1_{nG}}{\kappa^1_n} dG^1 + \frac{\beta_n w^2_{nG}}{\kappa^1_n} dG^2\right)\Bigg]
\end{aligned} \tag{A55}$$

Next, define $\widetilde{\lambda}_n = \lambda_n \kappa^1_n$. Also note that $\frac{\kappa^t_n}{\kappa^1_n}\beta_n^{t-1} = 1$ for $t = 1$. For $t = 2$, we use the modified Euler equation:

$$\kappa^1_n = \beta_n \frac{1+r^1}{1-\phi_n}\kappa^2_n \tag{A56}$$

where $\phi_n$ is a borrowing wedge. $\phi_n \geqslant 0$ is positive when households behave as if interest rates are higher than in reality, i.e. consume more in the future than they would like; this corresponds to borrowing constraints. This gives us

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 + \left( \vec{1}^T dc_n^1 - dl_n^1 \right) + \frac{1-\phi_n}{1+r^1} \left( \vec{1}^T dc_n^2 - dl_n^2 \right) \right. \\ \left. + \left( \frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left( \frac{1-\phi_n}{1+r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right] \tag{A57}$$

Differentiating the household's lifetime budget constraint (at constant $r^1$):

$$\vec{1}^T dc_n^1 - dl_n^1 + \frac{\vec{1}^T dc_n^2 - dl_n^2}{1+r^1} = -d\tau_n^1 - \frac{d\tau_n^2}{1+r^1} \tag{A58}$$

Plugging this in, we have:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 + \phi_n \left( \vec{1}^T dc_n^1 - dl_n^1 \right) - (1-\phi_n) \left( d\tau_n^1 + \frac{d\tau_n^2}{1+r^1} \right) \right. \\ \left. + \left( \frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left( \frac{1-\phi_n}{1+r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right] \tag{A59}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint binds:

$$\underline{s}_n^1 = l_n^1 - \tau_n^1 - \vec{1}^T c_n^1 \implies \vec{1}^T dc_n^1 - dl_n^1 = -d\tau_n^1 \tag{A60}$$

Defining the within-period willingness to pay for government expenditure $WTP_n^t = \frac{w_{nG}^t}{\kappa_n^t}$, we arrive at the final expression:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 - \left( d\tau_n^1 + (1-\phi_n) \frac{d\tau_n^2}{1+r^1} \right) \right. \\ \left. + \left( WTP_n^1 dG^1 + (1-\phi_n) \frac{WTP_n^2}{1+r^1} dG^2 \right) \right] \tag{A61}$$

□

## A.12. *Proof of Proposition 9*

*Proof.* Under the proposition's assumptions, Equation 34 reduces to:

$$dW = \mu^T dl^1 - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1+r^1} \tag{A62}$$

Moreover, by Equation 35, we have that:

$$\widehat{\mu}dl^1 = R^1 \left(I - C^1_{y^1} R^1\right)^{-1} \left(dG^1 - C^1_{y^1} \left(\widehat{\mu}d\tau^1 + \frac{\widehat{\mu}d\tau^2}{1+r^1}\right)\right) \tag{A63}$$

Combining these equations and rearranging:

$$\begin{aligned} dW &= \vec{1}^T R^1 \left(I - C^1_{y^1} R^1\right)^{-1} \left(dG^1 - C^1_{y^1} \left(\widehat{\mu}d\tau^1 + \frac{\widehat{\mu}d\tau^2}{1+r^1}\right)\right) - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1+r^1} \\ &= \vec{1}^T \left(I - C^1_{y^1} R^1\right)^{-1} dG^1 + \vec{1}^T \left[\left(I - R^1 C^1_{y^1}\right)^{-1} R^1 C^1_{y^1} + I\right] \left(-\widehat{\mu}d\tau^1 - \frac{\widehat{\mu}d\tau^2}{1+r^1}\right) \\ &= \vec{1}^T \underbrace{\left(I - C^1_{y^1} R^1\right)^{-1}}_{=dY^1/dG^1} dG^1 + \vec{1}^T \underbrace{\left(I - R^1 C^1_{y^1}\right)^{-1}}_{=dl^1/dy^1} \left(-\widehat{\mu}d\tau^1 - \frac{\widehat{\mu}d\tau^2}{1+r^1}\right) \end{aligned} \tag{A64}$$

Finally, one may add terms proportional to $\frac{dY^1}{dG^2} = 0$. □

## A.13. Proof of Corollary 4

*Proof.* To begin, recall the definition $m_n^{\text{next}} = m^T R^1 \overline{C}^1_{y^1}$, where $\overline{C}^1_{y^1}$ is the normalized matrix of spending directions, i.e. $C^1_{y_1} = \overline{C}^1_{y^1}\widehat{m}$. We have, therefore, by assumption that $m^T R^1 C^1_{y^1} = m^T R^1 \overline{C}^1_{y^1}\widehat{m} = \mathbb{E}_{y*}[m_n] \cdot m^T$.

Applying this fact to the multipliers in Equation 36, we have

$$\begin{aligned} \vec{1}^T \frac{dY^1}{dG^1} &= \vec{1}^T \left(I - C^1_{y^1} R^1\right)^{-1} = \sum_{k=0}^{\infty} \vec{1}^T \left(C^1_{y^1} R^1\right)^k \\ &= \vec{1}^T + \underbrace{\vec{1}^T C^1_{y^1}}_{=m^T} R^1 + \sum_{k=1}^{\infty} \underbrace{\vec{1}^T C^1_{y^1}}_{=m^T} \left(R^1 C^1_{y^1}\right)^k R^1 \\ &= \vec{1}^T + m^T R^1 + \sum_{k=1}^{\infty} \mathbb{E}_{y*}[m_n]^k m^T R^1 \\ &= \vec{1}^T + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m^T R^1 \\ &= \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T R^1 \end{aligned} \tag{A65}$$

Moreover, we have that:

$$\vec{1}^T \frac{dl^1}{dy^1} = \vec{1}^T \left(I - R^1 C^1_{y^1}\right)^{-1}$$
$$= \vec{1}^T + \underbrace{\vec{1}^T R^1 C^1_{y^1}}_{=m^T} + \sum_{k=1}^{\infty} \underbrace{\vec{1}^T R^1 C^1_{y^1}}_{=m^T} \left(R^1 C^1_{y^1}\right)^k$$
$$= \vec{1}^T + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m^T$$
$$= \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \tag{A66}$$

Plugging these into Equation 36, we obtain:

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1 + r^1}\right) \tag{A67}$$

Completing the proof. □

## A.14. Proof of Proposition 10

*Proof.* The proof of this result relies on material in Appendix B.6 on characterizing optimal fiscal policy; please consult this section and the results therein before proceeding with reading this proof.

We first prove the result for first-period transfers. At any optimum, we know that Equation A134 must hold for all policy variations $\tau^1_\varepsilon \in \mathbb{R}^N$ that only vary first-period transfers, keeping other instruments fixed. Taking $\tau^1_\varepsilon = e_n$, the $n$th basis vector, we see that:

$$\left(\widetilde{\lambda}^T - \gamma \vec{1}\right)^T_n = \left(\widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C^1_{y^1} R^1\right)^{-1} C^1_{y^1}\right)_n \tag{A68}$$

Stacking these over $n$, we obtain:

$$\left(\widetilde{\lambda} - \gamma \vec{1}\right)^T = \widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C^1_{y^1} R^1\right)^{-1} C^1_{y^1} \tag{A69}$$

Since $\{e_n\}$ is a basis and Equation A134 is linear, this equation fully encompasses the optimality condition of Proposition 19 with respect to first period transfers.

We can simplify this system of equations. First, see that:

$$\begin{aligned} R^1(I - C^1_{y^1} R^1)^{-1} C^1_{y^1} &= \sum_{k=0}^{\infty} R^1 (C^1_{y^1} R^1)^k C^1_{y^1} \\ &= \sum_{k=1}^{\infty} R^1 C^1_{y^1} \end{aligned} \tag{A70}$$

Adding $\widetilde{\lambda}^T \hat{\Delta}$ to both sides of Equation A69, we therefore obtain:

$$\begin{aligned} \left(\widetilde{\lambda}(1 + \widehat{\Delta}) - \gamma \vec{1}\right)^T &= \widetilde{\lambda}\widehat{\Delta} \left(I + \sum_{k=1}^{\infty} R^1 C^1_{y^1}\right) \\ &= \widetilde{\lambda}^T \widehat{\Delta} \left(I - R^1 C^1_{y^1}\right)^{-1} \end{aligned} \tag{A71}$$

Which can be rewritten as:

$$\left(\widetilde{\lambda}(1 + \widehat{\Delta}) - \gamma \vec{1}\right)^T \left(I - R^1 C^1_{y^1}\right) = \widetilde{\lambda}^T \widehat{\Delta} \tag{A72}$$

Now, express $R^1 C^1_{y^1} = R^1 \overline{C}^1_{y^1} \hat{m}$. Recognizing that all columns of the spending-to-income matrix $R^1 \overline{C}^1_{y^1}$ sum to one as total spending is equal to total factor income, and—by assumption—that $\widetilde{\lambda}_n(1 + \Delta_n)$ is constant across all households $n$ except for those for which the $n^{th}$ row of $R^1 C^1_{y^1}$ is zero, (A72) can be rewritten as:

$$\left(\widetilde{\lambda}(1 + \widehat{\Delta}) - \gamma \vec{1}\right)^T (I - \hat{m}) = \widetilde{\lambda}^T \widehat{\Delta} \tag{A73}$$

We therefore have all, for all $n$, that

$$\widetilde{\lambda}_n(1 + \Delta_n) - \gamma = \frac{1}{1 - m_n} \widetilde{\lambda}_n \quad \forall n \in N \tag{A74}$$

Which can be simply rearranged to yield the claimed expression:

$$\gamma = \widetilde{\lambda}_n \left(1 + \frac{m_n}{1 + m_n}(-\Delta_n)\right) \quad \forall n \in N \tag{A75}$$

We prove the result for first-period government spending in an analogous way. To begin, consider Equation A134 for policy variations $G^1_\varepsilon \in \mathbb{R}^{\mathcal{I}^1}$ that only vary first period expenditure. Again considering each basis vector of $\mathbb{R}^{\mathcal{I}^1}$ and stacking we obtain:

$$0 = \widetilde{\lambda}^T WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \widehat{\Delta} R^1) - \widetilde{\lambda}^T \widehat{\Delta} R^1 (I - C^1_{y^1} R^1)^{-1} C^1_{y^1} R^1 \tag{A76}$$

This can be rewritten as:

$$\widetilde{\lambda}^T WTP^1 - \gamma\vec{1}^T = \widetilde{\lambda}^T R^1 \widehat{\Delta}(I - C^1_{y^1} R^1)^{-1} \tag{A77}$$

From the assumption the the social gains from government expenditure equal $\tilde{v}$, we have that $\widetilde{\lambda}^T WTP^1 = \tilde{v}$. Moreover, by definition $\widetilde{\lambda\Delta} = \widetilde{\lambda}\widehat{\Delta}R$. Hence (A77) can be rewritten as

$$\tilde{v}\vec{1}^T - \gamma\vec{1}^T = \widetilde{\lambda\Delta}^T \left(I - C^1_{y^1} R^1\right)^{-1} \tag{A78}$$

Next, define $\widetilde{m}_i \equiv \left(m^T R^1\right)_i$ to be the rationing-weighted average MPC in the production of good $i$ and let $\widehat{\widetilde{m}}$ be the corresponding matrix with $\widetilde{m}$ on the diagonal. Moreover, define $\widetilde{C}_{ji} \equiv \frac{\left(C^1_{y^1} R^1\right)_{ji}}{\widetilde{m}_i}$ to be the average direction of consumption of workers producing $i$, weighted by their MPC and marginal rationing in $i$'s production.[39] Crucially, note that $\widetilde{C}\widehat{\widetilde{m}} = C^1_{y^1} R^1$ by construction and that $\vec{1}^T \widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T \widehat{\widetilde{m}}$:

$$\vec{1}^T \widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T C^1_{y^1} R^1 = m^T R^1 = \widehat{\widetilde{m}}^T \tag{A79}$$

The first order condition for expenditures (A78) is therefore equivalent to:

$$(\tilde{v} - \gamma)\vec{1}^T \left(I - C^1_{y^1} R^1\right) = (\tilde{v} - \gamma)\vec{1}^T \left(I - \widehat{\widetilde{m}}\right) = \widetilde{\lambda\Delta}^T \tag{A80}$$

But this holds iff and only if:

$$\gamma = \tilde{v} + \frac{1}{1 - \widetilde{m}_i}(-\widetilde{\lambda\Delta}_i) \quad \forall i \in \mathcal{I}^1, \tag{A81}$$

completing the proof. □

[39] For any $i$ with $\tilde{m}_i = 0$, define $\widetilde{C}_{ji}$ in any way satisfying $\sum_j \widetilde{C}_{ji} = 1$.

# B. Extensions of the Baseline Model

Here we extend baseline model to many periods (allowing for an infinite horizon) (B.1), allow for imperfect competition with fixed markups (B.2), study the structure of the multiplier in a more canonical flexible-wage equilibrium (B.3), provide a network reinterpretation of the multiplier at the zero lower bound (B.4), generalize our decomposition results to account also for supply shocks (B.5), provide first order conditions for optimal policy (B.6), and consider policy in the environment with imperfect competition (B.7).

## *B.1. Multiple Time Periods*

Consider the benchmark model from Section 2 but instead suppose that $t \in \mathbb{T} = \{1, ..., T\}$, where $T \in \mathbb{N} \cup \{\infty\}$. That is, in each $t$, firms use a vector of intermediates $X_i^t$, labor $L_i^t$ and a CRS production technology $F(X_i^t, L_i^t, z_i^t)$. The households have consumption $c_n^t$ and labor supply $l_n^t$ functions that satisfy the dynamic budget constraint:

$$\sum_{t \in \mathbb{T}} \frac{l_n^t}{\prod_{i \leqslant t} 1 + r^i} = \sum_{t \in \mathbb{T}} \frac{p^t c_n^t + \tau_n^t}{\prod_{i \leqslant t} 1 + r^i} \tag{A82}$$

The government chooses a sequence of lump-sum taxes and spending $\{\{\tau_n^t\}_{n \in N}, \{G_i^t\}_{i \in \mathcal{I}}\}_{t \in \mathbb{T}}$ subject to its lifetime budget constraint:

$$\sum_{n \in N} \mu_n \Big( \sum_{t \in \mathbb{N}} \frac{1}{\prod_{i \leqslant t} 1 + r^i} \tau_n^t \Big) = \sum_{t \in \mathbb{N}} \frac{1}{\prod_{i \leqslant t} 1 + r^i} p^t G^t \tag{A83}$$

The key difference in defining equilibrium here is the need to specify a rule that decides in which periods we have labor rationing. To this end define a set $\mathcal{T}(\omega) \subseteq \mathbb{T}$ which specifies time indices for which the economy is in a state of labor rationing, where $\omega$ is a vector of all exogenous parameters of the model.[40] In periods with rationing $t \in \mathcal{T}(\omega)$, instead of labor market clearing, we have that $l_n^t = l_n^t((L_i^t)_{i \in \mathcal{I}^t})$. An equilibrium of the model is then given by:

**Definition 2.** *(Dynamic rationing equilibrium) Given parameters $\omega$, a dynamic rationing equilibrium is a set of agent- and market-level variables $\{s_n^t, \{c_{ni}^t\}_{i \in \mathcal{I}^t}, l_n^t\}_{n \in N, t \in \mathbb{T}}$ and $\{r_t, \{p_i^t, \{X_{ij}^t\}_{j \in \mathcal{I}^t}, L_i^t, C_i^t, G_i^t\}_{i \in \mathcal{I}^t}\}_{t \in \mathbb{T}}$ that satisfy the following conditions. (1) Each household $n$ consumes according to its consumption function $c_n^t(\cdot)$ in all periods and supplies labor according to $l_n^t(\cdot)$ in all non-rationing periods i.e. $t \in \mathbb{T}/\mathcal{T}(\omega)$. (2) Firms choose $(X_i^t, L_i^t)$ to*

[40] For example, $\mathcal{T}$ can represent the set of periods in which the effective zero lower bound on real interest rates binds. Insofar as $\omega$ is sufficient to determine whether the zero lower bound binds, it is sufficient for it to determine $\mathcal{T}$.

*maximize profits for all $t \in \mathbb{T}$ (3) The market for all goods clears for all $t \in \mathbb{T}$ (4) The labor market clears in periods $t \in \mathbb{T}/\mathcal{T}(\omega)$ and is determined by rationing in all periods in periods $t \in \mathcal{T}(\omega)$, i.e. $l_n^t = l_n^t\big((L_i^t)_{i\in\mathcal{I}^t}\big)$ (5) The government spends according to its expenditure function $G^t(\cdot)$.*

For our dynamic equilibrium, we can again achieve an analogous Keynesian cross representation to our two period model, as the no-substitution theorem continues to hold. The dynamic fixed point equation for production is given by:

$$Q^t = \widehat{X}^t Q^t + G^t(\{r^t\}_{t\in\mathbb{T}}) + C^t(\{r^t, Q^t\}_{t\in\mathbb{T}}) \tag{A84}$$

Taking a first-order approximation following a partial equilibrium shock $\partial Q$ for both rationing and flexible periods yields:

$$dQ^t = \widehat{X}^t dQ^t + \sum_{\tau\geqslant 1}\Big[\big(G^t_{r^\tau} + C^t_{r^\tau}\big)dr^\tau\Big] + \sum_{\tau\in\mathcal{T}(\omega)}\Big[C^t_{y^\tau} l^\tau_{L^\tau}\widehat{L}^\tau dQ^\tau\Big] + \partial Q^t \tag{A85}$$

Stacking these relations yields the Keynesian cross representation:

$$dQ = \widehat{X}dQ + (G_r + C_r)dr + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A86}$$

where $J_{\mathcal{T}(\omega)}$ is a diagonal matrix with ones on the diagonal

Interestingly, via an appropriate relabelling, there is an heuristic isomorphism between the 2-period model and the $T$-period model whenever $\mathcal{T}(\omega) = \{t\}_{t=1}^{T_1}$, *i.e.* there is rationing for the first $T_1$ periods and non-rationing for the subsequent $T_2 = T - T_1$ periods. That is, In the $T$-period model, the rationing spell maps to the rationing period in the 2-period model. To this end, the formula in Proposition 8 corresponds to a dynamic generalization of the Miyazawa special case.

**Proposition 11.** *(Dynamic multipliers at the zero lower bound) Suppose that $r^t = \bar{r}^t$ for all $t \in T$. Then the general equilibrium effect on output $dY$ of a partial equilibrium shock $\partial Q$ is generically given by*

$$dY^{\mathcal{T}} = \left(I - C_y^{\mathcal{T}} l_L^{\mathcal{T}} \widehat{L}^{\mathcal{T}}\left(I - \widehat{X}^{\mathcal{T}}\right)^{-1}\right)^{-1}\partial Q^{\mathcal{T}} \tag{A87}$$

*where $dY^{\mathcal{T}}$ and $dQ^{\mathcal{T}}$ are $\mathcal{T}\times\mathcal{I}$-length vectors, $\widehat{L}^{\mathcal{T}}$ and $\widehat{X}^{\mathcal{T}}$ are diagonal matrices with entries corresponding to each rationing periods, and where $C_y^{\mathcal{T}}$ is the $(\mathcal{T}\times\mathcal{I})\times(\mathcal{T}\times N)$ matrix of intratemporal marginal propensities to consume, which maps changes in the household* *income distribution during rationing periods to changes in the consumption of each good during rationing periods.*

*Proof.* The dynamic fixed point equations for market clearing are given in matrix form as:

$$Q^t = \widehat{X}^t Q^t + G^t(\{r^t\}_{t\in\mathbb{T}}) + C^t(\{r^t, Q^t\}_{t\in\mathbb{T}}) \tag{A88}$$

Taking a first-order approximation following a partial equilibrium shock $\partial Q$ for both rationing and flexible periods yields:

$$dQ^t = \widehat{X}^t dQ^t + \sum_{\tau\geqslant 1}\Big[\big(G^t_{r^\tau} + C^t_{r^\tau}\big)dr^\tau\Big] + \sum_{\tau\in\mathcal{T}(\omega)}\Big[C^t_{y^\tau} l^\tau_{L^\tau}\widehat{L}^\tau dQ^\tau\Big] + \partial Q^t \tag{A89}$$

Stacking these relations yields the Keynesian cross representation:

$$dQ = \widehat{X}dQ + (G_r + C_r)dr + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A90}$$

where $J_{\mathcal{T}(\omega)}$ is a diagonal matrix with ones on the diagonal. Imposing $r^t = \bar{r}^t$ for all $t \in T$ simplifies this to:

$$dQ = \widehat{X}dQ + C_y l_L \widehat{L} J_{\mathcal{T}(\omega)} dQ + \partial Q \tag{A91}$$

Inverting this system to solve for the total change in production and solving for output:

$$dY = \left(I - C_y l_L \widehat{L} J_{\mathcal{T}(\omega)}\left(I - \widehat{X}\right)^{-1}\right)\partial Q \tag{A92}$$

Applying the selection matrix $J_{\mathcal{T}(\omega)}$ and taking the first $\mathcal{T} \times \mathcal{I}$ rows:

$$dY^{\mathcal{T}} = \left(I - C^{\mathcal{T}}_y l^{\mathcal{T}}_L \widehat{L}^{\mathcal{T}}\left(I - \widehat{X}^{\mathcal{T}}\right)^{-1}\right)^{-1}\partial Q^{\mathcal{T}} \tag{A93}$$

Which is the required expression. □

However, there is a subtle difference in the intuition behind the results in the two cases. In the $T$-period case, the shocks in each rationing period can influence the level of output in all other periods. As a result, it is no longer sufficient to consider the directed MPC of households, but rather the directed *intertemporal* MPC of households that represents marginal changes in consumption across goods and time. Indeed, if we set the response of the rationing function, the unit labor demands and the input-output matrix to the identity, we recover a $\mathcal{T}$-period version of the multiplier formula provided by Auclert *et al.* (2018):

**Corollary 5** (Intertemporal Keynesian Cross)**.** *In the environment of Proposition 11, if the* *rationing matrix and the input output matrix compose to the identity matrix, i.e.*

$$I = l_L^{\mathcal{T}} \widehat{L}^{\mathcal{T}} \left( I - \widehat{X}^{\mathcal{T}} \right)^{-1} \tag{A94}$$

*then the general equilibrium effect on output $dY^{\mathcal{T}}$ in response to a partial equilibrium shocks $\partial Q^{\mathcal{T}}$ is given by:*

$$dY^{\mathcal{T}} = \left( I - C_y^{\mathcal{T}} \right)^{-1} \partial Q^{\mathcal{T}} \tag{A95}$$

*Proof.* Simply imposing the given condition on Equation A87 yields the stated result. □

## B.2. Imperfect Competition

In this section we show how to incorporate imperfect competition in the form of fixed markups on marginal costs. We now return to the standard two period model $T = 2$ under rationing equilibrium. However, instead of each sector being populated by a continuum of perfectly competitive firms, we now suppose that for all $i \in \mathcal{I}^t$ there is a single monopolist producing each good, charging a fixed markup of $m_i^t$ over their marginal cost.[41] Of course, firms now have the capability of making profits $\pi_i^t$ and we must distribute these profits to households in equilibrium. Despite this, we argue that a no substitution theorem still holds and we can obtain analogous multiplier formulae once we augment labor income rationing with profit rationing. To do this, we have to slightly modify Assumption 1:

**Assumption 6.** *For each $t$ there exists some $\overline{p}^t \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i^t, L_i^t\}_{i \in \mathcal{I}^t}$ such that for all $i$, $F(X_i^t, L_i^t, z_i^t) \geqslant 1$ and $(1 + m_i^t)(\overline{p}^t X_i^t + L_i^t) \leqslant \overline{p}_i^t$*

Under this modified assumption, we can state and prove the modified no-substitution theorem with markups:

**Proposition 12.** *Under Assumptions 6 and 2, for a given $z^t$ and $m^t$, there exists a unique $p^t$ consistence with both flexible-wage and rationing equilibrium, independent of demand.*

*Proof.* We modify the proof of proposition 1 to accommodate markups. Each firm now sets a price $p_i = (1 + m_i^t)\kappa_i(p)$, where $\kappa_i$ is $i$'s unit cost function. That is, $i$ prices goods as though it were a competitive firm with production function $\frac{1}{1+m_i^t} F(X_i^t, L_i^t, z_i^t)$. Consider now a modified economy without markups and production functions given by the previously-stated markup-adjusted production functions. Assumption 6 implies that Assumption 1 holds in this modified economy. The result then follows by direct application of Proposition 1. □

[41]Note that this generalizes the more standard model in which each sector is composed of many differentiated firms, with each firm and household having the same CES aggregator for its demand from the firms making up each sector.

Having now established that the no substitution theorem continues to hold, we now proceed to establish our multiplier formulae in this setting. As previously mentioned, the key difference here is the need to apportion firm profits to households. To this end, suppose that profits from each firm are distributed to households according to an exogenous profit rationing function $\Pi^t : \mathbb{R}^{\mathcal{I}} \to \mathbb{R}^N$ satisfying $\sum_{i \in \mathcal{I}} \pi_i^t = \sum_{n \in N} \Pi^t(\pi^t)_n$ for all $\pi^t \in \mathbb{R}^{\mathcal{I}}$. We let $d_n^t = \Pi^t(\pi^t)_n$ represent household $n$'s total dividend income in period $t$.

As a result of profit distribution, household income is now comprised of rationed first-period labor income, chosen second-period labor income, and distributed (not chosen) dividend income in both periods. We therefore allow household consumption and labor supply functions to depend on $d_n^t$ directly.

We can now state a profit-inclusive Keynesian cross. Note that the only difference to Proposition 4 comes from the need to account for changes in profits, how these are distributed to households as dividends and their directed MPCs out of dividends.[42]

**Proposition 13.** *For any small shock to parameters there exist a pair of rationing equilibria production $Q$ and $Q + dQ$ before and after the shock. If the shock induces a partial equilibrium change in production $\partial Q$, the general equilibrium change $dQ$ is given to first order by:*

$$dQ = \widehat{X} dQ + (C_r + G_r) r_Q dQ + C_y l_{L^1}^1 \widehat{L}^1 dQ^1 + C_\pi \widehat{\Pi} dQ + \partial Q \tag{A96}$$

*where here $C_\pi$ is the matrix of household directed MPCs out of profit income, $\widehat{\Pi}$ is the block diagonal matrix composed of $\widehat{\Pi}^1$ and $\widehat{\Pi}^2$, and where $\widehat{\Pi}^t$ is the diagonal matrix with $i^{th}$ entry $m_i^t p_i^t$, and all quantities are evaluated at the initial equilibrium.*

*Proof.* This proof simply modifies the proof of Proposition 4. It is stated in full for clarity. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 2, we know that for each $\omega_n$ there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit $\omega$. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at $e$: household budget balance, government budget balance or market clearing. But by Assumption 3, continuity of the fiscal rule, continuity of the interest rate rule, continuity of the rationing function and continuity of the profit allocation function, we know that all functions in these expressions are continuous. It follows that there exists

[42] For the sake of generality, we distinguish between aggregate MPC out of dividend and labor income, i.e. $C_d^t \neq C_y^t$. Of course, for utility-maximizing households, these will be the same provided the income arrives in the same period.

$m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{A97}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions:

$$\begin{aligned} dQ \ &= \widehat{X} dQ + \widehat{X}_z dz Q + C_{\hat{p}} \hat{p}_z dz + C_{r^1} dr^1 + C_{y^1} dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}} \hat{p}_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G + C_\pi \hat{\Pi} dQ \end{aligned} \tag{A98}$$

Plugging in for $dr^1$ and $dy^1 = l^1_{L^1} \widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X} dQ + C_{y^1} l^1_{L^1} \widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1}) r^1_Q dQ + C_\pi \hat{\Pi} dQ + \partial Q \tag{A99}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}}) \hat{p}_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1 + (C_\tau + G_\tau) d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. □

## *B.3. Flexible-Wage Equilibrium*

In this appendix we consider a more standard flexible-wage equilibrium concept. In this context, we derive the multiplier and contrast it to the multiplier obtained in rationing equilibrium.

The notion of flexible-wage equilibrium is standard. The main difference relative to rationing equilibrium is that households now choose their labor supply in the first period. Household behavior can therefore be denoted by Marshallian consumption and labor supply functions $c^t_n(\varrho, \tau_n, \theta_n)$ and $l^t_n(\varrho, \tau_n, \theta_n)$. Firm optimality (Equation 2), household budget balance evaluated at their consumption demand and labor supply functions (Equation 3), and government budget balance (Equation 4) continue to hold. Now the first period labor market must clear in the standard fashion, so that Equation 6 is strengthened to:

$$F(X^t_i, L^t_i, z^t_i) = D^t_i \equiv \sum_{n \in N} \mu_n c^t_{ni} + \sum_{j \in \mathcal{I}^t} X^t_{ji} + G^t_i, \quad \sum_{i \in \mathcal{I}^t} L^t_i = \sum_{n \in N} \mu_n l^t_n \quad \forall i \in I^t, t \in \{1, 2\} \tag{A100}$$

We therefore define a flexible-wage equilibrium as:

**Definition 3.** *A flexible-wage equilibrium is a set of first and second period, agent- and market-level variables* $\{s^1_n, \{c^t_{ni}, l^t_{ni}\}_{t \in \{1,2\}, i \in \mathcal{I}^t}\}_{n \in N}$ *and* $\{r^t, p^t_i, \{X^t_{ij}\}_{j \in \mathcal{I}^t}, L^t_i, C^t_i, G^t_i\}_{t \in \{1,2\}, i \in \mathcal{I}^t}$ *that* *satisfy conditions (2), (3), (4), and (A100) given initial conditions.*

This flexible-wage equilibrium provides a baseline specification against which we will compare the rationing equilibrium results. Note that in flexible-wage equilibrium the real interest rate adjusts flexibly to clear the labor market; it is not controlled by a central bank. This owes to the fact that while the central bank could set the nominal rate, prices would adjust to maintain the real rate.

The no-substitution theorem used in the analysis of rationing equilibrium directly carries over to the environment with a flexible-price equilibrium.

**Proposition 14.** *Under Assumptions 1 and 2, for a given $z^t$, there exists a unique $p^t$ consistent with flexible-wage equilibrium, independent of demand.*

*Proof.* The proof follows exactly that of Proposition 1. See Appendix A.1 for the proof. □

Moreover, it can be established that a flexible-wage equilibrium exists under some mild technical conditions. In particular, we need to make some continuity and boundedness assumptions on consumption and labor supply:

**Assumption 7.** *The consumption and labor functions $c_n^t$ and $l_n^t$ are continuous in $r^1$. Moreover, for all $n$, $\lim_{r^1 \to -1} \sum_{i \in \mathcal{I}^1} c_{ni}^1(\varrho, \tau, \theta) \to \infty$, $\lim_{r^1 \to -1} l_n^1(\varrho, \tau, \theta)$ is bounded, $\lim_{r^1 \to \infty} \sum_{i \in \mathcal{I}^1} c_{ni}^1(\varrho, \tau, \theta)$ is bounded, and $\lim_{r^1 \to \infty} l_n^1(\varrho, \tau, \theta) \to \infty$.*

With this additional structure we are now able to prove existence of flexible-wage equilibria for the economy under consideration.

**Proposition 15.** *Under Assumptions 1, 2 and 7, there exists a flexible wage equilibrium.*

*Proof.* We prove the existence of an equilibrium by defining a fixed point map for $1 + r^1$, the gross real interest rate, such that at any fixed point the savings market clears. Given such an interest rate, we then explicitly construct an equilibrium.

Fix all exogenous parameters. Recalling that technology pins down prices and labor and input usage, we use the notation $p = p(z)$, $\widehat{X} = \widehat{X}(z)$, and $\widehat{L}^1 = \widehat{L}^1(z)$. To ensure the object over which we will construct the fixed point map lies in a compact set, we define the following transformation:

$$\tilde{r}^1(1 + r^1) = \frac{1 + r^1}{2 + r^1} \tag{A101}$$

where $\tilde{r}(0) = 0$, $\tilde{r}(\infty) = 1$, $\tilde{r}$ is continuous and invertible. We now define a correspondence $\Phi : [0,1] \rightrightarrows [0,1]$ by

$$\Phi(\tilde{r}^1) = \begin{cases} [0,1], & p^1(C^1(\tilde{r}^1) + G^1) = l^1(\tilde{r}^1) \\ \{1\}, & p^1(C^1(\tilde{r}^1) + G^1(\tilde{r}^1)) > l^1(\tilde{r}^1) \text{ or } \tilde{r} = 0 \\ \{0\}, & p^1(C^1(\tilde{r}^1) + G^1(\tilde{r}^1)) < l^1(\tilde{r}^1) \text{ or } \tilde{r} = 1 \end{cases} \tag{A102}$$

where here $C^1(\tilde{r}^1)$ is shorthand for $C^1(p^1, p^2, r^1, \theta, \tau)$, with the same conventtion for $l^1(\tilde{r}^1)$ and $G^1(\tilde{r}^1)$. Notice that $\Phi$ is non-empty-valued and convex-valued. In order to apply Kakutani's theorem, it suffices to show that $\Phi$ is UHC. To show that $\Phi$ UHC, it suffices to show that for any selection $\phi \in \Phi$:

$$\lim_{\tilde{r}^1 \to 0} \phi(\tilde{r}^1) = 1 \quad \lim_{\tilde{r}^1 \to 1} \phi(\tilde{r}^1) = 0 \tag{A103}$$

To this end, by Assumption 7, see that as $\tilde{r}^1 \to 0$, $p^1 c_n^1 \to \infty$ while $l_n^1$ is finite for all types $n \in N$; meanwhile, government expenditures are (always) weakly positive. Thus, as $\tilde{r}^1 \to 0$, it must indeed be that $p^1(C^1 + G^1) > l^1$. Now suppose that $\tilde{r}^1 \to 1$. Again, by Assumption 7, it must be that $p^1 c_n^1 \to 0$ while $l_n^1 \to \infty$ and so $p^1(C^1 + G^1) < l^1$; here we use that first-period government expenditures are bound by the government budget constraint. We have therefore established that $\Phi$ is UHC. Thus, Kakutani's fixed point theorem implies that $\Phi$ has a fixed point. That is there exists $1 + r^1 \in [0, \infty]$ such that:

$$p^1\big(C^1(\varrho, \theta, \tau) + G^1(\varrho, \theta, \tau)\big) = l^1(\varrho, \theta, \tau) \tag{A104}$$

Also note that by construction of $\Psi$, the resulting fixed point $1 + r^1$ is finite and strictly positive.

Using this $r^1$, we will now construct a flexible-price equilibrium, i.e. a set of first and second period, agent- and market-level variables:

$$\{s_n^1, \{c_{ni}^t, l_{ni}^t\}_{t \in \{1,2\}, i \in \mathcal{I}}\}_{n \in N} \quad \text{and} \quad \{r^1, p_i^t, \{X_{ij}^t\}_{j \in \mathcal{I}}, L_i^t, C_i^t\}_{t \in \{1,2\}, i \in \mathcal{I}} \tag{A105}$$

satisfying the conditions of Definition 3. We set within-period prices $p^t = p^t(z^t)$. For all $t, n$, let $c_n^t$, $l_n^t$, and $G^t$ be given by the household consumption and labor and government expenditure functions at real interest rate $r^1$. Let firms produce quantities $Q^t = (1 - \widehat{X}^t)^{-1}(C^t + G^t)$, demand inputs $X^t = \widehat{X}^t Q^t$, and demand labor $L^t = \widehat{L}^t Q^t$.

We now verify that the equilibrium conditions hold: household and government budget constraints follow by assumption on the consumption, labor, and expenditure functions. Firm optimization holds since firms make zero profits at the no-substitution theorem prices, so long as they demand inputs and labor optimally, according to $\widehat{X}^t$ and $\widehat{L}^t$. $Q^t = (1-\widehat{X}^t)^{-1}(C^t+G^t)$ and $X^t = \widehat{X}^t Q^t$ imply the goods market clears. Labor supplied equals $p^t(C^t+G^t)$, by—for $t = 1$—the selection of the interest rate, and by—for $t = 2$—combining the household and government budget constraints with this condition at $t = 1$; labor demanded equals $\vec{1}^T \widehat{L}^t(1-\widehat{X}^t)^{-1}(C^t+G^t)$. Using firms' zero profit condition to substitute for $\widehat{L}^t$, labor demand can be rewritten as $(p^t)^T(1-\widehat{X}^t)(1-\widehat{X}^t)^{-1}(C^t+G^t)$, so the labor market clears. This completes the proof that a flexible-price equilibrium exists. □

We now obtain a representation of the partial equilibrium effect on demand of any shock to primitives. We begin by parameterizing aggregate demand. Recognizing that each household's decisions depend only on real quantities, we can represent type $n \in N$'s Marshallian demand for good $j \in \mathcal{I}^t$ at time $t \in \{1,2\}$ as $c^t_{nj}(\varrho, \tau_n, \theta_n)$, where $\varrho = (p^1, p^2, r)$, and $\tau_n = (\tau^1_n, \tau^2_n)$. Aggregate consumption demand $C^t_j$ is then given by:

$$C^t_j(\varrho, \tau, \theta) = \sum_{n \in N} \mu_n \; c^t_{nj}(\varrho, \tau_n, \theta_n) \tag{A106}$$

where $\theta = (\theta_1, ..., \theta_N)$ and so forth. We define aggregate labor supply $L^t(y^1\varrho, \tau, \theta)$ and government expenditure analogously.

To find the partial equilibrium effect of each type of shock, we totally differentiate the goods market clearing condition:

$$Q^t = \widehat{X}^t Q^t + C^t + G^t \tag{A107}$$

We then collect the terms corresponding to changes in demand for goods before accounting for the way that direct changes in $Q^t$ cause higher-order, "multiplier" effects. Doing so yields the following partial equilibrium effect of each shock:

**Proposition 16.** *The following shocks have partial equilibrium effects on aggregate demand given by:*

1. *A change in government preferences $\theta_G$ by $d\theta_G$:*

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G) d\theta_G \tag{A108}$$

2. *A change in household preferences $\theta$ by $d\theta$:*

$$\partial Q = C_\theta(\varrho, \tau, \theta) d\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\theta d\theta \tag{A109}$$

*3. A change in taxes or transfers by $d\tau$:*

$$\partial Q = C_\tau(\varrho, \tau, \theta)d\tau + G_\tau(\varrho, \tau, \theta_G)d\tau - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_\tau d\tau \tag{A110}$$

*4. A change in productivity $z$ by $dz$:*

$$\partial Q = \left(C_p + G_p - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_p\right) p_z dz + \left(\widehat{X}_z + (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\widehat{L}_z\right) dzQ \tag{A111}$$

*Proof.* We consider each shock case by case. For each, we totally differentiate the goods market clearing condition and group all terms that have no dependence on resulting changes in equilibrium output. To this end, recall that the goods market clearing condition is given by:

$$Q = \widehat{X}Q + G + C \tag{A112}$$

In the flexible-wage case, total differentiation of this system of equations yields:

$$dQ = \widehat{X}dQ + \widehat{X}_z dzQ + C_p p_z dz + C_{r^1}dr^1 + C_\tau d\tau + C_\theta d\theta + G_p p_z dz + G_{r^1}dr^1 + G_\tau d\tau + G_{\theta_G}d\theta_G \tag{A113}$$

Similarly, we can expand the labor market clearing conditions to write

$$\widehat{L}dQ + \widehat{L}_z dzQ = L_p p_z dz + L_{r^1}dr^1 + L_\tau d\tau + L_\theta d\theta$$

Substituting for $dr^1$, we obtain

$$\begin{aligned} dQ =& \widehat{X}dQ + \widehat{X}_z dzQ + C_p p_z dz + C_\tau d\tau + C_\theta d\theta + G_p p_z dz + G_\tau d\tau + G_{\theta_G}d\theta_G \\ & (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\left(\widehat{L}dQ + \widehat{L}_z dzQ - L_p p_z dz - L_\tau d\tau - L_\theta d\theta\right) \end{aligned} \tag{A114}$$

We now consider the partial equilibrium effect of each type of shock by zeroing all general equilibrium effects through changes in output and by zeroing all other shocks:

1. A change in government preferences $\theta_G$ by $d\theta_G$:

$$\partial Q = G_{\theta_G}(\varrho, \tau, \theta_G)d\theta_G \tag{A115}$$

2. A change in household preferences $\theta$ by $d\theta$:

$$\partial Q = C_\theta(\varrho, \tau, \theta)d\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1}L_\theta d\theta \tag{A116}$$

3. A change in taxes or transfers by $d\tau$:

$$\partial Q = C_\tau(\varrho,\tau,\theta)d\tau + G_\tau(\varrho,\tau,\theta_G)d\tau - (C_{r^1}+G_{r^1})(L_{r^1})^{-1}L_\tau d\tau \tag{A117}$$

4. A change in productivity $z$ by $dz$:

$$\partial Q = \left(C_p + G_p - (C_{r^1}+G_{r^1})(L_{r^1})^{-1}L_p\right)p_z dz + \left(\widehat{X}_z + (C_{r^1}+G_{r^1})(L_{r^1})^{-1}\widehat{L}_z\right)dzQ \tag{A118}$$

□

Having understood how primitive shocks map into partial equilibrium changes in demand, we now explore how these shocks map into changes in output in general equilibrium. To do this, we examine the impact of a small shocks or, equivalently, the impact of large shocks to first order. We represent the general equilibrium mapping of any of these shocks by a matrix that we label the multiplier. Our strategy is simply to totally differentiate the market clearing conditions in matrix form.

**Proposition 17.** *For any small shock to parameters, there exist a pair of flexible-wage equilibria with production $Q = (Q^1, Q^2)$ and $Q + dQ$ before and after the shock. Assume $L^t_{r^1} \neq 0$. Then if the shock induces a partial equilibrium change in production $\partial Q$, the general equilibrium change $dQ$ is given to first order by:*

$$dQ = \widehat{X}dQ + \begin{bmatrix} C^1_{r^1}+G^1_{r^1} & 0 \\ 0 & C^2_{r^1}+G^2_{r^1} \end{bmatrix} \begin{bmatrix} \left(L^1_{r^1}\right)^{-1} & 0 \\ 0 & \left(L^2_{r^1}\right)^{-1} \end{bmatrix} \widehat{L}dQ + \partial Q \tag{A119}$$

*where all quantities above are evaluated at the initial equilibrium. Moreover, the impact on ouput is generically given by:*

$$dY = \left(I - \begin{bmatrix} C^1_{r^1}+G^1_{r^1} & 0 \\ 0 & C^2_{r^1}+G^2_{r^1} \end{bmatrix} \begin{bmatrix} \left(L^1_{r^1}\right)^{-1} & 0 \\ 0 & \left(L^2_{r^1}\right)^{-1} \end{bmatrix} \widehat{L}\left(I - \widehat{X}\right)^{-1}\right)^{-1} \partial Q \tag{A120}$$

*Proof.* The existence of two nearby equilibria is a consequence of UHC of the equilibrium set in the parameters. We formally show that the equilibrium set is UHC. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 2, we know that for each $\omega_n$ there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit $\omega$. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at $e$: household budget balance, government budget balance or market clearing. But by Assumption 7, and continuity of the fiscal rule, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contraction. This completes the proof that the equilibrium set is UHC.

We can relate these two nearby equilibria by totally differentiating the goods market clearing conditions. Stacking the vectors that represent the two periods, we have:

$$\begin{aligned} dQ =& \widehat{X} dQ + \widehat{X}_z dz Q + C_p p_z dz + C_{r^1} dr^1 + C_\tau d\tau + C_\theta d\theta \\ &+ G_p p_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G \end{aligned} \tag{A121}$$

Similarly, we can expand the labor market clearing conditions to write $\widehat{L}^t dQ^t + \widehat{L}^t_{z^t} dz^t Q^t = L^t_p p_z dz + L^t_{r^1} dr^1 + L^t_\tau d\tau + L^t_\theta d\theta$.[43] Substituting for $dr^1$, we have:

$$\begin{aligned} dQ =& \widehat{X} dQ + (C_p + G_p) p_z dz + (C_\tau + G_\tau) d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G + \widehat{X}_z dz Q \\ &+ (C_{r^1} + G_{r^1})(L_{r^1})^{-1}(\widehat{L} dQ + \widehat{L}_z dz Q - L_p p_z dz - L\tau d\tau - L_\theta d\theta) \\ =& \left(\widehat{X} + (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\widehat{L}\right) dQ + \partial Q \end{aligned} \tag{A122}$$

where:

$$\begin{aligned} \partial Q =& \Big(C_p + G_p - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_p\Big) p_z dz \\ &+ \left(\widehat{X}_z + (C_{r^1} + G_{r^1})(L_{r^1})^{-1}\widehat{L}_z\right) dz Q \\ &+ \Big(C_\tau + G_\tau - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\tau\Big) d\tau \\ &+ \Big(C_\theta - (C_{r^1} + G_{r^1})(L_{r^1})^{-1} L_\theta\Big) d\theta + G_{\theta_G} d\theta_G \end{aligned} \tag{A123}$$

□

A classical dichotomy holds in our flexible-wage model, such that the expressions above do not depend on monetary policy. Also notice that matrices are block diagonal: we can solve out each period in isolation. To fix ideas, consider an increase in first-period government spending (matched by a decrease in the second period), and suppose that substitution effects dominate. Then the exogenous increase in demand for goods (a) causes firms to demand more inputs and (b) increases the real wage in period 1 through an increase in the real interest rate, dampening consumption demand in period 1; this generates some "second-order" change in first-period demand. The equilibrium effect on first-period production

[43] While using this condition for both $t = 1$ and $t = 2$ might appear to over-determine $dr^1$, the two determinations are actually equivalent by Walras' law.

occurs in the limit as these higher-order responses die out (which they do, if we start from a stable equilibrium). Here, the second period is implicit in households' choices, but we need not consider it directly.

We now compare the flexible-wage multiplier to the generalized Keynesian multiplier. There are three main differences between the two multipliers. First, the two multipliers tell very different stories for how the interest rate responds to shocks. In the flexible case, interest rate changes are mediated through the labor market while in the rationing case they are determined through monetary policy.

Second, whereas in the flexible-wage case income is determined according household labor supply, in the rationing case income is determined by exogenous rationing functions not chosen by households. This implies that some households may want to supply more labor while others want to supply less; the equilibrium is in general inefficient. Insofar as preexisting employment relationships determine which of these households are which, they are essential for understanding how shocks propagate in the economy. Crucially, the fact that—in rationing equilibrium—households do not choose their labor supply opens the door to shocks that have extremely disparate impacts on the wealth of different households.[44] Indeed, in the rationing case as consumers have both endogenous and exogenous labor income sources, there are two channels through which consumers respond to shocks. In the flexible-wage case they respond only through changes to their endogenous labor income.

Finally, notice that in the flexible-wage case, the first-period effect of a pure demand shock can be assessed without referring to the second period; in the rationing case, it is necessary to consider intertemporal transmission channels. In the former, the interest rate is determined simply by labor market clearing condition within either period (the two are equivalent). In the latter, the interest rate—which affects first-period consumption—is determined by the endogenous policy response of the central bank, which in turn depends on the second-period shock, as well the amount of income that agents earn in the first period.

The consequence of these three differences is that demand shocks propagate very differently in the rationing price case and the flexible-wage case. These formulae therefore indicate that shock propagation hinges strongly on the level of price rigidity in the economy, even down to the relevant channels that need to be considered. For example, labor supply elasticities are crucial in understanding the output response under flexible-wages but irrelevant in the rationing case and consumers' MPCs out of income are important in understanding the rationing case but play no part in determining the response under flexible-wages.

[44] This in principle possible in flexible-wage equilibrium (if households have very different labor supply elasticities) but far less realistic.

## B.4. A Network Interpretation of the Multiplier

The multiplier formula in Corollary 3 that forms the backbone of our analysis in this paper also appears in the regional economics literature on social accounting matrices dating back to Miyazawa (1976). Our result therefore provides the first formal economic analysis that provides a microfoundation for this formula which receives widespread use in the regional economics literature and applied work to compute expenditure multipliers (such as the BEA's RIMS II system). This relationship motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households.

Formally this corresponds to an input-output matrix given by:

$$\widehat{X}^1 = \begin{bmatrix} \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} & \widehat{X}^1_{\mathcal{I}^1 N} = C^1_{y^1} \\ \widehat{X}^1_{N\mathcal{I}^1} = l^1_{L^1}\widehat{L}^1 & 0 \end{bmatrix} \tag{A124}$$

The multiplier at the zero lower bound can then be expressed as:

$$\begin{aligned} \begin{bmatrix} dQ^1_{\mathcal{I}^1} \\ dQ^1_N \end{bmatrix} &= \left(I - \begin{bmatrix} \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} & \widehat{X}^1_{\mathcal{I}^1 N} \\ \widehat{X}^1_{N\mathcal{I}^1} & 0 \end{bmatrix}\right)^{-1} \begin{bmatrix} \partial Q^1_{\mathcal{I}^1} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \left(I - \widehat{X}^1_{\mathcal{I}^1\mathcal{I}^1} - \widehat{X}^1_{\mathcal{I}^1 N}\widehat{X}^1_{N\mathcal{I}^1}\right)^{-1} & \cdots \\ \cdots & \cdots \end{bmatrix} \begin{bmatrix} \partial Q^1_{\mathcal{I}^1} \\ 0 \end{bmatrix} \end{aligned} \tag{A125}$$

One sees immediately that this recovers our generalized Keynesian cross of Corollary 3. We can therefore think of households as firms who, in order to supply labor, demand a consumption bundle as inputs. On top of the assumption that households only interact through firms, this representation also relies on the assumption that households do not choose their labor supply in the first period; this makes them analogous to firms, who must meet market demand.

## B.5. Network Decompositions for Supply Shocks

We now derive network decompositions of the multiplier as in Section 3.4 that are valid for both demand and supply shocks, extending the earlier analysis. To this end, we see that changes in GDP when we consider a supply shock have two distinct components:

$$d(GDP) \equiv d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{A126}$$

Where it is without loss to redefine units of consumption goods and evaluate at an initial equilibrium with $p^{1T} = \vec{1}$. Propositions 6 and 7 already decomposed the first term $\vec{1}^T dY^1$. To achieve our decomposition for supply shocks, we therefore need only compute $dp^{1T}Y^1$. To this end, we can employ Corollary 2, where we derived prices in closed-form as a function of $z$:

$$p^1(z) = \left(1 - \hat{X}^1(z)^T\right)^{-1}\hat{L}^1(z)\vec{1} \tag{A127}$$

It follows that the change in GDP can then be decomposed as before but with a new term which depends only on the IO matrix and labor shares and not labor rationing or household consumption. This is stated formally below:

**Proposition 18.** *The total change in first-period output due to a demand shock with unit-magnitude labor income incidence $\partial y^1$ can be approximated as:*

$$\begin{aligned} d(p^{1T}Y^1) =& \frac{1}{1-\mathbb{E}_{y*}[m_n]}\Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\textit{Incidence effect}} \\ &+ \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{next}] - \mathbb{E}_{y*}[m_n]\right)}_{\textit{Biased spending direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}}\Bigg) \\ &+ \underbrace{d\left[\left(1 - \hat{X}^1(z)^T\right)^{-1}\hat{L}^1(z)\vec{1}\right]^T Y^1}_{\textit{Price Effect}} + O^3(|m|) \end{aligned} \tag{A128}$$

*where $y^*$ is any reference income weighting of unit-magnitude and $m^i_{next}$ is the average MPC of households who receive as income $i$'s marginal dollar of spending.*

*Proof.* Recall that we have:

$$d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{A129}$$

Which we can always take as:

$$d(p^{1T}Y^1) = \vec{1}^T dY^1 + dp^{1T}Y^1 \tag{A130}$$

through an appropriate renormalization of the initial units of the goods. By Proposition 6, we have that:

$$
\begin{aligned}
1^T dY^1 = & \frac{1}{1-\mathbb{E}_{y*}[m_n]} \Bigg( 1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\text{Incidence effect}} \\
& + \underbrace{\mathbb{E}_{\partial y^1}[m_n] \left( \mathbb{E}_{\partial y^1}[m_n^{\text{next}}] - \mathbb{E}_{y*}[m_n] \right)}_{\text{Biased spending direction effect}} + \underbrace{\mathbb{C}\text{ov}_{\partial y^1}[m_n, m_n^{\text{next}}]}_{\text{Homophily effect}} \Bigg) + O^3(|m|)
\end{aligned} \tag{A131}
$$

We now need only compute the term $dp^{1T}Y^1$. To this end, from Corollary 2 we have that:

$$
p^1(z) = \left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1} \tag{A132}
$$

Differentiating yields:

$$
dp^{1T}Y^1 = \underbrace{d\left[\left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1}\right]^T Y^1}_{\text{Price Effect}} \tag{A133}
$$

Adding the two terms yields the claimed expression and completes the proof. □

## *B.6. Optimal Policy at a Global Optimum*

In the main text, we focused primarily on small changes in welfare corresponding to small changes in policy. In this section, we specialize to the case of small changes in policy *at an optimum*. Thus, the corresponding changes in welfare are second order.

Our first result decomposes the first-order condition for optimal government spending and transfers into five distinct mechanisms. This is closely related to Proposition 8 in the main text, which considers the change in welfare away from the global optimum.

**Proposition 19.** *Suppose taxes $\tau^{1*}, \tau^{2*}$ and expenditures $G^{1*}, G^{2*}$ solve the planner's problem. Now consider a change in policy $\tau^t = \tau^{t*} + \varepsilon\tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$, indexed by $\varepsilon$. The* *following first-order condition holds:*

$$
\begin{aligned}
0 = & \underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \widehat{\Delta} R^1)\right) G_\varepsilon^1}_{\text{Opportunistic government spending}} \quad + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G_\varepsilon^2}{1 + r^1}}_{\text{Short-termist government spending}} \\
& \underbrace{- (\widetilde{\lambda} - \gamma \vec{1})^T \widehat{\mu} \left(\tau_\varepsilon^1 + \frac{\tau_\varepsilon^2}{1 + r^1}\right)}_{\text{Pure redistribution}} \qquad \underbrace{+ \widetilde{\lambda}^T \frac{\widehat{\phi} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}}_{\text{Relaxation of borrowing constraints}} \\
& \underbrace{- \widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C_{y^1}^1 R^1\right)^{-1} C_{y^1}^1 \left(R^1 G_\varepsilon^1 - \widehat{\mu} \tau_\varepsilon^1 - \frac{1_{\phi_n = 0} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}\right)}_{\text{Keynesian stimulus (alleviation of involuntary unemployment)}}
\end{aligned}
\tag{A134}
$$

*where $\gamma$ is the marginal value of public funds.*

*Proof.* The planner takes prices and—locally—the interest rate as given. Goods and labor market clearing and first-period rationing determine the change in first-period employment as a function of $G_\varepsilon^1$ and $\tau_\varepsilon^1$. We are left with the following first-order condition of the planner's problem:

$$
0 = dW + \gamma \left[\mu^T \tau_\varepsilon^1 + \frac{\mu^T \tau_\varepsilon^2}{1 + r^1} - \vec{1}^T G_\varepsilon^1 - \frac{\vec{1}^T G_\varepsilon^2}{1 + r^1}\right] \tag{A135}
$$

where $dW$ is as in Equation 34. This gives an expression for the change in welfare in terms of $\tau_\varepsilon$, $G_\varepsilon$, and $l_\varepsilon^1$, the change in first-period employment. By Equation 22, $\widehat{\mu}\ l_\varepsilon^1 = R^1(I - C_{y^1}^1 R^1)^{-1} \partial Q^1$, where $R^1 \equiv l_{L^1}^1 \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ and $\partial Q^1 = G_\varepsilon^1 - C_{y^1}^1 \widehat{\mu} \tau_\varepsilon^1 - C_{y^2}^1 \widehat{\mu} \tau_\varepsilon^2$. For borrowing-constrained households, $C_{y^2}^1 = 0$; they would already like to substitute additional consumption toward the first period but are constrained not to do so. Other households are Ricardian, implying $C_{y^2}^1 = \frac{C_{y^1}^1}{1 + r^1}$. Plugging in for $dW$, and using matrix notation, we have

$$
\begin{aligned}
0 = \widetilde{\lambda}^T & \Bigg[ - \widehat{\Delta} R^1 (I - C_{y^1}^1 R^1)^{-1} \left(G_\varepsilon^1 - C_{y^1}^1 \widehat{\mu} \left(\tau_\varepsilon^1 + \frac{1_{\phi_n = 0}\ \tau_\varepsilon^2}{1 + r^1}\right)\right) \\
& - \left(\widehat{\mu} \tau_\varepsilon^1 + \frac{\widehat{\mu}(I - \widehat{\phi}) \tau_\varepsilon^2}{1 + r^1}\right) + \left(\widehat{\mu} WTP^1 G_\varepsilon^1 + \widehat{\mu}(I - \widehat{\phi}) \frac{WTP^2}{1 + r^1} G_\varepsilon^2\right) \Bigg] \\
& + \gamma \left(\mu^T \tau_\varepsilon^1 + \frac{\mu^T \tau_\varepsilon^2}{1 + r^1} - \vec{1}^T G_\varepsilon^1 - \frac{\vec{1}^T G_\varepsilon^2}{1 + r^1}\right)
\end{aligned}
\tag{A136}
$$

Now, observe that the term on the first line can be rewritten:

$$
\begin{aligned}
&R^1(I - C^1_{y^1}R^1)^{-1}\left(G^1_\varepsilon - C^1_{y^1}\widehat{\mu}\left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1}\right)\right) \\
=&R^1\left(\sum_{k=0}^{\infty}(C^1_{y^1}R^1)^k\right)\left(G^1_\varepsilon - C^1_{y^1}\widehat{\mu}\left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1}\right)\right) \\
=&\left(R^1G^1_\varepsilon + \left(\sum_{k=0}^{\infty}(C^1_{y^1}R^1)^k\right)C^1_{y^1}R^1G^1_\varepsilon\right) \\
&- R^1\left(\sum_{k=0}^{\infty}(C^1_{y^1}R^1)^k\right)C^1_{y^1}\widehat{\mu}\left(\tau^1_\varepsilon + \frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1}\right) \\
=&R^1G^1_\varepsilon + R^1\left(I - C^1_{y^1}R^1\right)^{-1}C^1_{y^1}\left(R^1G^1_\varepsilon - \widehat{\mu}\tau^1_\varepsilon - \widehat{\mu}\frac{1_{\phi_n=0}\ \tau^2_\varepsilon}{1+r^1}\right)
\end{aligned}
\tag{A137}
$$

Substituting this back in and rearranging, we obtain Equation A134. □

To better understand the form of the implied optimal policy, we discuss each term of Equation A134 in turn. The opportunistic government spending term is as in Werning (2011) and Baqaee (2015). It augments the standard first-order condition for government spending with a labor-wedge term, reflecting that the social cost of additional government purchases is lower than the market cost when they are produced using underemployed labor. The second term is also an augmented version of the standard expression for government spending—this time in the second period. The borrowing wedge reflects that households with binding borrowing constraints implicitly discount the future at a higher-than-market rate; the planner must account for this when deciding whether to make purchases on their behalf.

The third term of Equation A134 is a standard, pure redistribution term, weighing the private benefits of transfers against the social cost (the MVPF). The fourth term augments this, when there are borrowing constraints. In particular, taxes in the second period are less costly to borrowing-constrained households, since they discount the future more heavily than the market rate indicates.

Finally, the last line captures the value of stimulus brought on by changes in income—those corresponding to pure income transfers via taxes and labor market income earned by government employees producing expenditures.[45] $C^1_{y^1}$ maps income changes to changes in consumption. Then the output multiplier $(I - C^1_{y^1}R^1)^{-1}$ maps this partial equilibrium change in consumption to the general equilibrium change in output. Finally, $R^1$ maps the change

[45] If second period expenditures are held constant, then the net income transfer is zero, i.e. this term operates solely through redistribution to different households (who may spend differently).

in output to the change in labor supplied to meet income-induced demand changes, loading onto the labor wedges.

## B.7. Optimal policy with imperfect competition

In this section, we extend the optimal policy results of section 4 to the more general environment with constant, non-zero markups. As is section 4 we normalize prices $p_i^t$ to one throughout, without loss of generality.

To highlight as clearly as possible the parallels to the case without profits, we make two important assumptions. First—although in the first period, profit-creation is uninternalized by households—we assume that the government incentivizes second-period profit-creation with Pigouvian subsidies funded lump-sum by shareholders.

**Assumption 8.** *There is an ad-valorem subsidy $s_i^2$ on the purchase of i (for consumption or production), set equal to the profit rate $m_i^2$. It is funded directly by an additional lump-sum, second-period tax $\hat{\tau}_n^2$ defined by $\mu_n \hat{\tau}_n^2 = \sum_{i \in I} \left( \widehat{\Pi}_{ni}^2 \Big/ \sum_{n' \in N} \widehat{\Pi}_{n'i}^2 \right) s_i^2 Q_i^2$.*

Second, we assume that the MPC out of future profits is zero. This is a rather weak assumption, as the MPC out of even *current* capital income is small empirically.

**Assumption 9.** *For all households n, $C_{\pi^2}^1 = 0$.*

### B.7.1. Planner's problem

We begin by defining the household's problem. It is the same as Equation 32 in section 4.1, except that households now also receive profit income.

$$
\begin{aligned}
\max_{\tilde{c}^t, \tilde{l}^t} \quad & \sum_{t=1,2} \beta_n^{t-1} \left[ u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t) \right] \\
\text{s.t.} \quad & p^1 \cdot \tilde{c}^1 + \frac{p^2 \cdot \tilde{c}^2}{1+r^1} + \tau_n^1 + \frac{\tau_n^2}{1+r^1} \leqslant \tilde{l}^1 + \frac{\tilde{l}^2}{1+r^1} + \pi_n^1 + \frac{\pi_n^2 - \hat{\tau}_n^2}{1+r^1} \\
& \tilde{l}^1 + \pi_n^1 - p^1 \cdot \tilde{c}^1 - \tau_n^1 \geqslant \underline{s}_n^1 \\
& \tilde{l}^1 = l_n^1
\end{aligned}
\tag{A138}
$$

Note that this microfoundation implies $C_y = C_\pi$. That is, additional income from rationed labor has the same effects on consumption as additional income from profits.

As in section 4, we study the policy problem of a planner at the zero lower bound. Formally, the planner's problem is the same as in Equation 33 except that household behavior solves Equation A138 and aggregate variables evolve according to Equation A96 with $r_Q = 0$.

*B.7.2. Policy changes away from the optimum*

This section considers changes in welfare due to small changes in not-necessarily-optimal policies, as in section 4.2. The only difference now is the presence of profits.

This setup in mind, we now consider the change in welfare induced by changes in transfers and government expenditure, analogously to Proposition 8.

**Lemma 2.** *Under assumptions 8 and 9, the change in welfare $dW$ due to a small change in taxes and government expenditure—at a constant interest rate—can be expressed:*

$$\begin{aligned} dW = \sum_{n\in N} \widetilde{\lambda}_n \mu_n \Bigg[ & -\Delta_n dl_n^1 + d\pi_n^1 - \left( d\tau_n^1 + (1-\phi_n)\frac{d\tau_n^2}{1+r^1} \right) \\ & + \left( WTP_n^1 dG^1 + (1-\phi_n)\frac{WTP_n^2}{1+r^1} dG^2 \right) \Bigg] \end{aligned} \tag{A139}$$

*where $\widetilde{\lambda}_n$ is the value the planner places on the marginal transfer of first-period wealth to a household of type $n$, $\Delta_n$ and $\phi_n$ are $n$'s implicit first-period labor wedge and borrowing wedge, and $WTP_n^t$ is the vector of $n$'s marginal willingness to pay for period $t$ government expenditures on each good, in period $t$ dollars. The changes in first-period employment and profits, in turn, are given by*

$$\begin{aligned} & \hat{\mu} dl^1 = l_{L^1}^1 \widehat{L}^1 \left(1 - \widehat{X}^1\right)^{-1} dY^1, \quad \hat{\mu} d\pi^1 = \widehat{\Pi} \left(1 - \widehat{X}\right)^{-1} dY^1, \\ & dY^1 = \left( I - C_{y^1}^1 \left( l_{L^1}^1 \widehat{L}^1 + \widehat{\Pi}^1 \right) \left( I - \widehat{X}^1 \right)^{-1} \right)^{-1} \partial Q^1 \end{aligned} \tag{A140}$$

*Proof.* We follow the same steps as the proof of Proposition 8 (see appendix A.11) up to the substitution of the budget constraint, which now includes profits. With profits, differentiating the household's lifetime budget constraint (at constant $r^1$) gives:

$$p^1 dc_n^1 - dl_n^1 - d\pi_n^1 + \frac{p^1 dc_n^2 - dl_n^2}{1+r^1} = -d\tau_n^1 + \frac{d\pi_n^2 - d\hat{\tau}_n^2 - d\tau_n^2}{1+r^1} \tag{A141}$$

Note that since $\sum_{n'\in N} \widehat{\Pi}_{n'i}^2 = m_i^2 = s_i^2$:

$$d\hat{\tau}_n^2 = \frac{1}{\mu_n} \sum_{i\in I} \left( \widehat{\Pi}_{ni}^2 \Big/ \sum_{n'\in N} \widehat{\Pi}_{n'i}^2 \right) s_i^2 dQ_i^2 = \frac{1}{\mu_n} \widehat{\Pi}_{ni}^2 dQ_i^2 = d\pi_n^2 \tag{A142}$$

Plugging in the change in the differentiated budget constraint, we have:

$$dW = \sum_{n\in N} \widetilde{\lambda}_n \mu_n \Bigg[ -\Delta_n dl_n^1 + \phi_n \left(p^1 dc_n^1 - dl_n^1\right) + (1-\phi_n)\left(d\pi_n^1 - d\tau_n^1 - \frac{d\tau_n^2}{1+r^1}\right) \\ + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1-\phi_n}{1+r^1}\right)\frac{w_{nG}^2}{\kappa_n^2} dG^2\right)\Bigg] \tag{A143}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint $\underline{s}_n^1 = l_n^1 + \pi_n^1 - \tau_n^1 - p^1 c_n^1$ implies $p^1 dc_n^1 + d\tau_n^1 = dl_n^1 + d\pi_n^1$. Defining the within-period willingnesses to pay $WTP_n^t = \frac{w_{nG}^t}{\kappa_n^t}$, we arrive at the final expression:

$$dW = \sum_{n\in N} \widetilde{\lambda}_n \mu_n \Bigg[ -\Delta_n dl_n^1 + \left(d\pi_n^1 - d\tau_n^1 - (1-\phi_n)\frac{d\tau_n^2}{1+r^1}\right) \\ + \left(WTP_n^1 dG^1 + (1-\phi_n)\frac{WTP_n^2}{1+r^1} dG^2\right) \tag{A144}$$

Finally, the expressions for $dl, d\pi, dY$ come from rearranging Equation A96 under assumption 9 and using $dY = (1-\widehat{X})dQ$. □

Studying Equation A139 reveals a key insight: Under assumptions 8 and 9, the change in welfare due to a change in taxes and expenditures is the same as in an *as-if* economy without profits but where share-holders supply labor with a wedge −1. This labor supply wedge corresponds to complete under-employment; share-holders—who experience no marginal disutility of holding shares—would continue to be willing to hold shares until profits-per-revenue reached zero. Just like labor suppliers, share-holders do not choose their income but rather take it as given. This *as-if* representation of profits as under-employed labor allows us to carry over all of the results from Section 4 with minimal alterations.

**Proposition 20.** *Under assumptions 5, 8, and 9, the welfare change from a change in expenditures is proportional to the resulting change in output, whereas the welfare change from a change in transfers is proportional to the resulting change in income. Formally,*

$$dW = \vec{1}^T \frac{dY^1}{dG} dG + \vec{1}^T \frac{d(l+\pi)^1}{dy^1}\left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1}\right) \tag{A145}$$

*where* $\frac{dY^1}{dG^1} = (1 - C_{y^1}^1 R^1)^{-1}$ *and* $\frac{dY^1}{dG^2} = 0$ *are first-period output multipliers and* $\frac{d(l+\pi)^1}{dy^1} = (1 - R^1 C_{y^1}^1)^{-1}$ *is the first-period income multiplier; here* $R^1 = \left(l_{L^1}^1 \widehat{L}^1 + \widehat{\Pi}^1\right)\left(I - \widehat{X}^1\right)^{-1}$.

*Proof.* Reinterpret profit income as labor supply with wedge −1, as discussed above. The proof then follows from appendix A.12. □

The key here is that assumption 5's imposition that all marginal labor supplies have a labor supply wedge of $-1$ exactly matches with the shareholders' implicit labor supply wedge of $-1$. Indeed, both are indifferent to supply more of their factor. As a result, there is zero social cost to additional employment of either factor, so the optimal policy simply maximizes output.

As without markups, the output-maximizing policy is simply MPC-targetting when "network effects" are not present:

**Corollary 6.** *Suppose that all households' marginal spending is directed to households whose average MPC is equal to the incidence-weighted average MPC corresponding to a uniform output shock.*[46] *Formally,* $m_n^{next} = \mathbb{E}_{y*}[m_{n'}]$ *for all* $n$*, where* $m_i^{next} \equiv \left(mR^1 C^1_{y^1}\widehat{m}^{-1}\right)_i$ *and* $R^1 = \left(l^1_{L^1}\widehat{L}^1 + \widehat{\Pi}^1\right)\left(I - \widehat{X}^1\right)^{-1}$. *Then, under assumptions 5, 8, and 9, the welfare change from a policy is given by:*

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m\right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1 + r^1}\right) \tag{A146}$$

*Dollar-for-dollar, the best policy is the one most effectively targeting household MPC.*

*Proof.* Again, the proof follows from appendix A.13 after reinterpreting profit income as labor supply with wedge $-1$. □

*B.7.3. First-order conditions for optimal policy*

The same *as-if* representation of profits as under-employed labor also allows us to carry over results from section B.6 to the case of imperfect competition.

**Proposition 21.** *Suppose taxes* $\tau^{1*}, \tau^{2*}$ *and expenditures* $G^{1*}, G^{2*}$ *solve the planner's problem. Now consider a change in policy* $\tau^t = \tau^{t*} + \varepsilon\tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$*, indexed by* $\varepsilon$*. Then,*

[46] This ensures that the final two correction terms in Equation 28 are zero for all partial equilibrium shocks.

*under assumptions 8 and 9, the following first-order condition holds:*

$$
\begin{aligned}
0 = & \underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \check{\Delta} \check{R}^1)\right) G_\varepsilon^1}_{\text{Opportunistic government spending}} \quad + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G_\varepsilon^2}{1 + r^1}}_{\text{Short-termist government spending}} \\
& \underbrace{- (\widetilde{\lambda} - \gamma \vec{1})^T \widehat{\mu} \left(\tau_\varepsilon^1 + \frac{\tau_\varepsilon^2}{1 + r^1}\right)}_{\text{Pure redistribution}} \qquad \underbrace{+ \widetilde{\lambda}^T \frac{\widehat{\phi} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}}_{\text{Relaxation of borrowing constraints}} \\
& \underbrace{- \widetilde{\lambda}^T \check{\Delta} \check{R}^1 \left(I - \check{C}_{y^1}^1 \check{R}^1\right)^{-1} \check{C}_{y^1}^1 \left(\check{R}^1 G_\varepsilon^1 - \widehat{\mu} \tau_\varepsilon^1 - \frac{1_{\phi_n = 0} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}\right)}_{\text{Keynesian stimulus (alleviation of involuntary unemployment)}}
\end{aligned}
\tag{A147}
$$

*where $\gamma$ is the marginal value of public funds, $\check{R}^1 = \begin{bmatrix} l_{L^1}^1 \widehat{L}^1 \\ \widehat{\Pi}^1 \end{bmatrix} \left(I - \widehat{X}^1\right)^{-1}$, $\check{C}_{y^1}^1 = \begin{bmatrix} C_{y^1}^1 & C_{y^1}^1 \end{bmatrix}$, and $\check{\Delta}$ is the $N \times 2N$ matrix with entries $\check{\Delta}_{n,n} = \Delta_n$, $\check{\Delta}_{n,N+n} = -1$, and zeros elsewhere.*

*Proof.* This follows from reinterpreting profit income as labor supply with wedge $-1$ and then following the proof of Proposition 19. □

Intuitively, the planner targets "profit-wedges" in the same manner as labor supply wedges. These both reduce the social cost of government spending and provide a motive for Keynesian stimulus.

Finally, a similar network-irrelevance result holds as in the case without profits.

**Proposition 22.** *Impose Assumptions 8 and 9. Now, suppose that all households rationed to on the margin at the optimum have no marginal labor disutility, i.e. if $(R^1 C_{y^1}^1)_{n,-} \neq \vec{0}$ then $\Delta_n = 0$. Then Equation A147 holds with respect to variations in first-period transfers if and only if, for all $n \in N$,*

$$\gamma = \frac{\widetilde{\lambda}_n}{1 - m_n} \tag{A148}$$

*Alternatively, suppose that the social gains from first-period government expenditure are equal to some $\tilde{v}$ across goods and constraints bounding expenditures above zero do not bind. Then Equation A134 holds with respect to variations in first-period expenditures if and only if, for all $i \in I$,*

$$\gamma = \tilde{v} + \frac{1}{1 - \widetilde{m}_i} \left(-\widetilde{\lambda \Delta}_i\right) \tag{A149}$$

*where $\tilde{m}_i \equiv \left(m^T R^1\right)_i$ is the rationing-weighted average MPC in the production of good $i$ and $\widetilde{\lambda \Delta}_i \equiv \left(\widetilde{\lambda}^T \check{\Delta} \check{R}^1\right)_i$ is the rationing-and-welfare-weighted average rationing wedge in the* *production of good $i$, where $R^1$ is as in proposition 20 and $\check{R}^1$ and $\check{\Delta}$ are as in proposition 21.*

*Proof.* Again, this follows from reinterpreting profit income as labor supply with wedge $-1$ and then following Appendix A.14, plus imposing $\Delta_n = 0$ for marginal labor-suppliers in the transfer case. $\square$

# C. Validating the Model

The model that we develop and estimate in this paper makes stark predictions about the propagation of industry- and region-specific shocks. In this section, we attempt to empirically validate those quantitative predictions. Specifically, Corollary 3 provides an expression relating total output to spending in a state $s$ and industry $i$:

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 = M \partial Q^1 \tag{A150}$$

where $M$ is the generalized multiplier matrix. The $m_{s,r}$ entry gives the total change in output in state $s$ when there is a one-unit partial equilibrium shock to state $r$ distributed across industries in proportion to their share of total output in state $r$. Any identified partial equilibrium shock $G$ will be some component of the many partial equilibrium shocks hitting the economy, which we can express as $\partial Q^1 = G + U$, where $U$ is the partial equilibrium effect on demand of the unobserved shocks hitting the economy. Plugging this in, we arrive at the foundation for our estimating equation:

$$dY_t = M(G + U) = \beta M G_t + \epsilon_{i,t} \tag{A151}$$

where $G$ is the vector of identified industry-by-region shocks and $M$ is our estimated generalized multiplier. The strict prediction of our model is that $\beta = 1$, meaning that we have perfectly predicted the heterogeneous effects of the shocks on output growth. Note that the matrix $M$ includes not only heterogeneity in the response to a shock in one's own market, but also how each market will respond to other markets to spillovers arising from spending network effects. Therefore, in addition to testing $\beta = 1$, we also test separately for the existence of spillovers of the nature predicted by the model. More specifically, we run the following regression:

$$dY_t = M(G + U) = \alpha_0 \left(M_{diag}\right) G_t + \alpha_1 \left(M_{offdiag}\right) G_t + \epsilon_{i,t} \tag{A152}$$

where $M_{diag}$ is the diagonal entries of the multiplier matrix (i.e. all other entries are set to 0) and $M_{offdiag}$ are the off-diagonal entries of the multiplier matrix. $\alpha_0$ captures the degree to which the multiplier accurately captures the effect of a direct shock and $\alpha_1$ captures the degree to which the model accurately captures the nature of the spillovers across regions and industries.

In the following sections, we will use two different identified shocks for $G$ – state-level military spending shocks from Nakamura and Steinsson (2014) and a growth in industry imports from Autor et al. (2013). Of course, bringing this to the data presents several identification challenges particular to the shock in question. We address the challenges particular to each shock below as we slightly modify Equation A151 to fit the particular setting.

## *C.1. Government Spending Shocks from Nakamura and Steinsson (2014)*

The first shock that we consider is the local government spending shock developed by Nakamura and Steinsson (2014) to estimate local fiscal spending multiplier. We refer the reader to that paper for the details on the construction of the shock. We closely follow their original specification, using data on US states from 1966-2006. We restrict our attention to variation across states and our dependent variable is the 2-year change in state GDP per capita, divided by the level of state GDP lagged 2 periods. The state spending shock is the 2-year change in military spending per capita, also divided by the level of state GDP lagged 2 periods.Specifically, we run the following regression

$$\frac{y_{s,t} - y_{s,t-2}}{y_{s,t-2}} = \beta \frac{(MG)_{s,t} - (MG)_{s,t-2}}{y_{s,t-2}} + \gamma_s + \gamma_t + e_{s,t} \tag{A153}$$

where $\gamma_s$ and $\gamma_y$ are state and year fixed effects, respectively. The central concern is that military spending is not random and may be directed towards states based on their economic performance. Therefore, we follow Nakamura and Steinsson (2014) and instrument the state changes in spending with state dummies interacted with national changes in military spending. Table A1 shows the results. First, Column 1 shows the replication of the result in Nakamura and Steinsson (2014), which is the equivalent of imposing that $M$ has 1 on the diagonals but is 0 elsewhere (call this $M_1$). Column 2 shows the estimate of Equation A153. The estimates are noisy, but two small pieces of evidence suggest that including the multiplier provides a better fit for the data than the simple specification. First, while we cannot reject that the coefficient on either $M_1G$ or $MG$ is 1, the coefficient on $MG$ is closer to 1 than the coefficient on $M_1G$, suggesting that the heterogeneity embedded in $M$ is getting us closer to capturing all of the variation in the data. Second, the r-squared in Column 2 is slightly higher than that in Column 1. However, the estimates are noisy and largely inconclusive.

The remaining columns of Table A1 show the estimates separating the own and spillover effects as in Equation A152. A finding that the coefficient on the spillover term were positive and close to 1 would suggest that our measure was accurately picking up the experienced spillovers. Here, the estimates are also too noisy to be conclusive.

| | Baseline | | | Robustness | | |
|---|---|---|---|---|---|---|
| | | | | No State FE | post-1980 | post-1990 |
| State Spending ($M_1G$) | 1.474*** | | | | | |
| | (0.373) | | | | | |
| Model Prediction (MG) | | 1.189*** | | | | |
| | | (0.299) | | | | |
| Model Prediction ($M_{diag}G$) | | | 1.251*** | 1.166*** | 1.569** | 0.657 |
| | | | (0.355) | (0.309) | (0.611) | (0.908) |
| Model Prediction ($M_{nodiag}G$) | | | -0.145 | 0.496 | -7.112 | -8.899 |
| | | | (3.367) | (3.242) | (5.443) | (9.385) |
| Constant | | | | | | |
| Observations | 1989 | 1989 | 1989 | 1989 | 1377 | 867 |
| R-Squared | 0.316 | 0.319 | 0.316 | 0.309 | 0.305 | 0.308 |

Table A1: Reduced Form Validation: Government Spending from Nakamura and Steinsson (2014)

## C.2. *Chinese Import Shocks from Autor et al. (2013)*

We also explore the predictions of our model using import shocks constructed as in Autor et al. (2013). While the government spending shocks were primarily at the level of the state, import shocks are primarily at the level of the industry. Thus, as in Autor et al. (2013), we construct the state-level exposure to the China shock ($\Delta IP_{s,t}$) using the industry distribution in each state as:

$$\Delta IP_{s,t} = \sum_j \frac{L_{sjt}}{L_{st}} \frac{\Delta \text{Imports}_{j,t}}{L_{i,1991}} \tag{A154}$$

where $j$ is the industry, $s$ is the state, and $Imports_{j,t}$ are the imports from China to the US. Variation across states in import exposure stems from differences across states in their industry distribution. We assume that there are no imports to non-manufacturing industries. Using this measure as our state-level demand shock, we estimate

$$\Delta logY_{s,t} = \beta_1 M\ \Delta IP_{s,t} + \gamma_s + \gamma_t + \epsilon_{st} \tag{A155}$$

where $Y_{s,t}$ is state GDP and $\gamma_s$ and $\gamma_t$ are state and year fixed effects, respectively. We use stacked 5-year changes and utilize data from 1991-2011. The central concern is that imports grow most in areas that are already shrinking or growing, and therefore we instrument the China shock $\Delta IP_{s,t}$ with the imports from China in eight other developed countries as in Autor et al. (2013).

Table C.2 shows the results. Column 1 first shows the baseline estimate where $M = M_1$, where $M_1$ is a diagonal matrix of ones. As predicted given the results in Autor et al. (2013),

| | Baseline | | | Robustness | |
|---|---|---|---|---|---|
| | | | | Excluding GR | Rolling Window |
| China Shock G) | -1.847** | | | | |
| | (0.782) | | | | |
| Model Prediction (MG) | | -1.485** | | | |
| | | (0.596) | | | |
| Model Prediction ($M_{diag}G$) | | | -1.322* | -1.481** | -1.993*** |
| | | | (0.684) | (0.638) | (0.620) |
| Model Prediction ($M_{nodiag}G$) | | | -2.638 | -1.819 | -2.855 |
| | | | (1.814) | (1.637) | (1.903) |
| Constant | | | | | |
| Observations | 204 | 204 | 204 | 153 | 561 |
| R-Squared | 0.482 | 0.481 | 0.485 | 0.409 | 0.485 |

Table A2: Reduced Form Validation: China Shock from Autor et al. (2013)

states with a larger growth in imports experienced lower GDP growth rates. The following columns test for the ability of our estimated multiplier to predict the magnitude of the effect as well as the direction of the spillovers. The coefficient on $M_{nodiag}$ is generally negative and similar in magnitude to the coefficient on $M_{diag}$. This means that we generally find that the model correctly predicts the direction of the spillovers. However, the results are too noisy to draw any firm conclusions.

# D. Additional Tables and Figures

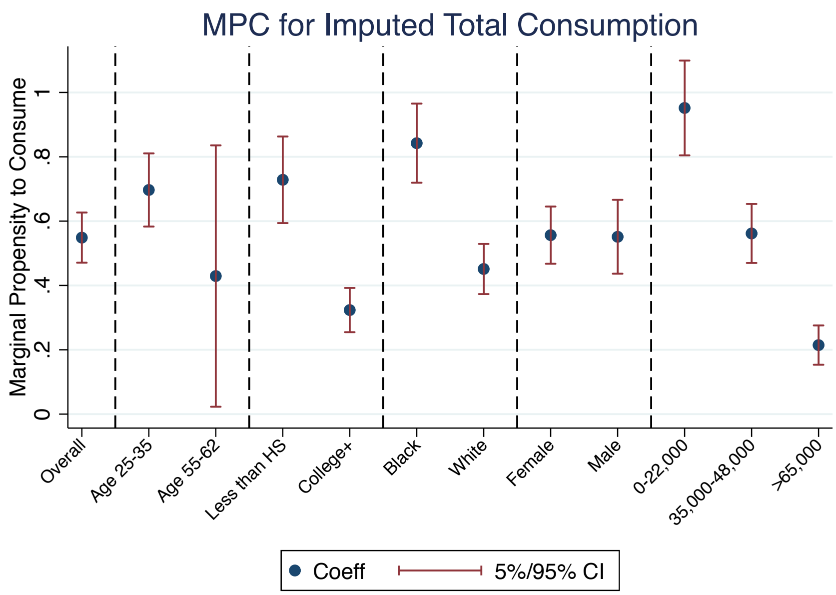


Fig. A1. Heterogeneity in estimated MPCs for total consumption across demographic groups.

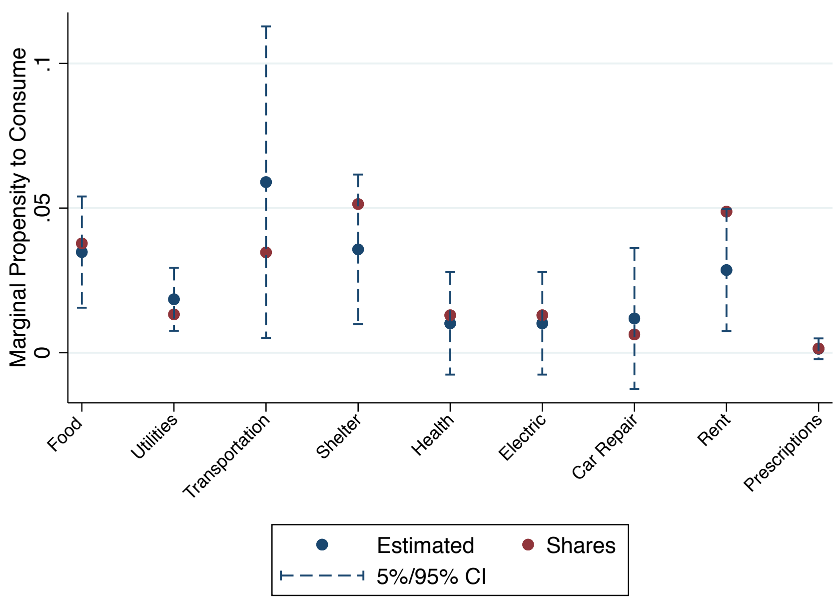


Fig. A2. Estimated Directed MPCs Vs. CEX basket-weighted MPCs

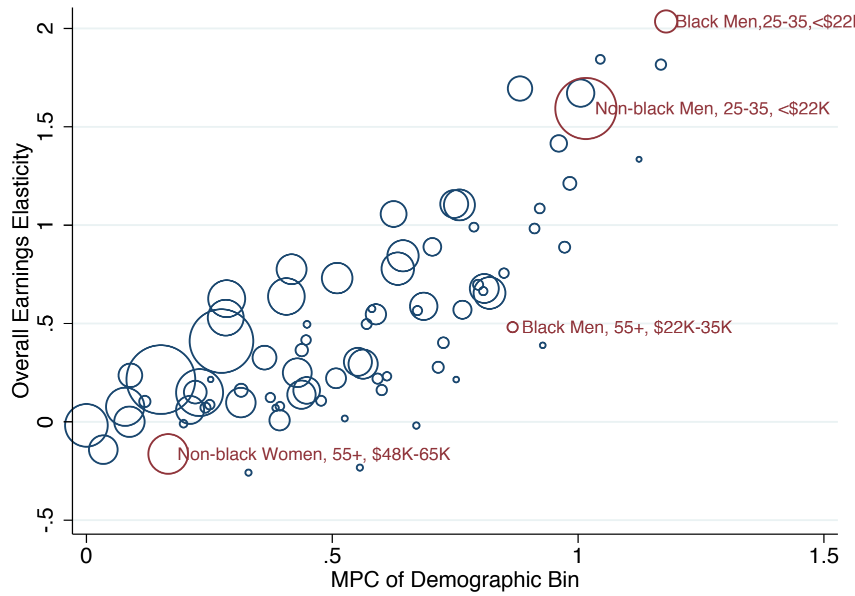


Fig. A3. Earnings elasticity to GDP shocks scattered against estimated MPC. See Patterson (2019) for more details.

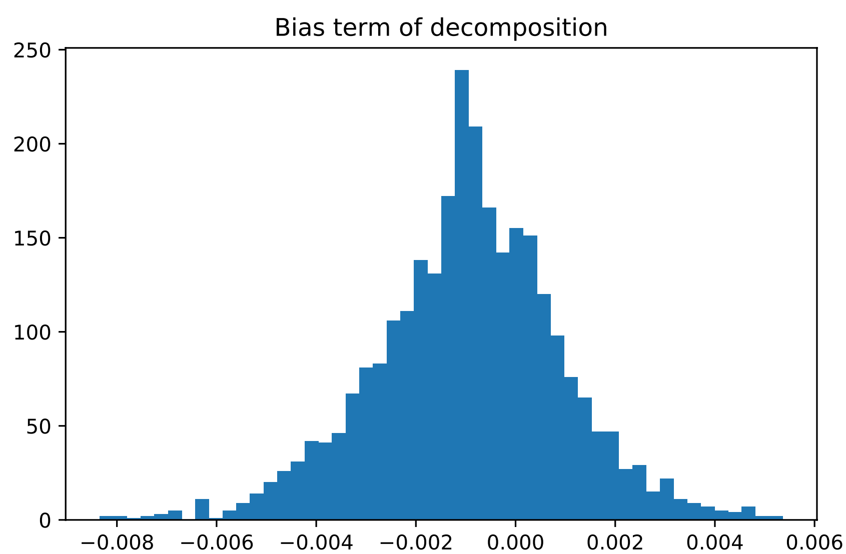


Fig. A4. Histogram of the bias terms from the decomposition in Proposition 6 for each unit demand shock to the 2805 sector-region pairs, with baseline $y^*$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

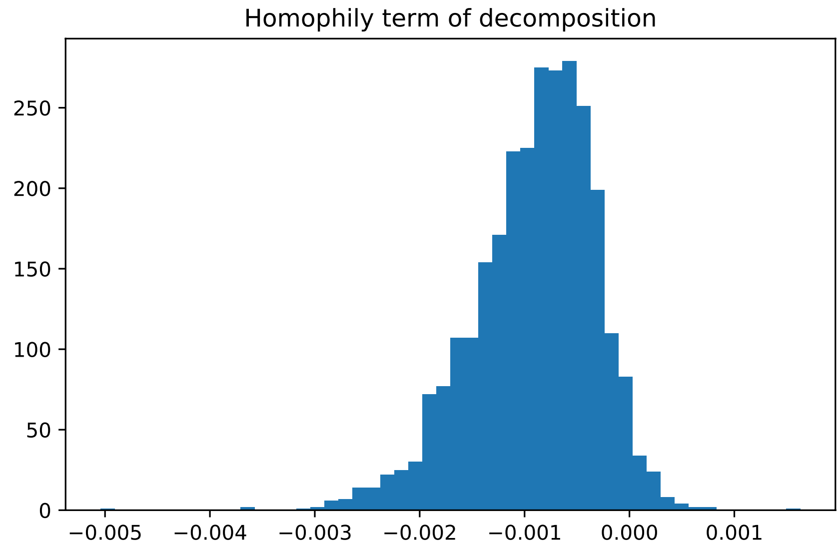


Fig. A5. Histogram of the homophily terms from the decomposition in Proposition 6 for each unit demand shock to the 2805 sector-region pairs, with baseline $y^*$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

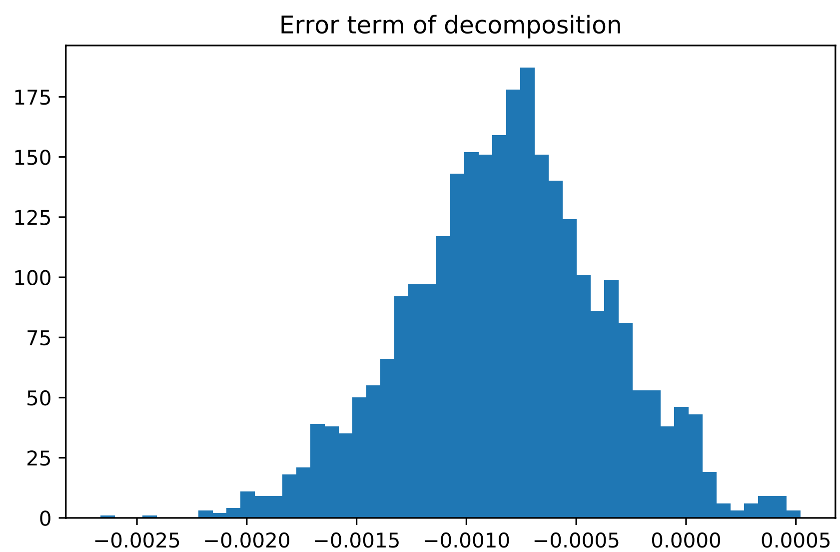


Fig. A6. Histogram of the error terms from the decomposition in Proposition 6 for each unit demand shock to the 2805 sector-region pairs, with baseline $y^*$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

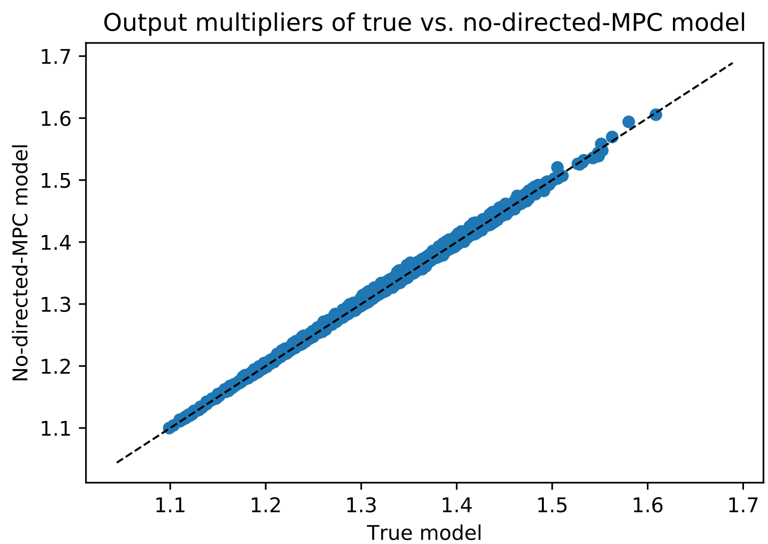


Fig. A7. Scatter plot of output multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which all households have homogeneous consumption baskets in proportion to aggregate consumption (y-axis).

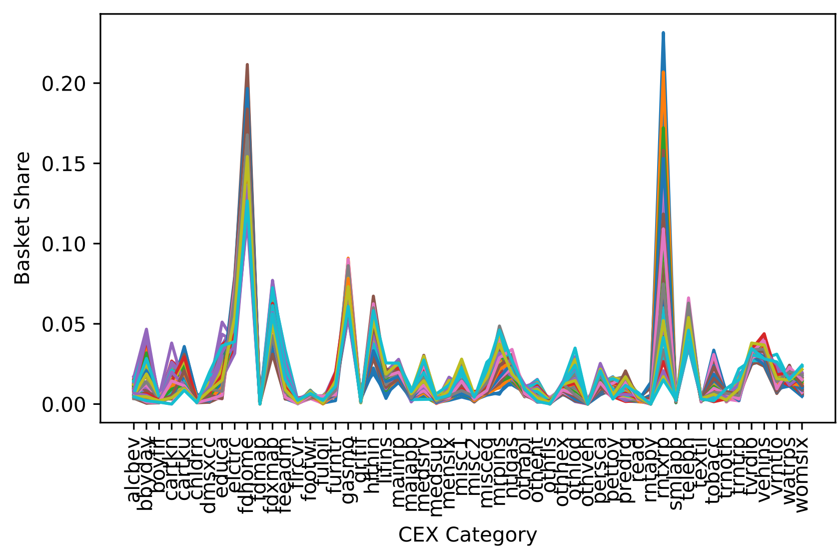


Fig. A8. Consumption basket weights for each demographic group (each line is a demographic group) across each CEX consumption category.

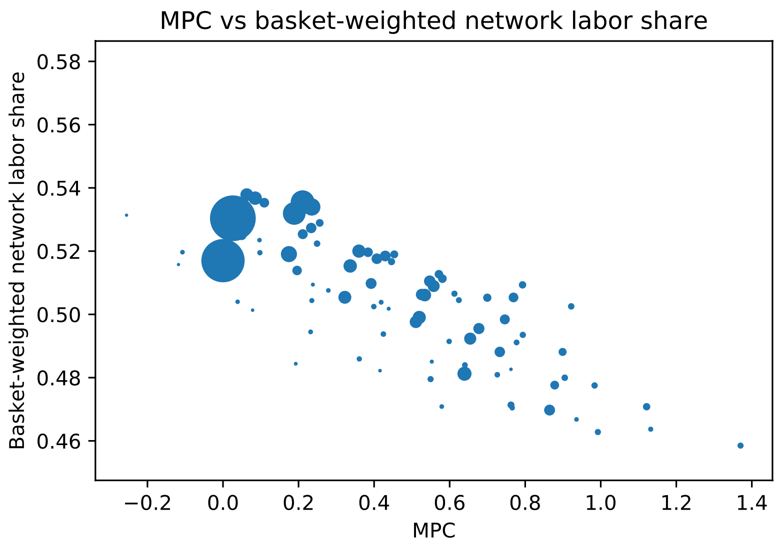


Fig. A9. Scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume.

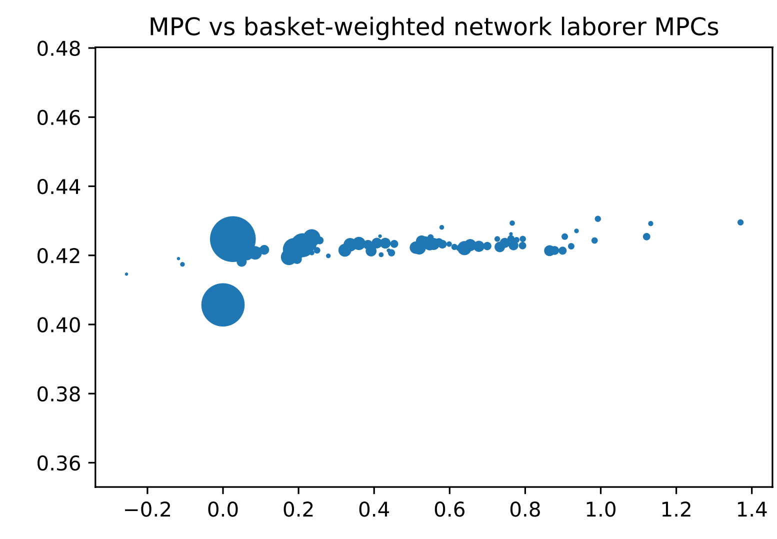


Fig. A10. Scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume.

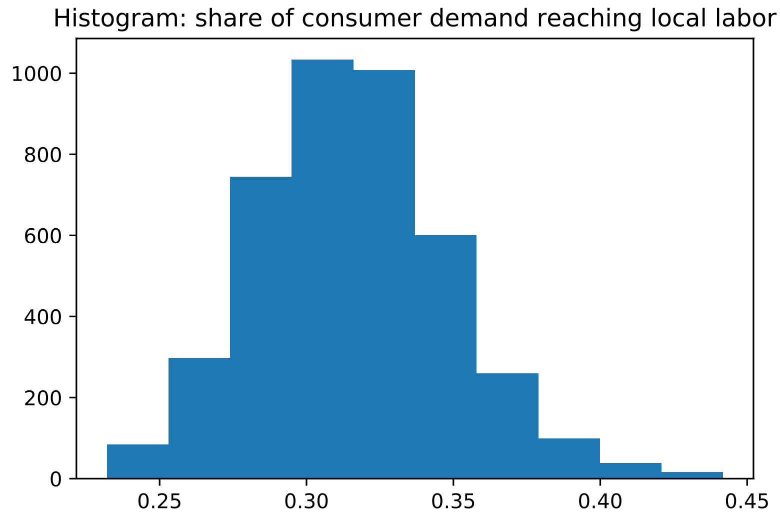


Fig. A11. Histogram of the fraction of consumer demand resulting in income for labor within the same state for each state-demographic pair.

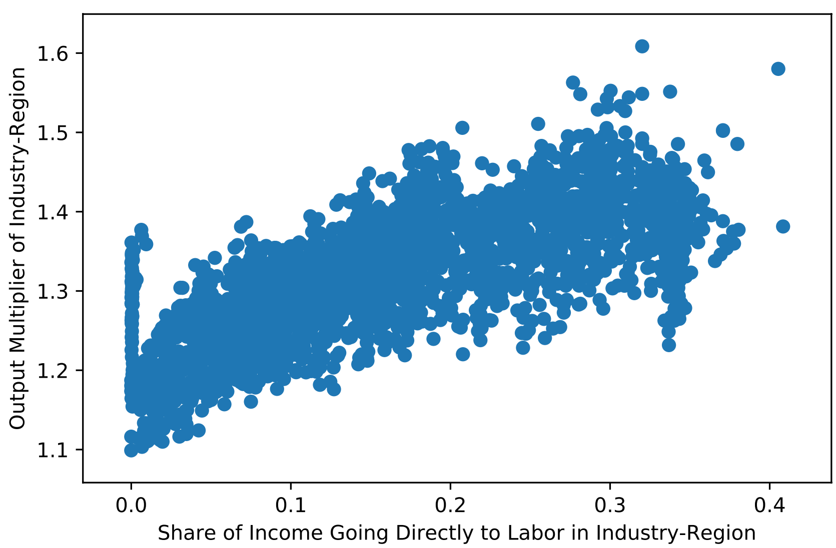


Fig. A12. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the share of income from production that goes directly to labor (as opposed to capital, foreigners, or inputs).

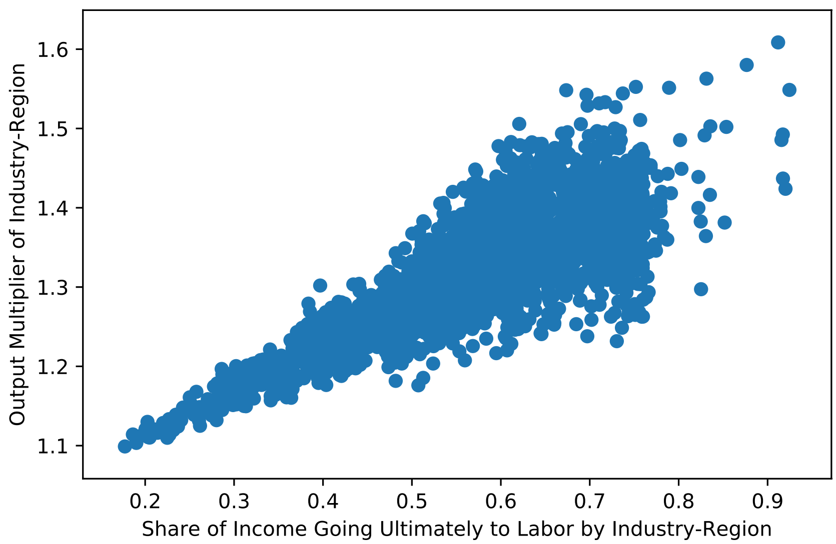


Fig. A13. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the ultimate labor share accounting for labor employed in the production of intermediates.

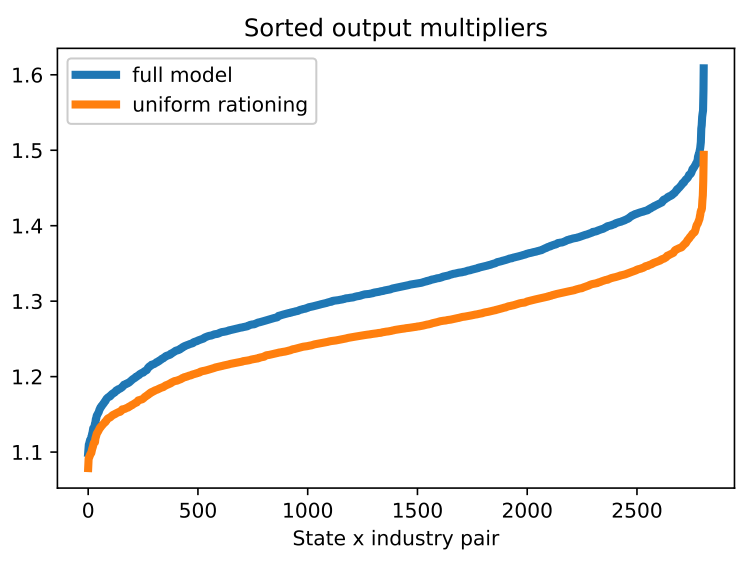


Fig. A14. Sorted change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their labor income.

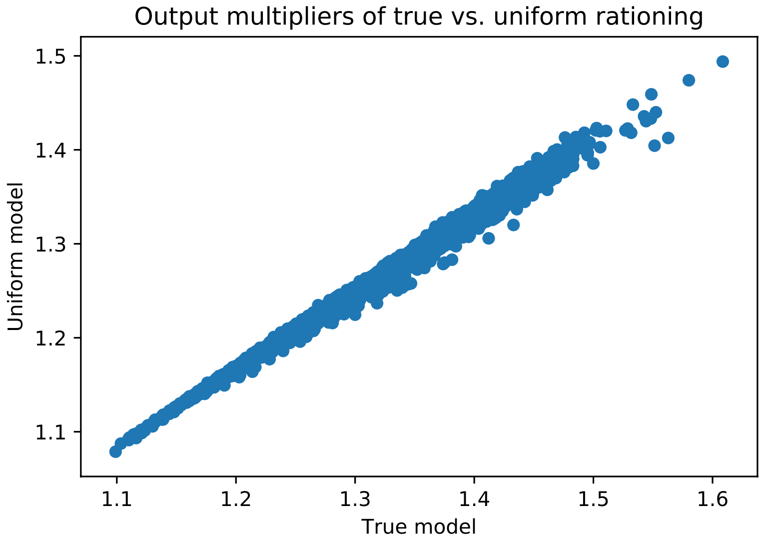


Fig. A15. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their income.

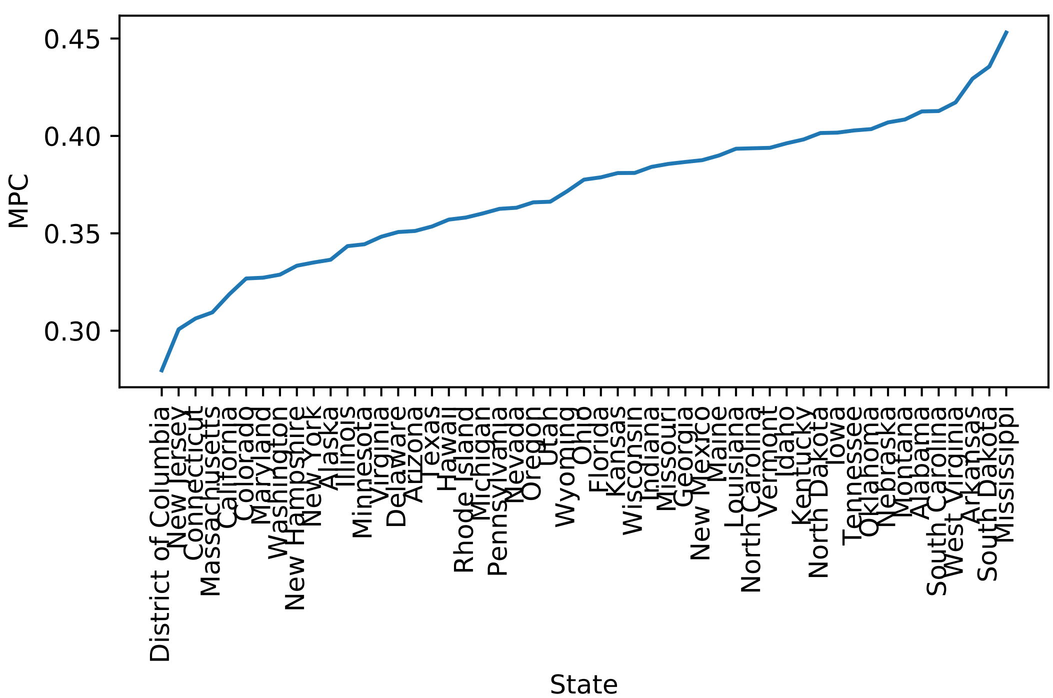


Fig. A16. Income-weighted average MPC by state.

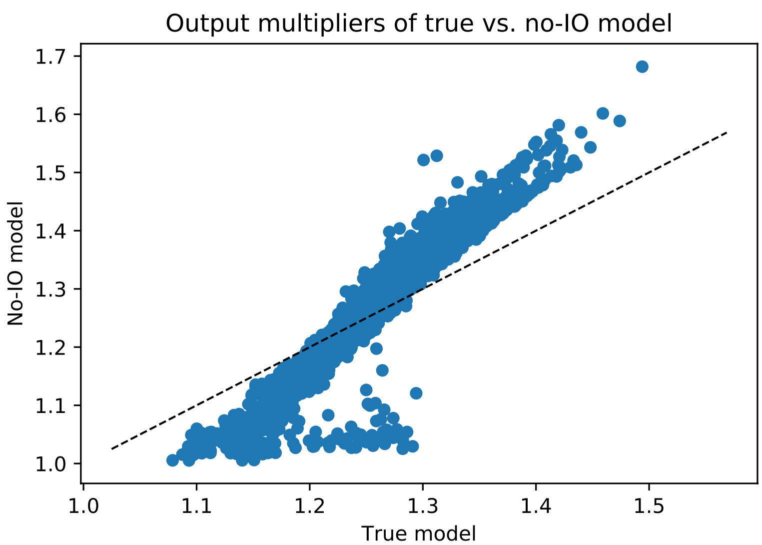


Fig. A17. Scatter plot of output multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which there is no intermediate goods use by firms (y-axis).

| Component | Incidence multiplier | Bias | Homophily | Total | Error |
|---|---|---|---|---|---|
| Magnitude | 1.302 | -0.002 | -0.001 | 1.298 | 0.000 |

Table A3: Homophily decomposition for a shock to demand proportional to 2012 GDP across sectors. "Incidence multiplier" includes the first and second terms in in Proposition 6. "Bias" is the bias correction and "homophily" is the homophily correction. Error is the difference between the sum of these terms and the exact multiplier.

| | No Directed MPC | Directed MPC |
|---|---|---|
| Uniform Rationing | 1.23 | 1.23 |
| MPC Rationing | 1.28 | 1.28 |

Table A4: Multiplier of a GDP-proportional output shock across model specifications. In this table, we eliminate regional structure and instead have 55 industries at the national level. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

| | No Directed MPC | Directed MPC |
|---|---|---|
| Uniform Rationing | 1.25 | 1.25 |
| MPC Rationing | 1.30 | 1.30 |

Table A5: Multiplier of a GDP-proportional output shock across model specifications. In this table, everything is as in the baseline except we eliminate regional trade and assume that all consumption and intermediate goods use is within each state. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

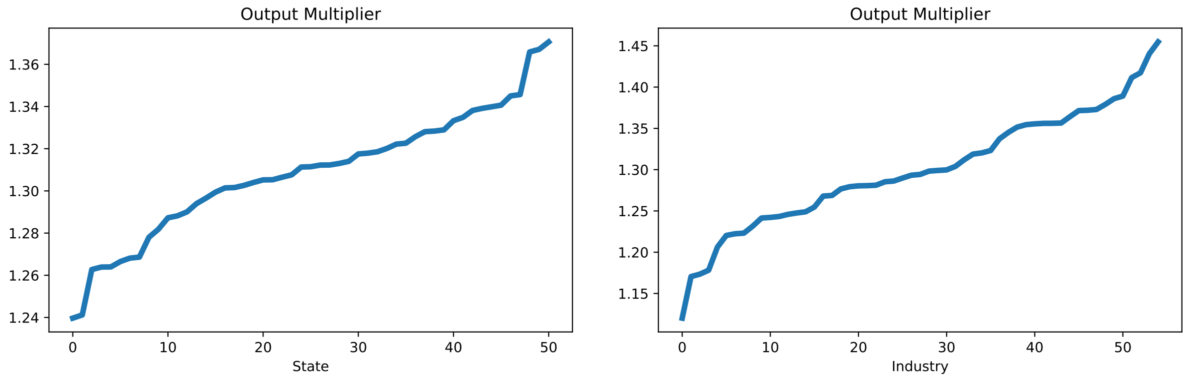


Fig. A18. Multipliers for state-level and industry level shocks. Formally, we take the shock for each state $r$ as $\partial Q_r = \left(\mathbb{I}[s = r]\frac{y_{sj}}{\sum_k y_{rk}}\right)_{sj}$, where $y_{rj}$ is BEA output for sector $j$ in state $r$ and each industry $j$ as $\partial Q_j = \left(\mathbb{I}[k = j]\frac{y_{rk}}{\sum_s y_{sj}}\right)_{rk}$. That is, we marginalize across each dimension according to output shares.

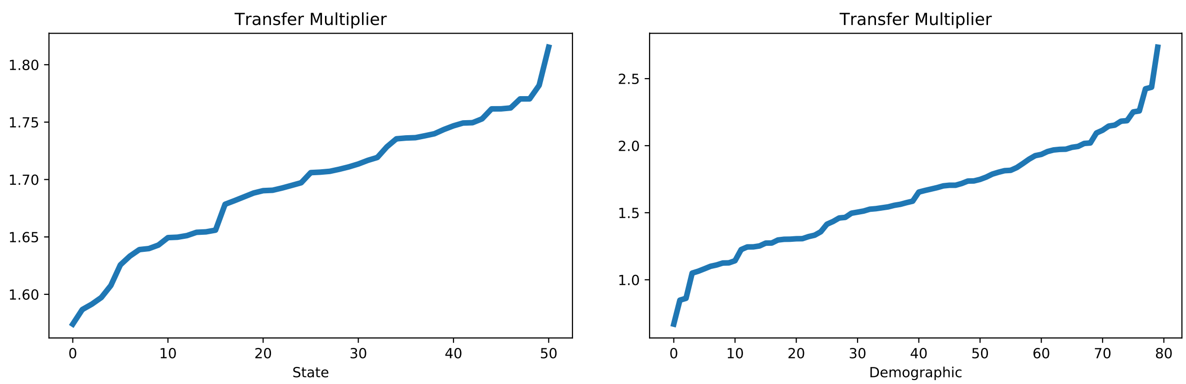


Fig. A19. Multipliers for state-level and demographic level transfer shocks. Formally, for the state-level shock, we transfer each state one dollar, in proportion to the demographic composition of that state. For the demographic-level shock, we transfer each demographic group one dollar, in proportion to the distribution of that demographic across states.

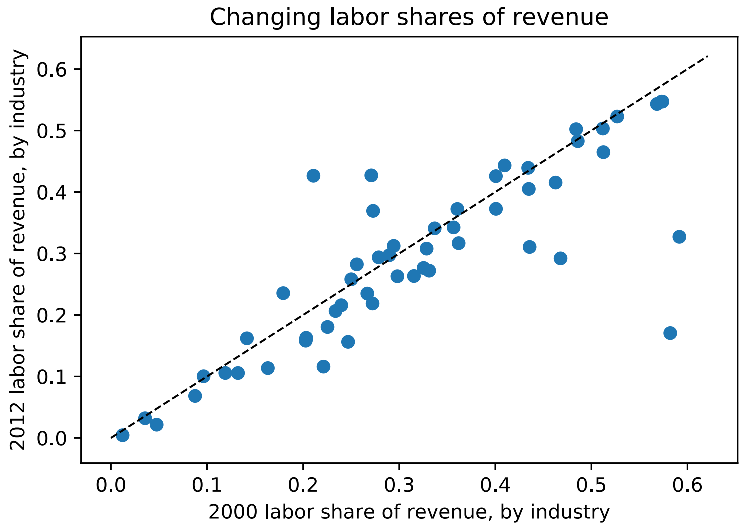


Fig. A20. Labor shares of revenue, by industry, in 2000 vs. 2012. Most industries experience a modest decline in labor share. The most dramatic decline is in the sector labelled “data processing, internet publishing, and other information services.” The most dramatic increase is in the sector labelled “apparel and leather and allied products.”

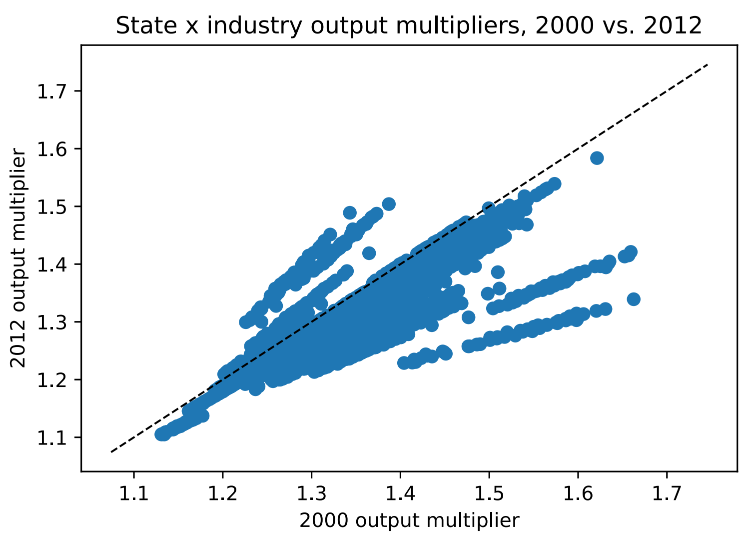


Fig. A21. Scatter plot of output multipliers in 2000 vs. 2012, by state-industry pair.