Advanced economies are permeated by complicated network structures that connect households and industries via employment, directed consumption, and input-output languages. How do these structures determine the effectiveness of the conditional transfers, unconditional transfers, and direct expenditures at the disposal of fiscal policymakers? We study this question in a model with household heterogeneity in MPCs, directed consumption patterns, and exposure to industry and regional shocks. Theoretically, we express fiscal multipliers in terms of estimable sufficient statistics, and we decompose them into three network effects on top of a standard Keynesian multiplier. Empirically, we find that fiscal multipliers vary substantially depending on how the government targets transfers and expenditure and that virtually all of the variation in multipliers stems from heterogeneous initial incidence of any policy across households with differing MPCs. These findings imply that targeting fiscal policy is important, but that maximally expansionary policy can be implemented by following a simple fiscal policy that targets households based solely on their MPCs.
1. Introduction

Economic shocks present policymakers with the challenge of designing stimulus programs that best prevent prolonged economic downturns. Most recently, in response to COVID-19, the United States Congress has implemented a broad spectrum of fiscal policies on an enormous scale. This response has included three major common categories of stimulus: undirected transfers (stimulus checks), targeted transfers (expanded unemployment insurance benefits), and targeted spending (industry programs, such as for the airlines). While COVID-19 presents unique challenges, this range of policy responses draws attention to questions that policymakers face during every major recession: which forms of fiscal stimulus are the most effective, whom do they help, and how should they be targeted?

These questions are complicated by the rich networks that make up present-day economies. Economic linkages – through supply chains, regional trade, and heterogeneous employment and consumption relationships – prevent a fiscal planner from conducting policy one household at a time. Rather, policymakers must consider the cascades of expenditure they set off, as expenditures in one industry in one state reach not only its workers but also others in its supply chain, those at firms where workers spend their marginal income, and so on. While such considerations may appear to greatly complicate the design of fiscal policy, we provide a number of theoretical and empirical results that indicate that a real-world policymaker can, in many cases, design maximally expansive fiscal policy by following simple rules that require only very limited information about the underlying structure of the economy.

We develop this argument in two parts. The first part of our paper provides a theory of how two key policy instruments – government purchases and fiscal transfers – propagate through supply chains, employment linkages, and the directed MPCs of heterogeneous households. While these channels interact in complex ways, we show how to decompose all of these interactions into three distinct effects, on top of a baseline representative firm and agent Keynesian multiplier. The second part of the paper takes this decomposition to the data and finds that all of the heterogeneity in multipliers is captured by the heterogeneous incidence of the fiscal shock onto households with different MPCs, whose particular patterns of consumption are by contrast irrelevant. As a result, maximally expansive fiscal policy in a widespread recession simply targets high MPCs, as in much simpler models. This MPC targeting is not only relatively simple, but also quantitatively important – for small stimulus policies, it can result in twice as much policy amplification as untargeted, GDP-proportional purchases.

Our starting point for this analysis is a semi-structural, general equilibrium model that incorporates heterogeneity among households and firms. On the household side, we allow
for heterogeneity in both the magnitude of households’ MPCs and their direction toward different goods. On the firm side, we allow for many sectors and regions, linked to one another through an arbitrary input-output structure. Finally, we allow for any pattern of household employment across the various firms, generating heterogeneous household income processes. Within this rich setting, we study a rationing equilibrium where wages are sticky and thus labor is rationed, so that households can be off their labor supply curves and be involuntarily un(der)employed.\footnote{We provide general, technical results on the existence of equilibria as well as a no-substitution theorem whereby prices are determined independently of demand.} This assumption, as well as a focus on the case where an effective lower bound binds, makes our model applicable to severe recessions.

The various microeconomic interconnections between households complicate the ways that fiscal shocks translate into output. Specifically, we provide an intuitive expression for the change in the vector of first period output across industries and regions in response to partial equilibrium shocks to final demand. First, the input-output network translates those shocks to final demand into changes in the production in each sector. Second, the rationing function captures how those changes in production translate into changes in labor income for each household. Finally, a matrix with the magnitude and direction of household MPCs across goods maps these changes in household income into changes in their demand for industries and regions. This loop repeats ad infinitum, generating our expression for the heterogeneity-adjusted multiplier.

Nevertheless, the total effect of any fiscal policy on aggregate GDP – or its fiscal multiplier – can be decomposed into three distinct effects on top of a baseline Keynesian multiplier that would exist in a model without heterogeneous agents and industrial linkages. First, the “incidence effect” captures that policies with incidence onto higher-MPC households change GDP by more. Second, the bias effect captures additional amplification that results when households directly affected by the policy disproportionately direct their marginal spending toward goods produced by higher-than-average-MPC workers. Third, the “homophily effect” captures the amplification that occurs when high (low) MPC households direct their spending to other high (low) MPC households, for instance due to geographic concentration. The magnitude of these three effects are themselves a function of the underlying structure of the economy, so that economies with higher MPCs and/or labor shares have higher multipliers whereas economies with less connected input-output networks have more dispersed multipliers.

In order to understand which features of the economy contribute to shock amplification and to gauge the quantitative importance of targeting fiscal policy, the second part of the paper takes our model to the data. We combine several public-use datasets describing 50
US states (plus DC), 55 sectors, and 80 demographic groups to estimate three key empirical objects: the regional input-output matrix describing the input-use requirements of every industry-region pair; the rationing matrix describing how much each demographic-region pair’s income changes in response to a one dollar change in production of each industry-region pair; and the directed MPC matrix describing how much each demographic-region pair consumes from each industry-region pair.

By combining these estimated matrices with our derived expressions for the fiscal multiplier, we uncover wide variation in government purchases and transfers multipliers depending on where in the economy a fiscal shock originates – our estimates imply that a dollar of purchases from the sector-region with the highest multiplier leads to twice as much amplification as spending that same dollar on a GDP-proportional basket of goods. We find that virtually all of the difference in multipliers is driven by differences in the incidence of the shock onto households with higher or lower MPCs, and that households’ patterns of directed consumption across sectors and regions—and so the bias and homophily effects—do not contribute meaningfully to the multiplier. We find that the large heterogeneity in shock incidence is driven primarily by dispersion in MPCs in the population and the sorting of workers of different types across sectors and regions.

The key implication of these empirical results for the conduct of fiscal policy is that targeted fiscal policy is both simple and important. Our empirical finding that the impact of directed household consumption on the multiplier (i.e. the bias and homophily effects) are quantitatively negligible implies that the multiplier of any fiscal shock depends only on its incidence onto households with higher or lower MPC. Thus, MPC targeting is the maximally expansionary policy. For fiscal transfers, this amounts to transferring money to those workers with the highest MPCs. Targeting government purchases is more complicated, as the planner hopes to allocate spending so as to affect the workers with the highest MPCs, which requires knowledge of the input-output network (as in Baqae (2015)) and the labor rationing process, both of which shape how changes in demand affects the income of workers. Our estimates suggest that a government that was going to transfer one thousand dollars to each employed worker would increase GDP by 69 cents for each dollar spent. If the government instead decided to transfer two thousand dollars to workers with above-median MPCs, they would spend the same amount but increase GDP by 96 cents for each dollar spent. Thus, even this relatively minor targeting of transfers to households with higher MPCs increases the amplification of the transfers by 40 percent.

We can use the structure of our exercise to provide theoretical results and counterfactual simulations that explore how the distribution of multipliers may vary across economies and across time within the US. We first construct a counterfactual economy with no input-output
linkages, finding that while this does not affect the amplification of a GDP-proportional shock, rich-IO connections do tighten the distribution of multipliers across industry-regions. Second, we consider the general decrease in sectoral labor shares between 2000 and 2012, finding that this decreases the multiplier of shocks to most, but not all, industry-region pairs. Third, we show that “hollowing-out” of the labor income distribution within occupations has negligible effects on fiscal multipliers.

A further contribution of our approach is that we can quantify which features of advanced economies matter for which macroeconomic questions. On the one hand, suppose first that a researcher is interested solely in understanding the response of aggregate GDP to fiscal policy. Our results suggest that accounting for heterogeneous incidence is sole important margin to consider. Thus, our results echo recent work that stresses amplification of shocks that load more heaving onto households with higher-than-average MPCs (Werning, 2015; Kaplan, Moll, and Violante, 2018; Auclert, 2019; Patterson, 2019), and the role of input-output linkages in distributing the incidence of shocks on various industries throughout the economy (Long and Plosser, 1987; Gabaix, 2011; Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012; Baqae and Farhi, 2019; Rubbo, 2019; Bigio and La’O, 2020). Conversely, we find that accounting for regional trade and the direction of consumption (in particular its within-region bias) as has been recently emphasized (Farhi and Werning, 2017; Caliendo, Parro, Rossi-Hansberg, and Sarte, 2018; Dupor, Karabarbounis, Kudlyak, and Mehkari, 2018) does not contribute in a quantitatively meaningful way to our understanding of the aggregate GDP impact of shocks. However, this is not to say understanding such features is not important. Suppose, on the other hand, that a researcher wishes to understand the distribution of the impact of shocks across households, industries, and space. Our results show that it is essential to model these features relating to the direction of consumption and the regional structure of the economy. Thus, our analysis may be of use for researchers deciding which model elements are essential for their analysis depending on the questions at hand and which ones can be omitted without compromising the integrity of the results.

Related Literature While each of the previously cited papers focuses on one or two dimensions of heterogeneity, we integrate them in a model rich enough to bring to the data through sufficient statistics. This more reduced-form approach is similar methodologically to Auclert, Rognlie, and Straub (2018), who use intertemporal MPCs to discipline macroeconomic models and study the implications for the financing and timing of fiscal stimulus. Our approach differs in its focus on heterogeneity and the targeting of stimulus in the cross-section. In fact, this approach dates back to the much earlier regional accounting literature

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2 Other papers adopting the sufficient statistics approach include Wolf (2019), and Koby and Wolf (2019).
which emphasized how demand may spill over across regions (Miyazawa, 1976). We micro-
found this literature’s focus on fixed prices in an environment with a single factor, sticky
wages, and a binding zero lower bound.

The theoretical part of our paper relates most closely to Baqee (2015). As we do in this
paper, Baqee emphasizes that shocks to an industry affect not only the factors employed
in that industry but also those used in producing its inputs, motivating a “network adjust-
ment” to the labor share of each industry. In more recent work, Baqee and Farhi (2018)
develop rich macroeconomic models featuring these channels as well as endogenous prices
and markups. By abstracting away from price movements, we are able to precisely charac-
terize – as well as empirically assess – the channels through which economic linkages affect
aggregate shock propagation and how these matter for optimal stimulus policy.³ Lastly,
our paper relates closely to several concurrent papers, motivated by the Covid-19 pandemic,
that explore the effects of fiscal and monetary policy when shocks are heterogeneous across
sectors. Woodford (2020) shares the theoretical insight that transfers multipliers vary de-
pending on targeting and allows for heterogeneity in spending patterns across constrained
and unconstrained households which affects the size of higher-order effects through a gen-
eralized Keynesian mechanism. Similarly, Baqee and Farhi (2020) find, in a model with
networks and nominal rigidities, that disparate demand and supply shocks blunt the power of
aggregate demand stimulus policies. We differ from both of these recent papers in our precise
decomposition of deviations from the Keynesian case and our focus on calibrating sufficient
statistics in order to demonstrating which features of the actual economy are empirically
important for shaping the distribution of multipliers.

Lastly, this paper also adds to a large empirical literature estimating multipliers from
fiscal shocks. Our structural estimates complement reduced-form empirical estimates of
open-economy multipliers – we calibrate an aggregate purchases multiplier of 1.30, which is
somewhat smaller than, but within the established confidence intervals of, those in Ramey
(2011), Nakamura and Steinsson (2014), Chodorow-Reich (2019) and Corbi, Papaioannou,
and Surico (2019). While most of the empirical literature has focused on identifying estimates
for the purchases multiplier, a few more recent empirical papers share our focus on uncovering
heterogeneity in purchases multipliers across space. These empirical papers leverage finer
geographical and sectoral data to explore fiscal spillovers. For example, Feyrer, Mansur, and
Sacerdote (2017) document geographical spillovers in demand from counties with increased

³Relatedly, Zorzi (2020) studies the interaction of cyclicality in durable consumption and investment
with sector-specific employment in a parametric environment. This paper and ours are related insofar as
they involve the interaction of directed demand and heterogeneous labor rationing. However, we abstract
away from the specific microfoundation of directed demand and take a more reduced form approach that
emphasizes richer connections between households and firms.
fracking production onto nearby regions. Auerbach, Gorodnichenko, and Murphy (2020) leverage rich data on Department of Defense contracts, finding reduced form evidence for both back-propagation of demand through supply chains and increased demand in other industries through income multipliers. Theoretically, we provide a framework consistent with the evidence presented in these papers and provide structural estimates detailing the distinct channels through which these spillovers operate.4

Outline The rest of the paper proceeds as follows. Section 2 introduces the model and defines the rationing equilibrium. Section 3 derives the multiplier and provides a decomposition characterizing the role of heterogeneity. Section 4 introduces the data and methodology we use to estimate the multiplier. Section 5 quantifies the distribution of fiscal multipliers and highlights the features of the economy that shape the nature of shock propagation. Section 6 explores the implications of these empirical findings for the design of fiscal policy and shows that maximally expansive fiscal policy can be achieved with relatively simple MPC targeting. Section 7 explores the stability of these conclusions across economies and over time, and. Section 8 concludes.

2. The Model and Rationing Equilibrium

To understand the propagation of fiscal policies, we build a semi-structural model. In the model, a continuum of heterogeneous households interact in a competitive, multi-sector, multi-region economy over two periods. We consider a rich class of household-level consumption and labor supply functions that accommodate arbitrary preference heterogeneity, household borrowing constraints, and most behavioral frictions, as well as a general, constant returns to scale input-output structure. We consider a rationing equilibrium, where first period wages are fixed and first period labor supply is determined by exogenous rationing functions rather than by household optimization. This allows households to lie off their labor supply curves, capturing classical involuntary unemployment.5 Our model is rich enough to capture many dimensions of household, industrial, and regional heterogeneity, but sufficiently tractable to deliver equations that we later bring directly to the data.

4Cox, Müller, Pasten, Schoenle, and Weber (2019) use the same procurement data and account for heterogeneity in price stickiness across sectors subject to fiscal shocks, which lies outside of our framework.

5In the Appendix and Supplementary Material, we provide a microfoundation for rationing equilibrium based on sticky wages, nominal interest rates, and inflation expectations; prove equilibrium existence results; extend this analysis to an arbitrary number of time periods and imperfect competition with fixed markups; and compare our rationing equilibrium to a flexible-wage equilibrium.
2.1. Model Primitives and Rationing Equilibrium

Time is indexed by $t \in \{1, 2\}$. In each period $t$, there is a finite set of goods $T_t$; we interpret goods as differing not only in kind but also in location. Each good is produced by a representative firm $i$ that uses a vector of intermediates $X^t_i = (X^t_{i1}, \ldots, X^t_{|T_t|}) \succeq 0$ and a single labor factor $L^t_i \succeq 0$ to produce gross output $Q^t_i = F^t_i(X^t_i, L^t_i)$, where $F^t_i$ is a CRS production. Each firm $i$ generates value added $Y^t_i = p^t_i F^t_i(X^t_i, L^t_i)$ in each period, firms take the vector of prices $p^t = \{p^t_i\}_{i \in I}$ and wages $w^t$ as given and maximize profits. We normalize the wage to one within every period, i.e. $w^t = 1$; intertemporal price comparisons are possible via the real interest rate $r^1$. Firms choose labor and intermediate inputs to maximize profits in each period:

$$\max_{X^t_i, L^t_i \succeq 0} p^t_i F^t_i(X^t_i, L^t_i) - L^t_i - p^t_i X^t_i \tag{1}$$

There is a continuum of households on the interval $[0, 1]$, and a finite set of types $N$, where each type $n \in N$ has mass $\mu_n > 0$ such that $\sum_{n \in N} \mu_n = 1$. $N$ can capture demographic factors such as age, sex, and race as well as location of residence. Households consume a vector of goods $c^t_n = \{c^t_{ni}\}_{i \in T^t}$ in each period $t$, and they save an amount $s^1_n$ between periods at a real rate $1 + r^1$; households have no initial savings or debt. The government levies (possibly negative) lump sum taxes on each household in both periods of $\tau_n = (\tau^1_n, \tau^2_n)$. Each household $n$ therefore has labor income $y^t_n = l^t_n$ in period $t$. Households’ first period labor income is determined by a differentiable rationing function that maps the vector of labor demands $(L^1_i)_{i \in I^1}$ to a length-$N$ vector $R^1((L^1_i)_{i \in I^1})$ of labor supplied to each firm for each household type. For example, firms may ration labor from workers that inhabit the same region. The rationing function treats all households within each type identically, and is such that the labor market clears:

$$\sum_{n \in N} R^1((L^1_i)_{i \in I^1}) = \sum_{i \in I^1} L^1_i \tag{2}$$

Rather than explicitly microfounding households’ decision problems, we simply assume there exist exogenous functions that describe their consumption and labor supply as a function of variables outside their control. Formally, the vector of household consumption demands $c^t_n(\varrho, y^1_n, \tau_n)$ and second period labor supply $l^2_n(\varrho, y^1_n, \tau_n)$ are functions of prices $\varrho = (p^1, p^2, r^1)$.

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6 Many of our formal results can accommodate shocks to the production function; see Appendix A.

7 This is without loss, as we can replace initial debt between agents with heterogeneous lump-sum taxes and transfers.
first-period income $y^1_n$, and taxes $\tau_n$. This allows us to nest non-homothetic preferences, behavioral frictions and borrowing constraints. These functions are always such that households satisfy their lifetime budget constraint:

$$l^1_n + \frac{p^1 c^1_n}{1 + r^1} = p^2 c^2_n \frac{1}{1 + r^1} + \tau^1_n + \frac{\tau^2_n}{1 + r^1}$$  \hspace{1cm} (3)$$

In addition to levying lump-sum taxes $\tau^i_n$ on households, the government purchases $G^i_t$ units of good $i \in \mathcal{I}$ subject to running a balanced budget over the two periods. To finance its fiscal spending and tax/transfer programs, the government issues bonds at a real interest rate of $r^1$ in the first period. The real lifetime government budget constraint is therefore

$$\sum_{n \in N} \mu_n \left( \tau^1_n + \frac{1}{1 + r^1} \tau^2_n \right) = p^1 G^1 + \frac{1}{1 + r^1} p^2 G^2$$  \hspace{1cm} (4)$$

So that the government budget constraint continues to hold when prices or the interest rate changes, we assume purchases are given by an exogenously specified function of real prices, taxes, and a government purchases preference parameter $\theta_G$. In particular, $G^i_t = G^i_t (p, (\tau^i_n)_{n \in N}, \theta_G)$, and is such that Equation 4 always holds.

We assume that the nominal interest rate set by the central bank directly pins down the real interest rate that enters into both the government and household budget constraints. We therefore suppose that the central bank sets real interest rates directly, potentially as a function of gross output of any good in any period:\footnote{Our formal results also accommodate preference shocks to these functions; see Appendix A.}

$$r^1 = r^1(Q)$$  \hspace{1cm} (5)$$

Finally, the prices of all goods and wages in the second period are set so that all other markets clear in the usual fashion:

$$Q^i_t = F^i_t (X^i_t, L^i_t) = \sum_{j \in \mathcal{I}^i} X^i_{jt} + \sum_{n \in N} \mu_n c^i_{nt} + G^i_t, \quad \sum_{i \in \mathcal{I}^2} L^2_i = \sum_{n \in N} \mu_n l^2_n$$  \hspace{1cm} (6)$$

A rationing equilibrium is therefore defined as follows:

**Definition 1.** A rationing equilibrium is a set of first and second period, agent- and market-level variables $\{s^i_n, \{c^i_{nt}, l^i_n\}_{t \in \{1, 2\}, i \in \mathcal{I}^i} \}_{n \in N}$ and $\{r^1, \{p^1_t, \{X^i_{jt}\}_{j \in \mathcal{I}^i}, L^i_t, G^i_t\}_{t \in \{1, 2\}}\}_{i \in \mathcal{I}^i}$ that satisfy conditions (1) – (6).
In Appendix C.1, we establish a number of properties of our rationing equilibrium that both eliminate any nuisance terms and ensure that our analysis is well-posed. In particular, we provide mild technical assumptions under which – as a consequence of the single labor factor – intratemporal prices are determined independently from demand (a no-substitution theorem) and an equilibrium exists. We will maintain these assumptions throughout the analysis.

2.2. Interpreting the Model

The concept of rationing equilibrium we study here has a rich intellectual tradition in Keynesian macroeconomics stretching back to Patinkin (1949), Clower (1965) and Barro and Grossman (1971). Indeed, the key idea that price rigidities or other frictions may cause a household to lie off its labor supply curve is a staple of many modern macroeconomic approaches to understanding involuntary unemployment and the business cycle, with our exact formulation via a rationing function being closest to that employed by Werning (2015). In Appendix B, we microfound our equilibrium concept in a model where wages are downwardly rigid and nominal interest rates are fixed. We therefore argue that our equilibrium notion corresponds well to an environment with wage rigidity of the kind commonly observed in the data (Solon, Barsky, and Parker, 1994; Grigsby, Hurst, and Yildirmaz, 2019; Hazell and Taska, 2019).

Our extremely reduced-form representation of the rationing function allows us to nest a number of empirically important phenomena. First, it allows us to capture classical involuntary unemployment, wherein – particularly during economic downturns – there are households who would like to work but cannot because firms are unwilling to hire them. Second, although our model only features a single labor type, it may be reinterpreted to accommodate arbitrarily many flexible factors to the extent that their relative wages are completely rigid. For example, each household type may represent a different type of labor or workers inhabiting a different region; to the extent that a firm marginally demands workers of various types in different proportions, the rationing function will employ them accordingly. The role of relative wage rigidity is to rule out responses of relative wages (and therefore prices) to shocks, which would induce additional margins of substitution by firms and households. Finally, our rationing-function approach allows us to accommodate regional migration driven solely by changes in labor demand. Since employment is demand-rather than supply-determined, the same total income is rationed to each household type in each region regardless of the size or composition of the demographic group in that region. The approach can even accommodate the possibility that labor rationing may respond to
migration-induced changes in the prevalence of different groups, so long as the vector of firms’ labor demands fully determines workers’ incentives to migrate.\textsuperscript{10} The extension to many labor types and labor markets is sketched in Appendix B.5.1.

Finally, our model has so far abstracted away from multiple time periods and imperfect competition. In Appendix D.1 and D.3, we respectively allow for arbitrarily many time periods and imperfect competition and show that suitably modified versions of all of our main results continue to hold.

3. The Multiplier

Within the setting outlined in Section 2, we next explore the general equilibrium impact of government purchases and transfers shocks. Our goal is to derive an expression for the general equilibrium multiplier that maps the effect of shocks in partial equilibrium to their general equilibrium impact. This is both of independent interest for understanding shock propagation and a key step toward understanding the efficacy of fiscal policy. Importantly, we derive a representation of the multiplier in terms of sufficient statistics that we can both use to understand how network structure in the macroeconomy matters, and that we later take to the data to study implications for the design of fiscal policy.

3.1. The Fiscal Multiplier in a Networked Economy

Our main results express the economy’s general equilibrium responses to fiscal shocks as a function of their partial equilibrium effect on goods demand. The partial equilibrium effect $\partial Y$ is the change in on final goods demand in response to a shock before interest rates or incomes have been allowed to adjust.

We begin by parameterizing aggregate demand. Recall that we can represent type $n \in N$’s Marshallian demand for good $j \in \mathcal{I}^t$ at time $t \in \{1, 2\}$ as $c_{nj}^t(y_{n}^{1}, \varrho, \tau_n)$, where $\varrho = (p^1, p^2, r^1)$, and $\tau_n = (\tau_n^1, \tau_n^2)$. Aggregate consumption demand $C_j^t$ is then given by:

$$C_j^t(\varrho, y^1, \tau) = \sum_{n \in N} \mu_n c_{nj}^t(\varrho, y_{n}^{1}, \tau_n).$$

\textsuperscript{10}This specification allows us to capture any demand-driven migration mechanism, for example: if there is a drop in demand in region A but not region B, and – in response – workers move from A to B, firms in B may marginally demand more workers of the types initially prevalent in A. This migration would be reflected in both the labor demands in region A and region B. Since the rationing function takes the full vector of labor demands across regions and returns a vector of labor supplies for worker types, a stable rationing function would still capture these dynamics. One set of models not accommodated are those in which amenities are endogenous to the shock and do not depend solely on labor demand.
To find the partial equilibrium effect of each type of shock, we compute the effect of a shock to government purchases or transfers before incomes or the interest rate adjusts:

$$\partial Y = G_{\theta_G} d\theta_G + C_\tau d\tau$$  \hspace{1cm} (8)

where $G_{\theta_G}$ and $C_\tau$ stack goods demands across the two time periods. These and all other partial derivatives throughout the analysis are evaluated at an equilibrium before the change in parameters.

In a rationing equilibrium, these partial equilibrium shocks propagate through the economy via two fundamental mechanisms. First, as firms ration additional labor demand to workers, households respond to increased income with greater spending on goods, generating an *income multiplier*. Second, as interest rates respond to changing output, households respond with different savings and consumption behavior, generating an *interest rate multiplier*. In deep recessions, the latter effect is likely to be weak, both because the consumption response to interest rates is small and because real interest rates may not be able to respond to output in the presence of a zero lower bound on nominal interest rates and sticky inflation expectations (Campbell and Mankiw, 1989; Kaplan, Violante, and Weidner, 2014; Vissing-Jorgensen, 2002). Away from this case, our results can still be interpreted as capturing the effect of fiscal policy in the absence of a countervailing response via monetary policy and/or inflation expectations. Therefore, for the rest of the analysis, we focus on the income multiplier; for results on the more general case, see Appendix A.1.

**Assumption 1.** At least one of the following statements is true:

1. The consumption and government responses to real interest rates sum to zero:

$$C^1_{r1} + G^1_{r1} = 0$$  \hspace{1cm} (9)

2. The central bank response of real interest rates to production is zero:

$$r^1_Q = 0$$  \hspace{1cm} (10)

Under Assumption 1, the income multiplier takes a particularly interpretable form. Below, we let $C^1_{y1}$—i.e. the derivative of aggregate consumption with respect to the vector of household incomes—be written as the product $C^1_{y1} \cdot m$ of a diagonal matrix $m$ of household MPCs (the column sums of $C^1_{y1}$) and a matrix $C^1_{y1}$ capturing the direction of the marginal spending. We moreover let $\hat{X}^1$ and $\hat{L}^1$ denote unit-production input-output and
labor demand matrices, respectively, in the first-period.\footnote{More formally, define the unit input demands for any firm \( i \) as those that solve the following program:
\[
(\hat{X}_i^t, \hat{L}_i^t) = \arg \min_{(X_i^t, L_i^t)} \text{ s.t. } p^t(X_i^t, L_i^t) \geq 1 \quad p^tX_i^t + L_i^t
\]
where \( p^t \) is the unique price vector consistent with firm optimization; we show that this price vector and these demands exist and are unique in Proposition 5 and Corollary 1 in the Appendix. The input-output matrix then stacks \( \hat{X}_i^t \) across firms.}

**Proposition 1.** For any small shock to fiscal policy inducing a partial equilibrium effect \( \partial Y \), there exists a selection from the equilibrium set\footnote{The fact that we write the statement as holding for a selection from the equilibrium set reflects the potential for multiple equilibria in the highly reduced-form model we have written. Despite the potential for multiplicity, we show that the equilibrium correspondence is upper hemi-continuous, implying one can nevertheless find a selection from the equilibrium set such that the given relationship holds. Of course, as our model nests many unique equilibrium models, in any such model the result can be stated without this qualification.} such that—under Assumption 1—the general equilibrium response of first period value added \( dY^1 \) is given by:

\[
dY^1 = \left( I - \overline{C}_{y^1} \bar{m} \overline{R}_{L^1} \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \right)^{-1} \partial Y^1 \tag{11}
\]

**Proof.** See Appendix A.1. \( \square \)

This is the key formula of the paper and can be understood as a generalization of the traditional Keynesian multiplier \((1 - MPC)^{-1}\) to the case of input-output networks, heterogeneous households, and arbitrary firm-household employment linkages. The term

\[
\left( I - \hat{X}^1 \right)^{-1}
\]

is the analog of the \( MPC \) in the traditional multiplier formula. Below each term, we have noted its dimensions, where recall \( N \) is the set of household types and \( \mathcal{I}^1 \) is the set of goods / firms in the first period. In this economy, following a demand shock to firms, the term \( \left( I - \hat{X}^1 \right)^{-1} \) maps changes in final demand to changes in production via the input-output network. Having pinned down the change in required production, \( \hat{L}^1 \) maps these to changes in firms’ demand for labor. Next, the rationing function \( \overline{R}_{L^1} \) maps these to changes in each household’s income. The MPC matrix \( \bar{m} \) maps these changes in income into changes in spending. Finally, the spending direction matrix \( \overline{C}_{y^1} \) maps changes in each household’s consumption spending to changes in aggregate consumption of each good. The final multiplier is the Leontief inverse of this object as this loop repeats \textit{ad infinitum}.
The crucial difference relative to the traditional Keynesian multiplier is that the structure of production, employment and consumption matters. First, it is important whether shocks load onto low or high MPC households, as studied by Patterson (2019). Moreover, the interaction between the input-output network and the directed consumption network matters: the multiplier is largest when it is not only partial equilibrium shocks but also higher order responses that load onto high MPC households, due to those households spending their marginal dollars at firms that hire high MPC workers or at firms that buy inputs from firms hiring high MPC workers, and so forth.\footnote{This same multiplier expression appears in the regional economics literature on social accounting matrices, dating back to Miyazawa (1976). To our knowledge, our result provides the first fully-microfounded justification of this formula, which receives widespread use in the regional economics literature and applied work to compute purchases multipliers (such as the BEA’s RIMS II system). The connection to the social accounting literature motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households. See Appendix C.3 for a formal description of this interpretation.}

In Appendix C.2, we provide comparative statics that demonstrate how the distribution of multipliers in the economy depends on the underlying structure of the economy. Specifically, we show that the multipliers are higher for any partial equilibrium shock when MPCs rise for all individuals or if the labor share rises, thus reallocating income from zero-MPC factors to high-MPC workers. Similarly, when shocks in the labor market are borne more heavily by high-MPC workers or when consumption is more directed toward goods produced by low-MPC households, the multiplier increases for all possible partial equilibrium shocks. Less obviously, we also show that when the IO matrix becomes more connected, meaning that firms use inputs from a more diverse set of industries, the distribution of multipliers shrink (i.e. the maximum multiplier falls and minimum multiplier rises). This arises because a more connected input-output matrix implies that shocks to any given industry affect a wider array of households, effectively distributing the shock to households with a more diverse set of MPCs.

Throughout the rest of the paper, in analogy to the assumption that the aggregate MPC is less than one in the simple Keynesian multiplier, we assume the moduli of $\bar{C}_y^1, mR_L^1, \hat{L}^1 \left( I - \bar{X}^1 \right)^{-1}$ and $R_L^1, \hat{L}^1 \left( I - \bar{X}^1 \right)^{-1} \bar{C}_y^1m$ are less than one, which guarantees that the fiscal multiplier is well-defined.\footnote{We later verify this assumption empirically. Also note that the modulus is less than one whenever all households have MPC less than one.} We will also always consider the equilibrium selection such that our multiplier formula applies.
3.2. Decomposing the Role of Heterogeneity

While the comparative statics help inform the importance of individual dimensions of heterogeneity, these many dimensions of heterogeneity also interact in potentially complicated ways to produce the multiplier in Proposition 1. In this section, we explain how these many dimensions can be understood through three key effects that lead to greater or lesser amplification relative the basic Keynesian case.

To begin, we simplify notation by renormalizing the units of all goods in each period so that all pre-shock, intratemporal prices are equal to one, i.e. \( p_t^i = 1 \). Given our focus on fiscal purchases and transfers shocks, which do not affect prices, this is without loss of generality.\(^{15}\)

Toward decomposing the role of heterogeneity, we now define the aggregate spending-to-income network

\[
\mathcal{G} = R^1_L \hat{L}^1 (I - \hat{X}^1)^{-1} C^1_y, \tag{13}
\]

as the map from an additional dollar of spending by one household to the vector of income changes in generates for each other household. Since every dollar spent eventually becomes income, every column of \( \mathcal{G} \) sums to one. Lastly, define

\[
\partial y^1 = R^1_L \hat{L}^1 (I - \hat{X}^1)^{-1} \partial Y^1 \tag{14}
\]

as the partial equilibrium incidence of a shock on labor income.

We now illustrate the mechanisms through which heterogeneity and network structure affects shock propagation through a series of examples, before characterizing them formally in Proposition 2. In each of the four examples below, there are two households: one with low MPC \( m_L = 0.1 \) and one with high MPC \( m_H = 0.5 \). What differs between the examples is the incidence \( \partial y^1 \) that a shock has onto the respective incomes of these households and the structure of their economic interactions through the spending-to-income network \( \mathcal{G} \).

Our first example illustrates a neutral case in which—despite the presence of heterogeneous households—the propagation of a partial equilibrium shock is “as if” the economy had a single household with MPC \( m = \frac{m_L + m_H}{2} \). In the example, the partial equilibrium shock has incidence \( \frac{1}{2} \) on each household, and each household divides its marginal spending equally between itself and the other household (see the top-left panel of Figure 1). Such a change in incomes could be generated by a government transfer of \( \frac{1}{2} \) to each household in the first period. As a result, the incidence of spending induced by the income earned in meeting the partial equilibrium demand shock is exactly \( m \) times the shock’s incidence for

\(^{15}\)Our results also accommodate preference shocks, see Appendix A; for results on supply shocks, see Appendix C.4
Fig. 1. Example 1: “Neutral” shock and spending-to-income network. Example 2: Shock directed toward high-MPC household (“incidence”). Example 3: Typical HH’s marginal spending directed toward HHs with higher than own MPC (“bias”). Example 4: Each HH directs marginal spending toward HHs with same MPC (“homophily”).

each household; similarly for spending induced by income earned in meeting this secondary demand, and so on. Thus, the multiplier is given by \( \frac{1}{1 - \frac{1}{2}} = 1.43 \).

In the second example, the structure of the economy is unchanged, but the partial equilibrium shock \( \hat{y}_1 \) is directed entirely to the high-MPC household (see the top-right panel of Figure 1). As a result of this differential incidence, the partial-equilibrium change in household income induces a greater increase in spending. However, since the high-MPC household’s divides its spending evenly between household types, subsequent “rounds” of spending still propagate at the baseline, Keynesian multiplier. In this case, the multiplier is given by \( 1 + \frac{\frac{1}{2} H}{1 - \frac{1}{2}} = 1.71 \), so shocks feature 65% more amplification than the baseline. Thus, a transfer solely to the high MPC household rather than a uniform transfer of the same size is much more effective.

In the third and fourth examples, we return to the neutral income incidence \( \hat{y}_1 = \left( \frac{1}{2}, \frac{1}{2} \right) \) of the first example and instead consider changes to the spending-to-income network \( \mathcal{G} \). In the third example, each household directs all of its marginal spending to the sector employing the high-MPC household (see the bottom-left panel of Figure 1). Unsurprisingly, this generates higher amplification, as household’s induced spending all propagates at a multiplier corresponding only the the higher-MPC households’ MPC. In particular the multiplier is given by \( 1 + \frac{\frac{1}{2} H}{1 - \frac{1}{2}} = 1.60 \), 43% larger than the neutral baseline.

In the final example, each household directs all of its marginal spending toward itself (see the bottom-right panel of Figure 1). In this case, each household’s share of the shock incidence propagates separately, at \( \frac{1}{1 - \text{MPC}} \) with that household’s MPC. Mathematically speak-
ing, since \( \frac{1}{1 - \text{MPC}} \) is convex in MPC, two isolated economies generate higher average amplification of \( \frac{1}{2} \left( \frac{1}{1 - \text{MPC}_L} + \frac{1}{1 - \text{MPC}_H} \right) = 1.56 \), which is 33% higher than the neutral integrated economy. Intuitively, since the high-MPC household spends more of its increase in income, it increases GDP more by directing its spending toward its own, high, MPC than the low-MPC household decreases GDP by directing its spending toward its own, low, MPC.

The second, third, and fourth examples each illustrate three distinct channels by which the characteristics of and connections between heterogeneous households affect amplification. First, one must account for the incidence of a shock onto households of higher or lower MPC. Second, the multiplier is higher when households’ marginal spending is biased toward households with higher MPCs than their own. Third, homophily in the spending network – in the form of correlation between household MPCs and MPCs of households on which they spend – also generates amplification. Proposition 2 establishes that these three channels exactly capture the deviations of shocks amplification away from the Keynesian baseline, to second order in MPCs.\(^{16}\)

**Proposition 2.** The total change in first-period aggregate value added due to a demand shock with unit-magnitude labor income incidence \( \hat{y}_1 \) can be approximated as:

\[
\mathbb{1}^T dY^1 = \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} \left( 1 + \mathbb{E}_{\hat{y}_1}[m_n] - \mathbb{E}_{y^*}[m_n] \right) \text{Incidence effect} \\
+ \mathbb{E}_{\hat{y}_1}[m_n] \left( \mathbb{E}_{\hat{y}_1}[m_n^{\text{next}}] - \mathbb{E}_{y^*}[m_n] \right) + \text{Cov}_{\hat{y}_1}[m_n, m_n^{\text{next}}] \text{Biased spending direction effect} \\
+ O^3(|m|) \text{Homophily effect}
\]

where \( y^* \) is any reference income weighting of unit-magnitude and \( m_n^{\text{next}} = (m^T \mathcal{G})_n \) is the average MPC of households who receive as income i’s marginal dollar of spending.\(^{17}\)

**Proof.** See Appendix A.2.

The above proposition holds for all reference partial equilibrium changes in labor earnings \( y^* \) of unit size, but naturally the choice of this \( y^* \) affects the accuracy of the approximation. Each of these effects are relative, meaning that they capture the amplification relative to some reference shock to earnings. In our later empirical analysis, we take \( y^* \) as the change in income induced by a GDP proportional demand shock. In this case, we show that the error term accounts for less than 0.3% of the multiplier, so that this approximation is very tight.

\(^{16}\)We provide an exact decomposition in terms of Bonacich centralities of \( \mathcal{G} \) (Appendix A.2).

\(^{17}\)For any \( N \)-length vectors \( z \) and \( x \), \( \mathbb{E}[x_n] \) denotes the average of \( x_n \) across household types, weighted by \( z_n \); similarly for Cov.

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In Appendix C.5 we discuss how Proposition 2 applies to several important benchmark economies, highlighting cases in which the various alterations to the Keynesian multiplier are zero. One important benchmark is a “homothetic economy” where both consumption and labor rationing functions are homothetic. In this case, a GDP-proportional shock has no bias effect, but heterogeneity in household consumption baskets and sectoral employment can still generate network effects through incidence and homophily. A truly “neutral” case occurs when all firms in the economy employ workers at the margin who have the same average MPC as one another. In this case, the bias and homophily effects are zero and the income multiplier for any shock is simply the Keynesian multiplier evaluated at this average MPC. Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm’s marginal employees have the population average MPC.

Clearly, the conditions required to eliminate the network adjustments are knife-edge. In general, the distribution of shocks does affect aggregate responses, and the IO and directed consumption networks affect both the size and direction of these responses. Moreover, one might reasonably expect that each effect formalized by Proposition 2 is empirically relevant based on existing empirical research. Indeed there is wide variation in household MPC and spending and transfer policies may be disproportionately directed toward certain sectors or households (Lewis, Melcangi, and Pilossoph, 2019; Cox et al., 2019). Meanwhile, Patterson (2019) documents that higher-MPC workers are more exposed to aggregate fluctuation, suggesting an aggregate bias effect in the spending-to-income network. Additionally, Hubner (2019) documents that higher-income households tend to consume more labor intensive goods, while at the same time a growing regional literature emphasizes that much of consumption is done locally while regions are heterogenous in both income and wealth levels—both suggesting a sizable role for the (anti-)homophily effect. In the following sections we assess each channel empirically, finding—contrary to the observations above—that only the incidence effect is qualitatively significant.

4. Data and Estimation Methodology

Using our framework, we have so far derived a simple sufficient statistics expression for generalized income multiplier for fiscal shocks. We also demonstrated theoretically how rich household, industry and regional heterogeneity can interact to potentially amplify shocks and shape optimal policy. We now take our multiplier to the data to quantify the gains from targeting fiscal stimulus and understand how a planner should target such stimulus in practice. To do this, we directly estimate the sufficient statistics that comprise the multiplier.
using a variety of datasets. In this section, we describe both the datasets we use to estimate these sufficient statistics and the methodology we employ to calculate the components of the multiplier.

First, recall from Proposition 1 that in the case of zero interest rate responsiveness the response of GDP to the partial equilibrium response of demand $\tilde{Y}$ to any primitive shock is given by:

$$dY = \left( I - \bar{C}y \cdot m R L \cdot \hat{L} (I - \hat{X})^{-1} \right)^{-1} \hat{Y}$$  \hspace{1cm} (16)

To estimate the multiplier, we therefore need estimates of three key objects: the regional input-output matrix $\hat{X}$ describing the input use requirements of every region-industry pair, the rationing matrix $R L \cdot \hat{L}$ describing how much each demographic-region pair’s income changes in response to a one dollar change in revenue of each region-industry pair, and the directed MPC matrix $\bar{C}y \cdot m$ describing how much each demographic-region pair consumes from each region-industry pair when they receive a one dollar income shock.

In going to the data, we must also account for three empirically-relevant factors that were absent from our baseline model – capital, profit, and foreign income. At a high level, our strategy is to (1) model capital as an input, (2) model profits by assuming constant markups, as in Appendix D.1, and (3) model foreign factors as a type of “labor” with zero MPC, reflecting that payments leaving the economy do not re-enter through income effects.

The following subsections describe in detail how we estimate each of the three components of our multiplier: the input-output, rationing, and directed consumption matrices. Table 1 shows which datasets are used in the estimation of each object. We restrict our attention to the United States in 2012, which is the most recent year for which we have several of the key datasets.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Input-Output</th>
<th>Rationing</th>
<th>Directed MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Community Survey (ACS)</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>BEA Make and Use Tables (IO)</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>BEA Regional Accounts (RA)</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Consumer Expenditure Survey (CEX)</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Commodity Flow Survey (CFS)</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Consumer Price Index (CPI)</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Internal Revenue Service Statistics of Income (IRS SOI)</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Panel Study of Income Dynamics (PSID)</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Summary of Datasets Used in the Estimation of Our Sufficient Statistics.
4.1. The Regional Input-Output Matrix

The regional input-output matrix $\hat{X}^1$ is an $(S \times I) \times (S \times I)$ matrix where $I$ is the number of industries and $S$ is the number of regions. The $(r_i, s_j)$ component of this matrix corresponds to the amount of sector $i$ in region $r$’s good required to produce a single unit of sector $j$ in region $s$’s good. To estimate this object, we must first take a stand on the level of granularity at which to model sectors and regions. Guided by the level at which input-output data are available, we largely follow the BEA’s collapsed input-output sector classification, leaving us with 55 sectors which loosely correspond to the 3-digit NAICS classification. Similarly, to take full advantage of the CFS microdata on interstate trade, we set regions at the level of the state (including Washington D.C.), leaving us with 51 regions. This leaves us with 2805 sector-regions.

We construct the regional input-output matrix in three steps. First, following others in the literature, we use data from the 2012 BEA make, use, and imports tables to construct the domestic, national input-output matrix, which measures the dollar value of products from industry $j$ that are used by industry $i$. For commodities produced by multiple industries, we assume that all users of such commodities source them from the various producing industries in the same proportions. We also make an adjustment to account for linkages across industries in capital investment. This is necessary as the standard use table accounts only for changes in intermediate goods usage. To impute each industry’s expenditure on investment goods, we assume that all industries invest the same fraction of their gross operating surplus (available in the use table) in capital. To compute the direction of this investment toward different industries, we assume that each firm demands the same investment good and compute its industrial composition with the same procedure – using the use, make, and import tables – as we use for inputs. We then add this investment correction to the previously constructed input-output matrix.

Second, we use the 2012 public-use microdata from the Commodity Flows Survey (CFS) to construct a matrix describing how much each state imports from all other states. The CFS is a survey conducted by the US Census Bureau and includes data on 4,547,661 shipments from approximately 60,000 establishments. The data records the location of the shipping establishment, the commodity being shipped, the value of the shipped commodity, and the location to which the commodity is being shipped. The public use microdata file modifies this underlying data by introducing noise and top-coding extremely large shipments. Using this information, we calculate the total value of shipments between each pair of states for each tradable industry using the mapping between commodities and industries outlined in
the BEA’s make table.\textsuperscript{18} For all nontradable industries, we assume that the commodity is sourced entirely within the state.

Finally, we construct the regional input-output matrix by combining the national industry-level input-output with state-by-state trade flows. Specifically, the amount of industry $i$ in state $r$ used by industry $j$ in state $s$ is the product of the share of industry $j$’s inputs that come from industry $i$ and the fraction of sector $i$ goods flowing to $s$ from $r$ (out of all origin states). This yields a matrix describing, for each industry-region pair, how much of each other industry-region pair’s production is used to produce a single unit of output.

\subsection*{4.2. The Directed MPC Matrix}

The directed MPC matrix $\mathbf{C}^1_y \mathbf{m}$ corresponds to an $(S \times I) \times (S \times N)$ matrix where $N$ is the number of demographic groups. The $(ri, sn)$ component of this matrix maps how a one dollar change in demographic $n$ living in region $s$’s income changes that household’s consumption of good $i$ in region $r$. Again, this first requires us to take a stand on the level of granularity at which to model demographic groups. Guided by the level at which precise estimation of MPCs is possible in the PSID, we set the number of demographic groups at 82, comprising 80 baseline groups (five initial income groups, four age groups, two gender groups, two race groups) and two dummy groups for the owners of capital and foreigners.\textsuperscript{19}

We construct the directed MPC matrix in three steps. First, we construct MPCs for total consumption expenditure for each of our 80 demographic groups using the PSID, CPI and CEX following the methodology in Patterson (2019). Specifically, we follow the procedure of Gruber (1997), using the panel structure of the PSID to estimate the equation:

\begin{equation}
\Delta C_{ht} = \sum_x (\beta_x \Delta E_{ht} \times x_{ht} + \alpha_x \times x_{ht}) + \delta_{s(h)t} + \varepsilon_{ht}
\end{equation}

where $C_{ht}$ is household $h$’s consumption at time $t$, $E_{ht}$ is household $h$’s labor earnings at time $t$, $x_{ht}$ is a demographic characteristic of the individual, and $\delta_{s(i)t}$ is a state by time fixed effect. Estimating Equation 17 we then obtain the following estimate of the MPC for household $h$ at time $t$:

\begin{equation}
\overline{\text{MPC}}_{ht} = \sum_x \hat{\beta}_x x_{ht}
\end{equation}

However, there are two challenges in performing this estimation. The first issues arises as there are a wide range of factors that could simultaneously move income and consumption.

\textsuperscript{18}Caliendo et al. (2018) use a similar methodology to construct their regional input-output matrix.

\textsuperscript{19}Our five income groups correspond to: less than $22,000$, $22,000$-$35,000$, $35,000$-$48,000$, $48,000$-$65,000$ and more than $65,000$. Our four age groups correspond to those 25-35, 36-45, 46-55 and 56-62. Our race groups are black and non-black. Our gender groups are men and women.
To address this, we instrument for changes in labor market earning using transitions into unemployment. This is desirable as such shocks are both large and persistent. Unemployment shocks therefore capture that variation most important to understanding recessions. Indeed, if recessions can be seen as shocks of the same persistence as unemployment, then this MPC is exactly the right object to capture shock propagation in the manner suggested by the model.\(^{20}\)

The second issue stems from measurement in the PSID: for most of the PSID sample, only expenditure on food consumption is measured. Using only this measure is problematic as food is a necessity and expenditure on food is likely to be distorted by the provision of food stamps (Hastings and Shapiro, 2018). To overcome this issue, we use overlapping information in the PSID and CEX to impute a measure of total consumption expenditure, following the methodology of Blundell, Pistaferri, and Preston (2008) and Guvenen and Smith (2014). Concretely, we use the CEX to estimate demand for food expenditure as a function of durable consumption, non-durable consumption, demographic variables and relative prices from the CPI. Under the assumption of monotone food expenditure, this function can be inverted to predict total consumption as a function of food expenditure and demographics in the PSID. This procedure generates substantial heterogeneity across households in estimated MPCs (see Figure A1 in Appendix G).

Next, we estimate the consumption basket shares in each of our 55 industries for each of our 80 demographic groups using the CEX and CPI. We first deflate consumption over the 54 measured categories using the CPI and then compute the average consumption basket share of each demographic group. Using a concordance between NIPA goods and our industry classifications, we then map consumption at the household level in each category to the 55 industries used in our analysis.

We use these consumption basket shares and our estimated MPCs to construct an estimate of the directed MPC for each of the 80 demographic groups out of each of the 55 industries. We do this by assuming linear Engel curves of households for each category of consumption. Formally, we estimate the directed MPC of household \(h\) at time \(t\) as:

\[
\overline{\text{MPC}}_{n(ht)i} = \alpha_{n(ht)i}\overline{\text{MPC}}_{n(ht)}
\]

where \(n(ht)\) is the demographic group of household \(h\) at time \(t\) – which we from now on suppress when clear from context – and \(\alpha_{n(ht)i}\) is the demographic-specific consumption change in categories.

\(^{20}\)While the MPC out of an unemployment shock is relevant for the GE amplification of shocks, it is potentially not the right MPC for determining the response of consumption to targeted transfers. We return to this in Section 5, but note here that the MPCs estimated here are close in magnitude and have similar cross-demographic patterns as those estimated using tax rebates or lottery winnings (Parker, Souleles, Johnson, and McClelland, 2013; Fagereng, Holm, and Natvik, 2019).
basket weight of good \(i\). Naturally, the imposition of linear Engel curves may be overly restrictive. However, our estimates always lie in the 95\% confidence interval of estimates of good-specific MPCs from the PSID in the years in which this is possible (see Figure A2 in Appendix G), suggesting that we are capturing reasonable dimensions of heterogeneity with this assumption.

Finally, we use our estimated state-state gross flows in goods to arrive at the regionally-directed MPCs. Formally, for tradable goods, we assume that all households in a state consume from all other states in proportion to the fractions of imports of that good that originate from those states:

\[
\overline{MPC}_{risn} = \lambda_{irs} \overline{MPC}_{ni}
\]

(20)

where \(\lambda_{rs}\) is the fraction of shipments of good \(i\) from state \(s\) to state \(r\) as a function of the total shipments of good \(i\) to state \(r\), as we earlier computed to construct the regional input-output matrix.\(^{21}\) We assume all nontradable goods are consumed within the state.

The procedure above provides the directed MPC entries for the 80 demographic groups. It remains to estimate the directed MPCs for capitalists and foreigners. For foreigners, we simply set all entries to zero. This coincides with the assumption that, of all foreign recipients of income that leaves the US, none spend this income in the US or indirectly cause other spending in the US. For capitalists, we take the MPC out of stock market wealth as estimated by Chodorow-Reich, Nenov, and Simsek (2019) at 0.028. We then allocate this in the direction of the aggregate consumption basket as reported in the BEA use table.

Finally, we assume that the marginal consumption response out of first-period government transfers (i.e. negative taxes) is the same as that out of first period income earned through labor supply. That is,

\[
C_1^{\tau_1} = -C_1^{y_1}m.
\]

(21)

Theoretically, this follows if consumption and labor are additively separable in household utility, as in Section 6. Empirically, the documented cross-sectional patterns in MPCs in response to tax transfers are similar to those we uncover using employment shocks, suggesting that this assumption is not driving the patterns we uncover below (See Parker et al. (2013); Fagereng et al. (2019)).\(^{22}\)

\(^{21}\)This potentially sources too much consumption from outside the state given that the CFS comprises both consumption goods and intermediate goods flows. In section 5, we explore the robustness of this modelling assumption for how consumption is sourced by considering a model with total consumption autarky where all consumption is sourced within the state. This has a very small impact on the results.

\(^{22}\)Our framework is flexible enough that it would be easy to perform our empirical analysis with a different calibration of \(C_1^{\tau_1}\). Since the MPC estimates out of tax rebates are noisier than those using unemployment, we maintain this assumption in the current analysis.
4.3. The Rationing Matrix

The rationing matrix $\mathbf{R}^1 \mathbf{L}^1$ corresponds to an $(S \times N) \times (S \times I)$ matrix where $N$ is the number of demographic groups. The $(rn, si)$ component of this matrix maps how a one dollar change in the production of good $i$ in region $s$ translates to a change in labor income for demographic $n$ in region $r$.

We construct the rationing matrix in three steps. We first use the ACS to compute, within each state-industry pair, the total labor earnings by each demographic group in 2012. We also use state-level data from the BEA on compensation and gross output by industry to compute labor shares of value added for each state-industry pair.

Second, we use these two components, along with the estimated demographic group MPCs, to construct the the labor rationing entries for workers. Concretely, we employ the following formula:

$$
\left( \mathbf{R}^1 \mathbf{L}^1 \right)_{rnsi} = \mathbb{I}[r = s] \frac{y_{inr}}{\sum_n y_{inr}} \alpha_{ir} \beta_i \left( 1 + \xi \left( MPC_n - \overline{MPC}_{ir} \right) \right)
$$

where $y_{inr}$ is total earnings of demographic $n$ in industry $i$ in region $r$, $\alpha_{ir}$ is state-by-industry labor share of value added, $\beta_i$ is the national value added to gross output ratio in industry $i$, $\xi$ is the correlation between MPCs and earnings elasticities, and $\overline{MPC}_{ir}$ is the earnings-weighted MPC of all workers in industry $i$ in region $r$. The indicator function imposes the condition that all labor earnings are received within the state where production occurs. This is the unique functional form that both preserves a constant correlation between MPC and earnings elasticity, of which there is strong evidence from Patterson (2019) and preserves total income received across all demographic groups in each industry-region pair. We set $\xi = 1.332$, the correlation of MPC with earnings elasticity to aggregate shocks measured in Patterson (2019). While our model can in principle incorporate regional migration in response to shocks, we – by assuming that employment at each firm only depends on its own labor demand – only partially allow for this possibility. In particular, our calibration rules out the possibility that the share of labor each firm rations from each demographic group may depend on changes in the group’s share of the population due to migration.

Finally, it remains to allocate those factor payments that are not received by labor. These take two forms: payments made to the domestic owners of capital and payments made to foreign factors. We compute directly payments made to domestic owners of capital via the following procedure. We first compute profits in each region-industry pair. To do this, we compute the domestic profit share of production from the BEA use table and add this to the residual value added in each state-industry pair that is not paid to labor. We then allocate

\[23\] See Patterson (2019) for more details and discussion.
these profits to the capitalist demographic group in each state according to that state’s share of dividend income in the IRS SOI data. Finally, we compute payments made to foreigners as the residual of payments made to intermediate producers, payments made to labor and payments made to capitalists.\textsuperscript{24}

5. Empirical Exploration of Fiscal Multipliers

In this section, we study the propagation of fiscal shocks in our calibrated economy, exploring both changes in aggregate GDP and demand spillovers across regions. We begin by quantifying how the variation in the fiscal multiplier for government purchases or transfers – i.e. the total change in income\textsuperscript{25} per unit of fiscal spending – depends on how that shock is targeted. We demonstrate that these differences stem almost exclusively from differences in the initial incidence of shocks on households with different MPCs rather than from variation in which goods these households consume, and we accordingly study how various features of the economy determine the incidence of shocks. Finally, we quantify the extent of geographic spillovers through the income multiplier and contextualize our findings in light of recent empirical estimates.

5.1. Extent of Heterogeneity in Multipliers

We estimate that the response of GDP to a demand shock which is GDP-proportional across industries and regions, or \textit{aggregate purchases multiplier}, is equal to 1.30, a number consistent with the large literature (Ramey, 2011; Chodorow-Reich, 2019). However, the left panel of Figure 2 – which shows the effect on GDP of spending a dollar in a given industry within a specified state – documents wide dispersion depending on how a shock is targeted, with the extent of amplification beyond the original purchase varying by a factor of six. Indeed, our range of multipliers, from about 1.1 to 1.6, provides one rationalization for the variation in purchases multipliers estimated in the literature. Transfers multipliers, i.e. the effect on aggregate income of transferring one dollar to a household of a given demographic within a specified state, vary even more widely. The right panel of Figure 2 shows that the effect on aggregate income of transferring a dollar to a household ranges from slightly below one for some households (some types have negative MPCs) to nearly three dollars for others.

\textsuperscript{24}In a small fraction of cases, this leads to a \textit{negative} foreign share of revenues, which is unrealistic. To avoid this, we could alternatively reduce the profit share of revenue in region-industry pairs with high labor shares. Insofar as we use similarly small MPCs for foreigners and capitalists, this alternative calibration would generate similar quantitative results.

\textsuperscript{25}For a government purchases shock, this is the same as the change in GDP.
Much of the heterogeneity in multipliers remain when targeting is constrained to be more granular: government purchases multipliers differ by a factor of more than three across industries and a factor of 1.5 across states (see Figure A17 in Appendix G); transfers multipliers differ by a factor of 1.3 across states and by nearly as much across demographic groups as across demographic-region pairs (See Figure A18 in Appendix G).

5.2. Sources of Heterogeneity in Multipliers

Recall from Proposition 2 in Section 3.2 that – for any fiscal shock – three adjustments to the basic Keynesian multiplier capture the effect of heterogeneity in household characteristics and interconnections on the total change in GDP. In particular, the dispersion in fiscal multipliers from Figure 2 could derive from differences in 1) the incidence effect, wherein shocks to some markets load more heavily on agents with higher MPCs, 2) the bias effect, wherein shocks in some markets load onto households who direct their spending to high-MPC households, or 3) the homophily effect, wherein shocks in some markets load onto households who direct their spending to similar-MPC households. However, we find that — as an empirical fact — all of the heterogeneity across groups in Figure 2 is driven by the differential direct incidence of those shocks onto agents with different MPCs.

5.2.1. Importance of Initial Incidence

To understand why only the incidence effect is empirically large, recall Proposition 2. In order for the bias and homophily terms to be large, there must be significant heterogeneity across households in basket-weighted MPCs $m_{n}^{\text{ext}}$ – that is, in the average MPC of the work-
ers ultimately employed in producing \( n \)'s marginal unit of consumption – and these basket weighted MPCs must differ from the benchmark \( \mathbb{E}_{y^*}[m_n] \). Indeed, if \( m_{n}^{\text{next}} \) is homogeneous and \( \mathbb{E}_{\phi^I}[m_{n}^{\text{next}}] = \mathbb{E}_{y^*}[m_n] \), then both the bias and homophily terms are zero as all households effectively direct their consumption in the same way for the purposes of amplification.

The left panel of Figure 3 documents that in the data, there is minimal heterogeneity in basket-weighted MPCs, shown by the very shallow slope between basket-weighted MPCs (y-axis) and household MPCs (x-axis). As a result, the homophily effects are very close to zero. Moreover, the scatterplot demonstrates that basket-weighted MPCs all lie very close to the benchmark average MPC (\( \mathbb{E}_{y^*}[m_n] \)). Consequently, bias effects are also very close to zero. Indeed, for any possible shock, the incidence term accounts for more than 99 percent of the multiplier.\(^{26}\) To drive this point home, the orange line in the right panel of Figure 3 shows multipliers from a counterfactual model without heterogeneous consumption in which the bias and homophily effects are identically zero. As one can see, there is effectively no difference in the full distribution of multipliers when we impose this condition, demonstrating that it plays no role in shaping the baseline estimates.\(^{27}\)

The lack of bias and homophily effects appears to be a real feature of the data, rather than a failure of our estimation approach to capture them. While the bias and homophily terms each operate to second order in the average MPC – which constrains them to be modest in size – it is easy to see from the examples in Section 3.2 that the combination of these terms can, in principle, be quantitatively large. Indeed, our estimates of consumption basket shares in the CEX do display substantial variation across households (see Figure A9 in Appendix G), allowing for the possibility of large network effects. The absence of bias and homophily effects, then, stems from two empirical observations. First, high MPC households disproportionately consume goods produced by low-labor-share industries (see Figure A5 in Appendix G), directing more spending toward capital, the owners of which have low MPCs.\(^{28}\) Second, our estimates feature substantial within-region non-tradeables demand, with around

\(^{26}\)Concretely, we construct any feasible \( \delta y^I \) via a linear combination of demand shocks to each sector-region pair. We then compute the bias and homophily effects from each of these shocks and plot the full distribution of bias and homophily terms (see Figure A4 in Appendix G, respectively). Across the full distribution of shocks, the contributions of the bias and homophily terms range between zero to four tenths of a percent increase in the multiplier – they are empirically negligible for all feasible demand shocks. We also compute the full distribution of error terms arising from the approximation in our decomposition result (the right panel of Figure A4 in Appendix G) and find that they are uniformly an order of magnitude smaller than the bias and homophily terms. Our approximation is therefore very tight for any feasible shock.

\(^{27}\)In Figure A8 of Appendix G we show a scatter plot of the multipliers from these two models. The correlation in multipliers across the two models is nearly perfect.

\(^{28}\)Conditional on reaching labor, the average MPC of workers producing consumption baskets is very slightly increasing across the MPC distribution (see Figure A5 in Appendix G), so labor share differences account for the bulk of differences in basket-weighted MPCs stemming from heterogeneous consumption baskets. This finding is also consistent with the empirical patterns in Hubmer (2019).
Fig. 3. The left panel shows a scatter of MPCs $m_n$ against basket-weighted MPCs $m_n^{\text{next}}$. The dashed line gives the average MPC $E_{\text{w}}[m_n]$ for $y^s$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP. The right panel shows the change in GDP for each industry-region pair according to a one dollar demand shock in each pair, sorted by the magnitude of the effect. The full model is the baseline and plotted in blue. No directed MPC assumes that all households direct their consumption in proportion to aggregate consumption. No IO assumes that there is no use of intermediate goods.

a third of total labor demand remaining within the state from which consumption originates (see Figure A10 in Appendix G). Moreover, there is spatial heterogeneity in MPCs, with income-weighted MPCs differing by a factor of 1.5 across states (See Figure A15 in Appendix G). Together, these regional forces generate a modest positive homophily effect whereby higher (lower) MPC workers direct their consumption more toward local labor which similarly features high (low) MPC. However, these labor share and local demand effects are both fairly weak, and they run in opposite directions. When combined, they partially cancel, so that all types spend on goods baskets produced by households of very close to the average MPC.

The empirical irrelevance of the bias and homophily effects is a robust feature across a wide range of alternative calibrations. Concretely, Appendix Figure A7 shows the size of the these effects in version of the model with and without input-output linkages, regional trade, and heterogeneous income rationing by MPC and location. In all cases, the bias and homophily effects constitute contribute less than 0.01 to the multiplier of a GDP-proportional shock. So while we have demonstrated the theoretical possibility of large bias and homophily effects (see Section 3.2), it is unlikely that they are empirically relevant in advanced economies.

5.2.2. Determinants of Initial Incidence

Since the heterogeneity in shock amplification in Figure 2 does not stem from higher order network effects, it must instead come from differences in the incidence of different shocks
onto the MPCs of households. For transfers, the initial incidence is immediately apparent and is driven solely by heterogeneity in MPCs in the population. However, for government purchases, three distinct factors widen the distribution of multipliers. First, differences in the demographic composition of the workforce across sectors and regions causes large differences in the average MPCs of workers across firms and regions. Second, differences in the share of labor that each sector directly employs cause large differences in the MPC of the ultimate recipients of factor income. In particular, firms employing lots of capital but little labor pass most factor payments on to the owners of capital who have very low MPC and therefore feature small purchases multipliers. This is shown in Figure A11 in Appendix G that plots the labor share of each industry-state pair against its purchases multiplier: there is substantial heterogeneity in labor use and low labor use is associated with a small purchases multiplier. Third, differences across firms in the covariance of worker MPC and exposure to changes in firm revenue generate additional widening of the distribution of multipliers. This is shown in Figures A13 and A14 in Appendix G where we compare the baseline model – which features greater rationing more to agents with higher MPCs – to a model with rationing to agents uniformly by income; there we observe both an upward shift in the distribution of purchases multipliers as well as an increase in range.

Conversely, input-output linkages serve an important role in narrowing the heterogeneity induced by these differences. This can be seen in the right panel of Figure 3, where the green line corresponds to the model without input-output linkages, which features a much more dispersed distribution of multipliers. The role of input-output linkages in reducing dispersion is intuitive. In the absence of inputs, when the firm directly employing the highest-MPC factors gets an additional dollar of revenue, it spends it all on those high-MPC factors. With inputs, this same firm spends a fraction of its revenue on goods produced by other firms, who in turn direct that money to their (by construction) less-than-highest-MPC factors – effectively diluting the MPC of the initial firm. This dilution effect attenuates the heterogeneity in industry multipliers. This same phenomenon explains why the distribution of transfers multipliers in Figure 2 is more dispersed that the distribution of purchases

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29 See Figure A16 in Appendix G for a scatter plot of the multipliers across both the full model and that without input-output linkages.

30 Our finding that IO linkages reduce heterogeneity in purchases multipliers is distinct from an existing literature that emphasizes the role of IO networks in amplifying economic shocks (Acemoglu et al., 2012; Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2016; Baqicee, 2018; Elliot, Golub, and Leduc, 2020). First and foremost, our finding is not that IO links attenuate amplification on aggregate, but rather than they reduce the dispersion in amplification across industries. In this sense, we simply have a different focus. Moreover, the key reasons that IO links generate aggregate amplification in the literature—namely, that supply shocks are more powerful when the input share of production is large (a la Hulten) and that supply and demand shocks can cause cascades of firm defaults when production has a fixed cost—play no role in our setting, as we focus on demand shocks and assume production is CRS.
multipliers: A transfer to the highest- or lowest-MPC household reaches it directly, rather than being spread across households with more moderate MPCs.

5.3. Regional Demand Spillovers

Finally, we turn our focus away from the effects of fiscal shocks on aggregate GDP and instead consider how income multipliers may propagate across state lines. Such spillovers are of direct policy relevance, as a planner may want to stimulate demand in a particular, depressed, area without “overheating” the economies of other nearby regions. They are also of interest to a recent empirical literature that uses quasi-random cross-regional variation in fiscal spending to estimate local fiscal multipliers (Nakamura and Steinsson, 2014; Chodorow-Reich, 2019). Regional demand spillovers complicate the relationship between these local estimates and the national multiplier, as most research designs only recover the effect of spending on \(i\) in GDP in \(i\) relative to GDP in \(j\) – which is not a suitable control group if the spending indirectly boosts \(j\)’s GDP.

The regional interlinkages embedded in our model allow us to provide an estimate for the magnitude of these cross-state spillovers. We quantify these spillovers within our model by considering a unit of government purchases in each state, which we assume is distributed across industries within the state in proportion those industries’ shares of GDP within the state. Averaging across states, we find that aggregate GDP increases by 1.3 units in response to 1 unit of additional spending. Of this 30 percent amplification, about 16 percentage points come within the state that received the additional government purchases, while 14 percentage points come from spillovers to other states – firms and households in the shocked state demand more goods and some of those are sourced from other states. The spillover to any given state is small and only about 2 percent as large as the effects within the shocked state. However, each state contributes to the total effect, and overall, the spillovers contribute meaningfully to the overall effect of the shock.

These estimates are in line with some recent empirical evidence estimating the magnitude of these spillovers directly. Specifically, Auerbach et al. (2020) use detailed geographic information on local defense spending and find that large positive spillovers across geographies, suggesting the importance of positive demand spillovers through input-output networks and directed MPCs. They also find that the spillovers are decreasing in the distance between cities. Our results are consistent with this, as our estimated spillovers are largest for the

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31We consider the problem of such a planner in Section 6.3.
32Of course, the shock itself all remains in the shocked state, so that the total change in GDP within the shocked state is 1.16, on average.
geographically closer states. These estimates suggest that demand spillovers across states are empirically important when evaluating the total effect of localized fiscal spending.

6. Implications for Design of Fiscal Policy

So far, we have studied how fiscal shocks propagate to affect GDP and income in general equilibrium. We found that fiscal multipliers vary wildly depending on where spending is targeted, and that this variation is driven entirely by the heterogeneous incidence of the shocks on workers with different MPCs.

In this section, we explore the implications of these results for the design of fiscal policy. We begin by characterizing the motives of the social planner and clarify how the estimated multipliers from Section 5 directly inform the design of policy. We break our subsequent analysis into two parts: First, we consider a setting with widespread underemployment. Here, the social planner seeks solely to maximize aggregate income and – due to empirically small role of directed spending in shaping multipliers – can achieve this objective with simple MPC targeting. Second, we consider the more general case when underemployment is more severe in some regions than others. We illustrate how geographically heterogeneous severity shapes policy in an application to the Great Recession.

6.1. Welfare and Fiscal Policy

In Section 2, we did not specify household utility functions, instead simply working with Marshallian demands. In order to compute the welfare effects of fiscal policy, we assume each household $n$ has an additively-separable utility function over consumption and labor supply. At time $t = 1$, households of type $n$ choose consumption but not labor supply, and face a borrowing constraint in the form of a minimum level $s_n$ of savings. At time $t = 2$,

In Appendix F, we more formally explore the extent to which our model predicts the cross-state spillovers in response to several identified demand shocks. While the estimates are under-powered, we find evidence suggesting that our structural estimates are qualitatively consistent with cross-state spillovers in response Chinese-import shocks as in Autor, Dorn, and Hanson (2013).

Separability between consumption and labor supply ensure that MPCs out of labor income and transfers are the same.
households are unconstrained. The household’s problem is therefore:

\[
W_n(t_n^1, r_n) = \max_{c_t^1, l_t^1} \sum_{t=1}^{T-1} \beta_{tn}^{t-1} \left[ u_n^t(c_t^1) - v_n^t(l_t^1) \right] \\
\text{s.t. } l_t^{T-1} + \frac{1}{1+r} + \frac{r}{1+r} l_t^0 \leq l_t^1 + \frac{1}{1+r} l_t^{T-1} \\
\bar{l}_t - l_t^{T-1} \geq s_n^1 \\
\bar{l}_t = l_t^1
\] (23)

We assume that the planner is utilitarian, placing some welfare weight \( \lambda_n \) on households of type \( n \). Social welfare is then given by:

\[
W(G, \tau) = \sum_{n \in N} \lambda_n \mu_n W_n(l_n^1(G, \tau), \tau_n)
\] (24)

where \( l_n^1(G, \tau) \) denotes household labor income consistent with rationing equilibrium with fiscal policy given by \( (G, \tau) \).

Our goal is to understand how changes in fiscal policy affect social welfare. To this end, we define \( \kappa_n^t \equiv u_n^t(c_t^1) \) to be \( n \)’s marginal value of additional expenditure in period \( t \) (recall prices are normalized to one), and let \( \tilde{\lambda}_n \equiv \lambda_n \kappa_n^1 \) denote the modified welfare weights that represent the planner’s marginal value of transferring a dollar to \( n \). We denote by \( \Delta_n \) each household’s labor wedge – i.e. \( v_n^1 = \kappa_n^1 (1 + \Delta_n) \) – which measures how far the household is off their intratemporal labor supply condition.

Proposition 3 characterizes the welfare impact of a change in fiscal policy in terms of these welfare statistics. In particular, the Proposition fixes second-period policy – as well as total spending in the first period – and considers how first period policies should be targeted at a given level of spending.

**Proposition 3.** The change in welfare \( dW \) due to a small change in taxes and government purchases in the first period—at a constant interest rate—can be expressed as:

\[
dW = \sum_{n \in N} \mu_n \tilde{\lambda}_n \begin{bmatrix}
-\Delta_n d l_n^1 \\
Address under-emp. Make transfers
\end{bmatrix}
\] (25)

**Proof.** See Appendix A.3.

Equation 25 clarifies that, in general, the social planner hopes to do more than simply...

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35 We suppress dependence on the interest rate as we assume it to be fixed throughout.

36 For a more formal definition, see Appendix A.3.
maximize aggregate GDP. First, she seeks to alleviate involuntary un(der)employment by changing the labor allocation so as to provide more employment to households with large negative labor wedges (i.e. the underemployed).37 Second, she may make transfers between households, in the name of pure redistribution. Finally, different households have differing marginal utility and potentially different social welfare weights, determining the relative weight placed on these effects.38

Of course, the planner cannot directly reallocate labor income, but rather must induce a reallocation of income through the multiplier effects on their spending and transfer policies. Nevertheless, as we have empirically estimated the equilibrium mapping between fiscal policy and income (as given in Proposition 1), we can calculate how any policy affects \( dl_1 \) and therefore \( dW \). We can now apply the insights from Section 5 to understand the welfare effects of targeting fiscal stimulus.

6.2. Simple MPC-Targeting

We first focus on the case where the social planner solely seeks to maximize aggregate income. This corresponds to the case where all labor is rationed on the margin to un(der)employed households, who have no marginal disutility of labor. We view this as a sensible condition in the context of a severe depression, where underemployment is widespread and not concentrated in particular demographic groups or regions. In the empirical case demonstrated above, where the bias and homophily effects are zero for all possible policies, Propositions 2 and 3 imply that, when the modified welfare weight on all households is equal,39 the change in welfare from first period fiscal policies is given by:40

\[
dW \propto \sum_{n \in N} m_n \hat{\gamma}_n^1
\]  

37 In Appendix D.2, we show that this result carries over directly to environments with non-zero markups in the first period. Intuitively, profit owners can be thought of as providing capital services with completely elastic supply. This allows us to treat capital owners “as if” they simply supply labor and are rationed to in proportion to firms’ markups. The only modification required to accommodate this broader interpretation is that the generalized income multiplier must be extended to include capital income. This interpretation contrasts sharply with Baqae (2015), who proposes that a labor-wedge-reducing planner should target the industry with the highest network-adjusted labor share. The difference comes from the fact that Baqae’s model features competitive firms (hence no markups) and efficiently-allocated capital (no capital wedge).

38 In Appendix C.6, we provide a further decomposition of these terms for small variations in policy starting at the global optimum, similarly to Werning (2011).

39 The modified welfare weight is equal across households when the social planner is, on the margin, indifferent to transferring a dollar from any one household to any other. While this formulation is consistent with borrowing constraints, a neutral utilitarian planner (i.e. \( \lambda_n \) constant) would, in many models, prefer to transfer money to households who are more borrowing constrained, since they have higher marginal utilities of income today.

40 For a formal statement and proof, see the proof of Proposition 3.
where $\delta y^1 = R_{L1}^1 \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} dG^1 - \mu dr^1$ is the partial equilibrium change in household incomes induced by the fiscal policy.\(^{41}\) Thus, the change in welfare is proportional to the inner product of household MPCs and how much fiscal policy directly changes household incomes. Intuitively, this implies that the social planner who seeks to maximize aggregate income should simply target policies to affect households with the highest MPCs. This intuitive result holds because in the absence of bias and homophily effects, all households direct their consumption in the same way for the purposes of amplification. Therefore, the best thing a planner that is aiming to maximize income can do is target households with the highest MPCs.

Figure 4 empirically demonstrates the effectiveness of simple MPC targeting. The left panel scatters household MPCs against the resulting transfers multiplier from giving them a dollar, revealing an effectively perfect relationship between the two.\(^{42}\) Note that for the design of fiscal transfers, even the IO network and industry labor shares are irrelevant. The social planner simply needs to know the distribution of households MPCs to design policy that maximizes aggregate GDP.

By contrast, the right panel shows that it is not sufficient to target the sectors employing the highest MPC workers – while there is a positive relationship between the MPC of a sector’s employees and the purchases multiplier in a sector, the correlation is well below 1.

\(^{41}\)See Appendix A.3 for a formal proof.

\(^{42}\)Note that the MPC that we use in Figure 4 is estimated using unemployment as the identifying shock, and therefore captures the consumption response to a potentially persistent shock. The MPC that is better suited for the analysis of fiscal policy would be the MPC out of a transitory shock. If the MPC out of these two shocks are highly correlated across demographic groups, this difference should be less important for the question of which demographic groups to target. While it is hard to test this explicitly, the cross-demographic patterns in MPCs that we utilize here have a correlation of above 0.5 with self-reported MPCs from survey data (Jappelli and Pistaferri, 2014) and have similar patterns as those in response to tax rebates (Parker et al., 2013).
Rather, maximally expansive purchases policy targets those sectors such that when their production expands, accounting for the intermediates goods they use and the intermediates used by the producers of those intermediates and so on, the resulting change in labor income ends up in the hands of the highest MPC agents. While this requires no knowledge of the direction of household spending, it does rely on an understanding of the structure of production – through the input-output network and labor rationing. The planner must work out the final labor income consequences of their spending and target according to the MPC of the workers receiving that terminal labor income. This echoes results in Baqae (2015), which emphasizes the need to adjust labor shares for the input-output structure of production. This difference is quantitatively important; the right panel of Figure 4 shows how naively targeting sectors employing the highest MPC workers is effective but leaves much of the gains from targeting on the table.

To the extent that transfer policy bypasses these complications by directly giving income to households, it is easier to target than government purchases. The clear caveat is that government purchases may have direct value. If this is the case, our analysis shows how much stimulus would have to be sacrificed to obtain that direct value, enabling a policymaker with knowledge of the value of direct government purchases to determine which policy to optimally pursue.

6.2.1. Quantifying Gains from MPC-Targeting

The large dispersion in multipliers across sectors in Figure 2 suggests that the potential gains from targeting both transfer and government purchase policy are quite large. We begin quantifying the potential gains from targeting the highest-multiplier segments of the economy by comparing the maximum purchases and transfers multipliers to those of untargeted purchases and untargeted transfers. For purchases, the change in GDP due to a shock that targets each industry-region pair proportionally to its value added is 1.30. While this number is sizeable, the estimates in Figure 2 demonstrate that had the policymaker instead spent on the state-industry pair with the highest multiplier – which we estimate to be 1.61 in the oil and gas extraction industry in Georgia – the additional GDP induced by the same amount of spending policy would be twice as large. For transfer spending, we find that uniformly distributing a dollar to all household would lead to an increase in GDP of 77 cents. In contrast, if the government instead gave that dollar to the group with the highest multiplier – which we estimate is black men in South Carolina aged between 25-35 who earn less than $22,000 – it would generate 1.78 additional units of value added, a 130% increase over the uniform baseline.

While it may be possible for the social planner to achieve the maximum multiplier for
small fiscal stimulus programs, for larger programs, the social planner likely will be constrained in the amount that she can transfer to any one segment of the economy. For example, with the recent CARES act, in addition to other stimulus payments, the government transferred approximately $1,200 to each individual making less than $75,000 annually.\footnote{This is a rough characterization of the CARES act, which included several additional details. Specifically, eligibility depended on household income in the case of married couples and payments depended on the number of dependents. See \url{https://home.treasury.gov/policy-issues/cares/assistance-for-american-workers-and-families} for the details of the stimulus payments.} Putting this stylized policy into our model\footnote{We transfer $1200 (2020 USD) to each employed worker with income below $65,000 (2012 USD).}, these transfers cost the government 106.4 billion dollars and increase GDP by 79 cents for each dollar spent. Figure 5 shows the multiplier that could be achieved in our model had the government spent the same amount but if they had made payments of different sizes and targeted those payments to households based solely on their MPCs. For example, the value at $2000 shows the multiplier that the model predicts would have resulted if the government gave $2000 dollars to each worker in order of their MPCs until they exhausted their 106.4 billion dollar budget. This calculation shows that, with a maximum transfer size of $1,200, the multiplier on the income-targeted in the CARES act (1.79) was very close to multiplier with maximal MPC targeting (1.8), suggesting that income-targeting was effective given the constraint of transferring no more than $1,200 to each individual. However, the government could have achieved a higher multiplier by transferring larger amounts to fewer but higher-MPC workers. Indeed, increasing the transfer to $2,500 would have produced a multiplier that was 2.02 for the same budget.\footnote{Of course, an important caveat is that the households MPCs could themselves be a function of size of the shock. Using lottery winnings in Norway, Fagereng et al. (2019) find that MPCs fall with the size of the award, with those receiving the equivalent of up to $2,000 US dollars having an average MPC close to 1, those receiving the equivalent of $5,000 having an MPC of around 0.9 and and those receiving the equivalent of 8000 dollars having an MPC of around 0.5. These estimates suggest that there is substantial scope to increase the size of transfer payments above the $1,200 threshold before substantially altering the magnitude of household MPCs.}

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**Fig. 5.** Transfer amounts are inflated to 2020 dollars.
6.3. Fiscal Policy with Localized Shocks

While our theoretical and empirical results imply very simple fiscal rules in the case where the planner seeks to simply maximize aggregate GDP, our framework also provides a flexible toolkit for evaluating the welfare effects of fiscal policy targeting more localized economic downturns, where not all marginal labor is supplied by underemployed households with zero marginal disutility from labor. For example, this would be the case with the initial shock to which the policymaker is responding is very concentrated in some areas and underemployment is less widespread. In this case, if the planner continues to have no redistributional preferences, the welfare effect of government purchases equals:

\[ dW = - \sum_{n \in N} \mu_n \Delta_n d l_n \]  

Equation 27 demonstrates that the planner does not simply wish to maximize aggregate income, but also wants to direct stimulus to those households who are most severely underemployed. This means it will no longer be optimal for the planner to target based on MPCs alone, but rather on a combination of MPCs and labor wedges. As an example of how our framework can be applied in this setting, we perform such an exercise using imputed labor wedges during the Great Recession, which, although widespread, had much more severe impacts on certain regions and demographic groups.

To assess the value fiscal policies under (27), we must augment our existing estimates with estimates of regional-demographic-specific rationing wedges. In Appendix E, we provide two microfoundations in which the rationing wedge for each demographic group in each state is given by the percentage change in labor hours worked by that group in the Recession relative to the preceding period.\(^{46}\) To compute the welfare effects of fiscal policy, we can then simply combine changes in hours worked at the state-demographic level in the ACS from 2005-6 and 2009-10, and take the product with the induced spending-to-labor-income map that we have already estimated. This delivers the welfare gain from the stimulus benefit associated with spending one dollar in a specific industry in a specific state in the middle of the Great Recession.

This analysis illustrates that in the presence of localized shocks, targeting industry-regions simply based on their multipliers or their labor wedges alone is somewhat effective but leaves significant gains on the table. To the first point, we find that – across all sector-region pairs – the purchases multiplier of dollar of fiscal stimulus is somewhat, but not

\(^{46}\)In particular, this is true if either (i) all households within a group are homogeneously employed, have quadratic labor disutility and apply a zero utility discount rate to the future or (ii) all households within a group are probabilistically totally unemployed or fully employed.
perfectly predictive of its welfare effect, with an estimated $R^2$ of 69% (see the left panel of Figure A6 in Appendix G). The average level of labor wedges of workers in a given region and industry is similarly predictive of the welfare effect of stimulus targeting that industry and region, with an $R^2$ of 72% (See the right panel of Figure A6 in Appendix G). Thus, while the planner can achieve large welfare gains by considering either sectoral-regional multipliers or underemployment in isolation, there are also considerable welfare gains that come from jointly considering more detailed information. A fully optimizing planner could benefit both from weighing the tradeoff between high-multiplier and high-wedge industries, as well as from using wedges that are “IO-network-adjusted” to account for employment wedges in an industry’s supply chain.\textsuperscript{47}

7. Historical and Counterfactual Exercises

In order to shed light on how our analysis might differ in other countries, or in other time periods within the US, we perform three counterfactual exercises. Each exercise varies one the three key blocks of the multiplier in Proposition 1—the IO network, employment linkages, or consumption patterns—while keeping the other two fixed at our estimates for the 2012 US economy. We alternately consider the effects of greater home bias in input usage, historical (i.e. higher) sectoral labor shares, and increased income inequality.

7.1. The Role of IO Linkages

A salient feature of our calibration is that firms have large input shares (the average is around 45%). As a result, demand shocks to one industry-region are spread across other, upstream sectors as well as other regions – even before reaching households. In this section we explore the effects of the rich IO linkages in our economy by considering a stark counterfactual in which firms have zero input shares, instead directing revenues only to primary factors.

More formally, we modify our calibration in the following way: First, we set $\hat{X}^1 = 0$, so that no firms use inputs. Second, so as to hold fixed the total cost of production, we rescale each firm’s marginal demand for each labor type, capital, and foreign factors proportionally to the reduction in input expenditure. Interpreting this alternative calibration as a “no-inputs” economy, we ask two questions. First, how does input usage shape the fiscal multiplier of a GDP-proportional shock? Second, how does input usage shape the degree of heterogeneity?

\textsuperscript{47}As the Great Recession featured widespread underemployment, it is perhaps unsurprising that around two thirds of the welfare gains from fiscal stimulus can be explained by the size of fiscal multipliers. In the case of a more localized shock or recessionary episode, heterogeneity in wedges would play a greater role; our framework still facilitates such an evaluation.
We begin with the aggregate question. To begin, recall we have already shown in Section 5.2.1 that our finding of negligible bias and homophily effects holds in a range of alternative calibrations (including this one). How IO linkages affect the fiscal multiplier of a GDP-proportional shock, then, only depends on how they determine the shock’s incidence onto high- or low-MPC households. As an empirical matter, we find that the presence of IO linkages has no effect on incidence: the aggregate multiplier is 1.30 in either case.

However, IO linkages do affect the distribution of multipliers around this aggregate average. In fact, starting from an economy with zero bias and homophily effects a weakening of IO linkages always increases the difference between the largest and smallest industry-region purchases multipliers, as we show in Appendix C.2. As discussed in Section 5.2.2, this is because IO linkages serve to dilute the incidence of a shock by spreading shocks to firms with the highest-MPC (or lowest-MPC) workers across others in their supply chain whose workers have more moderate MPCs. Figure 6 shows that, in practice, inputs spread out not only the highest and lowest industry-region multipliers, but also the whole distribution. At the same time, the distribution of transfers multipliers is unaffected, since transfers reach households directly, rather than through the IO network. One implication is that targeting government purchases (but not transfers) is relatively more effective in economies with weaker input linkages.

7.2. The Decline of the Labor Share

Our model allows us to consider not only hypothetical counterfactuals but also actual, historical changes in the structure of the economy. One salient change over the past several years is the well-documented decline in the labor share in the US (Karabarbounis and
Neiman, 2014; Dorn, Katz, Patterson, and Van Reenen, 2017). Indeed, Hazell (2019) provides empirical evidence that this reduction in the labor share has dampened unemployment fluctuations. In this section, we perform a similar exercise in our model, comparing the purchases and transfers multipliers as industry-specific labor shares change from their 2000 to 2012 levels. Intuitively, if spending is directed away from high-MPC workers and toward low-MPC capitalists, aggregate amplification should fall.

Our methodology is as follows. We assume that, within each year and each industry, the shares of employee compensation in revenue is constant across states. We obtain these shares from the BEA use tables in 2000 and 2012. The aggregate labor share of value added fell from 59.2% in 2000 to 54.9% in 2012; the aggregate labor share of revenue fell from 32.1% to 30.0%. Figure A19 shows the distribution of labor shares of revenue by industry in each year. We maintain our earlier, 2012-based, estimates of demographic-specific consumption baskets and MPCs, demographic employment by region, and input-output network. We allocate the difference in labor income between 2000 and 2012 to a factor with MPC zero; this can be understood as a foreign factor or as profits accruing to MPC-zero shareholders.

Unsurprisingly, in line with our theoretical results in Appendix C.2, the reduction in the labor share leads to a smaller fiscal multiplier, as revenues are directed to lower-MPC households. We estimate an aggregate multiplier — i.e. the GDP response to a shock proportional to the 2012 distribution of value added across states and industries — of 1.338 in 2000 and 1.300 in 2012. Figure 7 shows the sorted distributions of purchases and transfers multipliers across all shocks, for 2000 and 2012. Predictably, the distribution of purchases multipliers shifts down, as less of the income from a given change in demand flows to workers and more flows to low-MPC factors. Still, the multiplier does not fall for every state-industry pair. Figure A20 shows that a few industries — namely those with sufficiently increased labor shares, such as “apparel and leather and allied products” — have higher multipliers in 2012 than in 2000.

For transfers multipliers, the response to changing labor shares is almost zero. This is because transfers target households of each MPC directly, so that differences in the labor share only affect the multiplier through bias and homophily effects, which are small.

7.3. Rising Labor Income Inequality

While the previous exercise speaks to changes in the distribution of aggregate income along the wealth dimension, labor incomes have also become more unequally divided between US workers over the last several decades, seen both in a rise in the college wage premium and a steep rise in the labor incomes at the very top of the distribution (Smith, Yagan, Zidar,
Differences in labor shares are more relevant for purchases shocks. Perhaps surprisingly, changes in income inequality – at least narrowly construed – have almost no effect on the distribution of either purchases or transfers multipliers in our model.

To illustrate this point we consider a “hollowing out” of the income distribution within occupations, for example due to a within-industry decline in cognitive routine tasks. Concretely, within in each industry, within each race-sex-age group, we reallocate labor income initially earned by the middle income group evenly between the lowest and highest income earners of the same demographic and industry of employ. This has almost exactly zero effect on the distributions of multipliers, a consequence of the fact that within race-age-sex groups, the dependence of MPC on income is approximately linear in the income bins we consider (see Figure A21). We conclude that – while labor income inequality may in principle affect fiscal multipliers – it must do by changing MPCs conditional on income or re-sorting workers across industries, not simply redistributing income in a way that preserves incidence MPCs.

8. Conclusion

This paper develops expressions for how fiscal policies affect economic activity in the presence of heterogeneous households and firms and takes those formulae to the data to characterize the dimensions of heterogeneity that affect the efficacy of stimulus policy. We build a Keynesian model with rich household heterogeneity in MPC magnitudes and directions, industrial and spatial linkages, and differential employment sensitivity. All of these elements can be unified into a single, reduced-form network that maps the marginal spending of any given household to the marginal income of factor owners producing the goods the household consumes. We provide a novel decomposition to understand the importance
of these rich interconnections by providing three corrections to the standard representative-agent Keynesian multiplier.

Empirically, we find that despite a rich regional, input-output and consumption structure, the government can implement maximally expansive policy by simply targeting either their spending or transfers to households with the highest MPCs. Linkages through the direction of household spending are empirically unimportant, meaning that the effect of the fiscal shock on aggregate output only depends on the shock’s incidence onto the incomes of households of different MPCs. Indeed, we show that concentrating transfers among the highest MPC households can increase the effect of the policy on GDP by up to 130%.

This is a result with powerful implications for policymakers and researchers. First, governments should understand the opportunity costs associated with untargeted fiscal spending. While other important implementation or political constraints may have weighted in favor of uniform stimulus checks, the above analysis suggests that the untargeted fiscal policies in the Great Recession and the COVID-19 pandemic left substantial gains on the table. Second, the results suggest that measuring household MPCs and the degree to which they vary along dimensions that are easily observed by the policymaker is an important research priority.
References


A. Omitted Proofs

All proofs referred to in Sections 2, 3, and 6 are stated for the model presented in the text, but also hold in a generalization of the model in which households experience preference shocks (such as discount rate shocks) and—except where noted otherwise—firms experience technology shocks.

We model technology shocks in a reduced form way by allowing the production function of each firm $j$ in each period $t$ to depend on a technology parameter $z^t_j$, so that its output is given by $F^t_j(X^t_j, L^t_j, z^t_j)$. We assume that $F^t_j$ is differentiable in $z^t_j$.

We model household preference shocks in a similarly reduced form way by allowing each household type $n$'s first- and second-period household consumption and second-period labor supply to depend not only on prices, taxes, and the real interest rate, but also on a preference shock $\theta_n$. Consumption and second-period labor supply are therefore given by $c^t_j(\rho, y^1_n, \tau_n, \theta_n)$ and $l^t_j(\rho, y^1_n, \tau_n, \theta_n)$. We assume that $c^t_j$ and $l^t_j$ are differentiable in $\theta_n$. Throughout our proofs we let $\theta = (\theta_1, ..., \theta_n)$ and denote by $C^t_i(\rho, y^1, \tau, \theta)$ aggregate consumption demands as a function of, among other things, preference shocks:

$$C^t_j(\rho, y^1, \tau, \theta) = \sum_{n \in N} \mu_n c^t_{nj}(\rho, y^1_n, \tau_n, \theta_n). \quad (A1)$$

A.1. Proof of Proposition 1

We first derive the general multiplier in the presence of interest rate effects in response to the more general set of shocks considered in this appendix:

**Proposition 4.** There exists a matrix $M$ such that for any small shock to parameters contained in $\text{Span}\{d\theta_G, d\theta, d\tau, dz\}$, there exists a selection from the equilibrium set such that the general equilibrium response in value added is given by:

$$dY = M \partial Q \quad (A2)$$

where $\partial Q$ is the partial equilibrium change in gross output\textsuperscript{48} associated with the shock, stacked

\textsuperscript{48}$\partial Q$ generalizes the partial equilibrium effect on final goods demand $\partial Y$ used in the main text to shocks—in particular, technology shocks—which may directly affect demand for inputs.
over time periods, as given by \((A7)\). Moreover, the matrix \(M\) is given by:

\[
M = \left( I - D \left( I - \hat{X} \right)^{-1} \right)^{-1}, \quad \text{where} \quad D = \begin{pmatrix}
C_{y,1} R_{L,1} \hat{L}^1 + \left( C_{r,1} + G_{r,1} \right) r_{Q,1}^1 & C_{r,1} + G_{r,1} \\
C_{y,2} R_{L,1} \hat{L}^1 + \left( C_{r,2} + G_{r,2} \right) r_{Q,2}^1 & C_{r,2} + G_{r,2}
\end{pmatrix}
\]

\((A3)\)

**Proof.** The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters \(\{\omega_n\}\) such that \(\omega_n \to \omega\). By Proposition 6, we know that for each \(\omega_n\) there exists a non-empty corresponding set of equilibria \(E_n\). Moreover let \(E(\omega)\) be the set of equilibria corresponding to the limit \(\omega\). Now consider an arbitrary sequence of equilibria \(\{e_n\}\) such that \(e_n \in E_n\) for all \(n \in \mathbb{N}\) and \(e_n \to e\). Toward a contradiction, suppose that the set of equilibria is not UHC in the parameters, i.e. \(e \notin E(\omega)\). It follows that one of the following does not hold at \(e\): household budget balance, government budget balance or market clearing. But by Assumption 4, continuity of the fiscal rule, continuity of the interest rate rule and continuity of the rationing function, we know that all functions in these expressions are continuous. It follows that there exists \(m \in \mathbb{N}\) such that \(e_m \notin E_m\), a contradiction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

\[
\frac{dr^1}{dQ^1} = r_{Q,1}^1 dQ^1 + r_{Q,2}^1 dQ^2 = r_{Q}^1 dQ
\]

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions. Our differentiability assumptions allow us to express

\[
dQ = \hat{X} dQ + \hat{X}_z dz Q + C_p p_z dz + C_{r,1} dr^1 + C_{y,1} dy^1 + C_{r} d\tau + C_\theta d\theta + G_p p_z dz + G_{r,1} dr^1 + G_{r} d\tau + G_\theta d\theta
\]

Plugging in for \(dr^1\) and \(dy^1\) we get

\[
dQ = R_{L,1} \hat{L}^1 dQ^1 + R_{L,1} d\hat{L}^1 Q^1
\]

where here

\[
\hat{Q} = (C_p + G_p) p_z dz + \hat{X}_z dz Q + C_{y,1} R_{L,1} \hat{L}^1 dz Q^1 + (C_r + G_r) d\tau + C_\theta d\theta + G_\theta d\theta
\]

Recognizing that \(dY = (I - \hat{X}) dQ\) and substituting completes the proof.

\(\Box\)
The claimed result now follows as an immediate corollary:

Proof. Under Assumption 1, $D$ reduces to:

$$D = \begin{bmatrix} C_{y1}^1 R_{L1}^1 \hat{L}^1 & 0 \\ C_{y2}^2 R_{L1}^1 \hat{L}^1 & 0 \end{bmatrix}$$  \hfill (A8)

Simple matrix manipulations show that one may extract just the first $\mathcal{I}_1$ rows:

$$dY^1 = (I - C_{y1}^1 R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1})^{-1} \partial Q^1 \hfill (A9)$$

\[\square\]

A.2. Proof of Proposition 2

We first prove a lemma re-expressing the multiplier in terms of $\mathcal{G}$ and $\partial y^1$.

**Lemma 1.** The total change in first-period GDP due to a partial equilibrium demand shock with labor income incidence $\partial y^1$ can be expressed as

$$\mathbf{1}^T dY^1 = \underbrace{\mathbf{1}^T \partial y^1}_{\text{Direct effect}} + m^T \left( \sum_{k=0}^{\infty} (\mathcal{G}m)^k \right) \partial y^1 \hfill (A10)$$

Proof. Starting from Proposition 1 and using that the modulus of $C_{y1}^1 R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1}$ is less than 1, we can express:

$$dY^1 = \sum_{k=0}^{\infty} \left[ C_{y1}^1 R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1} \right]^k \partial Y^1$$

$$= \partial Y + C_{y1}^1 m \sum_{k=0}^{\infty} \left[ R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1} C_{y1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1} \right]^k R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1} \partial Y^1 \hfill (A11)$$

$$\mathbf{1}^T dY^1 = \mathbf{1}^T \partial Y + m^T \left( \sum_{k=0}^{\infty} (\mathcal{G}m)^k \right) \partial y^1$$

where the last line uses the definitions of $\mathcal{G}$ and $\partial y^1$, and the fact that $\mathbf{1}^T C_{y1}^1 = \mathbf{1}^T$ (by construction).

Finally, $\mathbf{1}^T \partial Y^1 = \mathbf{1}^T \partial y^1$ because $\mathbf{1}^T R_{L1}^1 \hat{L}^1 (I - \hat{X}^1)^{-1} = \mathbf{1}^T$, since firms earn zero profits.  \[\square\]
We now use this result to prove the claimed decomposition:

**Proof.** Let \( \mathbf{b} \equiv \mathbf{1}^T (\mathbf{I} - \mathbf{Gm})^{-1} \) be the vector of Bonacich centralities of households in the income-to-spending network; these are well-defined as we have assumed the modulus of \( \mathbf{Gm} \) is less than one. Let \( (b_{\text{next}})^T = b^T \mathbf{G} \) be the row vector with \( n^{\text{th}} \) entry equal to the average Bonacich centrality of the household to whom \( n^{\text{th}} \)’s marginal spending flows.

We begin by providing a lemma that exactly decomposes the general equilibrium change in GDP in terms of Bonacich centralities.

**Lemma 2.** For any \( x \in \mathbb{R} \), the total change in first-period GDP due to a partial equilibrium demand shock with unit-magnitude labor income incidence \( \hat{\gamma}^1 \) is equal to

\[
\mathbf{1}^T dY^1 = (1 + x \cdot \mathbb{E}_{\hat{\gamma}^1}[m_n]) + \mathbb{E}_{\hat{\gamma}^1}[m_n] \left( \mathbb{E}_{\hat{\gamma}^1}[b_{\text{next}}^n] - x \right) + \mathbb{Cov}_{\hat{\gamma}^1}[m_n, b_{\text{next}}^n] \tag{A12}
\]

Setting \( x \) equal to the \( \frac{1}{1 - \text{MPC}} \) multiplier with the MPC weighted by income \( y^* \), we obtain an exact decomposition in the spirit of Proposition 2.

\[
\mathbf{1}^T dY^1 = \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} + \frac{\mathbb{E}_{\hat{\gamma}^1}[m_n] - \mathbb{E}_{y^*}[m_n]}{1 - \mathbb{E}_{y^*}[m_n]}
\]

+ \( \mathbb{E}_{\hat{\gamma}^1}[m_n] \left( \mathbb{E}_{\hat{\gamma}^1}[b_{\text{next}}^n] - \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} \right) + \frac{\mathbb{Cov}_{\hat{\gamma}^1}[m_n, b_{\text{next}}^n]}{1 - \mathbb{E}_{y^*}[m_n]} \),

\[
\text{Keynesian multiplier} \quad \text{Incidence effect} \quad \text{Biased MPC direction effect} \quad \text{Homophily effect} \tag{A13}
\]

**Proof.** Note that Proposition 1 implies that the change in GDP resulting from some shock with unit incidence is given by

\[
\mathbf{1}^T dY^1 = b^T \hat{\gamma}^1 = \mathbf{1}^T \hat{\gamma}^1 + b^T \mathbf{G} \hat{\gamma}^1 \tag{A14}
\]

Let \( b_{\text{next}}^T = b^T \mathbf{G} \) be the row vector with \( i^{\text{th}} \) entry equal to the average Bonacich centrality of the household to which \( i^{\text{th}} \)’s marginal spending flows. We then have, for any \( x \in \mathbb{R} \):

\[
\mathbf{1}^T dY^1 = 1 + \mathbb{E}_{\hat{\gamma}^1}[m_n] b_{\text{next}}^n = 1 + \mathbb{E}_{\hat{\gamma}^1}[m_n] \cdot \mathbb{E}_{\hat{\gamma}^1}[b_{\text{next}}^n] + \mathbb{Cov}_{\hat{\gamma}^1}[m_n, b_{\text{next}}^n]
\]

\[
= \left(1 + x \cdot \mathbb{E}_{\hat{\gamma}^1}[m_n]\right) + \mathbb{E}_{\hat{\gamma}^1}[m_n] \left( \mathbb{E}_{\hat{\gamma}^1}[b_{\text{next}}^n] - x \right) + \mathbb{Cov}_{\hat{\gamma}^1}[m_n, b_{\text{next}}^n] \tag{A15}
\]

We can now prove Proposition 2. First, note that:

\[
b_n = 1 + m_n + O(|m|^2) = 1 + \frac{m_n}{1 - \mathbb{E}_{y^*}[m_n]} + O(|m|^2) \tag{A16}
\]
Substituting this into Equation A13, we have

\[ 1^T dY^1 = \frac{1}{1 - E_y[m_n]} + \frac{E_{\tilde{\theta}_y}[m_n] - E_\theta[m_n]}{1 - E_\theta[m_n]} + \frac{E_{\tilde{\theta}_y}[m_n^\text{next}] - E_\theta[m_n^\text{next}]}{1 - E_\theta[m_n^\text{next}]} + O(|m|^2) \]

Rearranging yields Equation 15.

A.3. Proof of Proposition 3

Proof. We provide results here for a more general welfare function in which households have instrumental value of government purchases given by \( w^i_t(G^t) \). To begin, we define \( \kappa^t_n \) to be \( n \)'s marginal value of additional expenditure in period \( t \), i.e. for all \( i \), \( u^t_{nci} = \kappa^t_n \) (recall prices are normalized to one). Therefore,

\[ dW = \sum_{n \in N} \lambda_n \mu_n \sum_{t=1,2} \beta_n^{t-1} \left( u^t_{nci} d\alpha_n^t - v^t_{nci} d\theta_n^t + w^t_{nci} dG^t \right) \]

\[ = \sum_{n \in N} \lambda_n \mu_n \sum_{t=1,2} \beta_n^{t-1} \left[ \kappa^t_n \left( 1^T d\alpha_n^t - \frac{v^t_{nci}}{\kappa^t_n} d\theta_n^t \right) + w^t_{nci} dG^t \right] \] (A18)

Next note that in the second period, free labor supply implies \( v^2_t = \kappa^2_n \). In the first, there may be some wedge \( \Delta_n \) such that \( v^1_t = \kappa^1_n (1 + \Delta_n) \); a positive wedge indicates that \( n \) works as if the wage was higher than it is, i.e. oversupplies labor; a negative wedge represents involuntary un(der)employment. In these terms, we have

\[ dW = \sum_{n \in N} \lambda_n \kappa^1_n \mu_n \left[ -\Delta_n d\alpha_n^1 + \sum_{t=1,2} \frac{\kappa^t_n}{\kappa^1_n} \beta_n^{t-1} \left( 1^T d\alpha_n^t - d\theta_n^t \right) + \left( \frac{u^1_{nci} dG^1}{\kappa^1_n} + \frac{\beta_n v^2_{nci} dG^2}{\kappa^1_n} \right) \right] \]

(A19)

Next, define \( \tilde{\lambda}_n = \lambda_n \kappa^1_n \). Also note that \( \frac{\kappa^t_n}{\kappa^1_n} \beta_n^{t-1} = 1 \) for \( t = 1 \). For \( t = 2 \), we use the modified Euler equation:

\[ \kappa^1_n = \beta_n \frac{1 + \gamma^1_n}{1 - \theta_n^1} \kappa^2_n \] (A20)
where $\phi_n$ is a borrowing wedge. $\phi_n \geq 0$ is positive when households behave as if interest rates are higher than in reality, i.e. consume more in the future than they would like; this corresponds to borrowing constraints. This gives us

$$dW = \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 + \left( 1^T dc_n^1 - dl_n^1 \right) + \frac{1 - \phi_n}{1 + r^1} \left( 1^T dc_n^2 - dl_n^2 \right) + \left( \frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left( \frac{1 - \phi_n}{1 + r^1} \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right) \right]$$  \hspace{1cm} (A21)

Differentiating the household’s lifetime budget constraint (at constant $r^1$):

$$1^T dc_n^1 - dl_n^1 + \frac{1^T dc_n^2 - dl_n^2}{1 + r^1} = -d\tau_n^1 - \frac{d\tau_n^2}{1 + r^1}$$  \hspace{1cm} (A22)

Plugging this in, we have:

$$dW = \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 + \phi_n \left( 1^T dc_n^1 - dl_n^1 \right) - (1 - \phi_n) \left( d\tau_n^1 + \frac{d\tau_n^2}{1 + r^1} \right) + \left( \frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left( \frac{1 - \phi_n}{1 + r^1} \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right) \right]$$  \hspace{1cm} (A23)

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint binds:

$$s_n^1 = l_n^1 - \tau_n^1 - 1^T c_n^1 \implies 1^T dc_n^1 - dl_n^1 = -d\tau_n^1$$  \hspace{1cm} (A24)

Defining the within-period willingness to pay for government purchases as $WTP_n^1 = \frac{w_{nG}^1}{\kappa_n^1}$, we arrive at the final expression:

$$dW = \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[ -\Delta_n dl_n^1 - \left( d\tau_n^1 + (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) + \left( WTP_n^1 dG^1 + (1 - \phi_n) WTP_n^2 dG^2 \right) \right]$$  \hspace{1cm} (A25)

This proves the first part of the proposition. To prove the second part of the proposition, see that we assumed: the modified welfare weights are uniform across households ($\tilde{\lambda}_n = 1$ for all $n \in N$), there is no willingness to pay for government purchases ($WTP_n^1 = 0$ for all $n \in N$), and all underemployed households have zero marginal disutility of labor (if $\Delta_n < 0$ then $\Delta_n = -1$). Thus, Equation 25 reduces to:

$$dW = \mu^T dl^1 - \mu^T d\tau^1$$  \hspace{1cm} (A26)

Moreover, by Equation ??, we have that:

$$\mu dl^1 = \Gamma^1 \left( I - C_{y^1}^1 \Gamma^1 \right)^{-1} \left( dG^1 - C_{y^1}^1 \left( \mu d\tau^1 + \right) \right)$$  \hspace{1cm} (A27)
Combining these equations and rearranging:

\[ dW = \Gamma^{1T} (I - C^{1}_{y} \Gamma^{1})^{-1} \left( dG^{1} - C^{1}_{y} \mu d\tau^{1} \right) - \mu^{T} d\tau^{1} \]

\[ = \Gamma^{1T} \left( I - C^{1}_{y} \Gamma^{1} \right)^{-1} dG^{1} - \Gamma^{1T} \left[ \left( I - \Gamma^{1} C^{1}_{y} \right)^{-1} \Gamma^{1} C^{1}_{y} + I \right] \mu d\tau^{1} \]

We now use the final assumption made that the bias and homophily effects are zero for all possible policies. To this end, we first show that, if the bias and homophily effects are zero for all purchases and transfers shocks relative to some baseline income incidence \( y^{*} \), then either \( m_{n} = 0 \) or \( m_{n}^{\text{next}} = E_{y^{*}}[m_{n}^{\prime}] \).

To start, fixing a single type \( n \in N \), consider the bias term corresponding to a transfers shock with direct incidence \( \dot{c} y^{1} = \dot{c}_{n} \) (i.e. only transferring to \( n \)).

\[
\text{bias}_{\dot{c} y^{1}} y^{*} = E_{\dot{c} y^{1}}[m_{n}] (E_{\dot{c} y^{1}}[m_{n}^{\text{next}}] - E_{y^{*}}[m_{n}^{\prime}]) = m_{n} (m_{n}^{\text{next}} - E_{y^{*}}[m_{n}^{\prime}])
\]

(A29)

The assumption that this is zero then implies that either \( m_{n} = 0 \) or \( m_{n}^{\text{next}} = E_{y^{*}}[m_{n}^{\prime}] \).

To apply this fact, recall the definition \( m_{n}^{\text{next}} = m^{T} \Gamma^{1} C_{y^{1}}^{1} \), where \( C_{y^{1}}^{1} \) is the normalized matrix of spending directions, i.e. \( C_{y^{1}}^{1} = \overline{C}_{y^{1}}^{1} m \). Our previous observation—that for all \( n \), \( m_{n} = 0 \) or \( m_{n}^{\text{next}} = E_{y^{*}}[m_{n}^{\prime}] \)—then implies that \( m^{T} \Gamma^{1} C_{y^{1}}^{1} = (m^{\text{next}})^{T} m = E_{y^{*}}[m_{n}^{\prime}] \cdot m \).

Applying this fact to the multipliers in Equation A28, we have

\[
\Gamma^{1T} \frac{dY^{1}}{dG^{1}} = \Gamma^{1T} \left( I - C_{y^{1}}^{1} \Gamma^{1} \right)^{-1} = \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma^{1T} \left( C_{y^{1}}^{1} \Gamma^{1} \right)^{k} = \Gamma^{1T} \left( \frac{m^{T}}{1 - E_{y^{*}}[m_{n}]} m \right) \Gamma^{1}
\]

(A30)

Moreover, we have that:

\[
\Gamma^{1T} \frac{dl^{1}}{dy^{1}} = \Gamma^{1T} \left( I - \Gamma^{1} C_{y^{1}}^{1} \right)^{-1} = \Gamma^{1T} + \sum_{k=1}^{\infty} \frac{1}{k!} \Gamma^{1T} \left( \Gamma^{1} C_{y^{1}}^{1} \right)^{k} \Gamma^{1}
\]

(A31)

Substituting (A30) and (A31) into Equation A28 shows that:

\[
dW = \left( \frac{1}{1 - E_{y^{*}}[m_{n}]} m \right)^{T} \left( \Gamma^{1} dG^{1} - \mu d\tau^{1} \right)
\]

(A32)
By budget balance, we moreover have that:

\[ \mathbb{1}^T dG^1 - \mathbb{1}^T \mathbf{\mu} d\tau^1 = 0 \quad (A33) \]

Thus, as the columns of \( \Gamma^1 \) sum to 1, we have that:

\[ \mathbf{1}^T \Gamma^1 dG^1 - \mathbf{1}^T \mathbf{\mu} d\tau^1 = 0 \quad (A34) \]

Thus, the change in welfare is simply given by:

\[ dW = \frac{1}{1 - \mathbb{E}_{g^*}[m_n]} m^T (\Gamma^1 dG^1 - \mathbf{\mu} d\tau^1) \quad (A35) \]

yielding the claim given in the text. \( \square \)
B. Rationing Equilibrium Microfoundation

In this section, we provide a micro-foundation for rationing equilibrium concept. Its key elements are downward short-term wage rigidity, sticky inflation expectations, and exogenous monetary policy. In light of these frictions, short-term labor markets clear through labor market rationing—which we model as an endogenous constraint on household labor supply—that constrains aggregate labor supply to the amount firms demand at the fixed wage. After showing how the multiplier of Proposition 1 obtains under these assumptions, we illustrate how the model may be extended to accommodate multiple labor types and/or multiple firms or regions.

B.1. Key macroeconomic assumptions

Our analysis relies on three key macroeconomic assumptions. First, we assume that the nominal interest rate $i^1$ is exogenously fixed. In this sense, the model describes the behavior of an economy where monetary policy is constrained at an effective lower bound. Alternatively, one may interpret our model as the one relevant to a fiscal planner who acts after monetary policy has already been set. Second, we assume that inflation (or at least household inflation expectations) is exogenous and homogenous, in the sense that there exists $\pi > 0$ so that

$$\frac{w^2}{w^1} = \pi.$$  \hspace{1cm} (A36)

We normalize the units of second-period labor supply (and so, implicitly, goods) so that $\pi = 1$. Third, we assume that first-period wages are downwardly rigid. That is, there exists $w^1$ such that

$$w^1 \leq w^1.$$  \hspace{1cm} (A37)

Our results all apply locally to equilibria in which the downward rigidity constraint on first-period wages strictly binds. An alternative assumption is that first-period wages are symmetrically rigid; in this case our results apply to all equilibria.

B.2. Household problem with a rationing constraint

Each household $n$ supplies labor $l^t_n$ and consumes a vector of goods $c^t_n$ in periods $t = 1, 2$, in order to maximize an utility function which is additively separable from any intrinsic value of government purchases. The household faces lump-sum taxes $\tau_n$, a borrowing constraint in the form of minimum savings $\sigma^1_n$, and an endogenous constraint on first-period labor supply $R^1_n((L^1_i)_{i \in Z^1})$, which binds only if the downward-wage-rigidity bound $w^1$ does as well; we
The household problem is therefore:

$$\max_{\tilde{c}, \tilde{l}} \sum_{t=1,2} \beta_n^{t-1} \left[ u_n^t(\tilde{c}, \tilde{l}) + w_n^t(G^t) \right]$$

subject to

$$p(t, z^t)\tilde{c}+\frac{p^2(w^2, z^2)^T\tilde{c}}{1+i^1} + \frac{\tau_n}{1+i^1} \leq w^1\tilde{l} + \frac{w^2\tilde{l}^2}{1+i^1} \quad (A38)$$

$$w^1\tilde{l} - (p(t)^T\tilde{c}) - \tau_n \geq \tilde{z}_n^1$$

if $w^1 = w^1$, then $\tilde{l}_n^1 = R_n^1((L_i^1)_{i\in I^1})$.

where $p(t, z^t)$ are prices as a function of wages and technology (by the no-substitution theorem, Proposition 5), $i^1$ is the nominal interest rate, $G^t$ is a vector of government purchases that we take as given, and $l^1_s$ is a selection from the solution to $n$'s problem without the final constraint.

The fact that labor is rationed only when downward rigidity binds reflects that in that case, households cannot simply obtain their desired level of labor supply by undercutting the market wage.\footnote{Also implicit in our formulation is that – in principle – some households may be rationed more, not less, labor than they would freely provide. Of course, whether this happens or not, depends on the rationing function, which one may think is unlikely to over-demand labor in the sorts of recessionary environments we consider. Alternatively, one may show that all of our results obtain if (a) the rationing function depends not only on firm demands but also on household’s unconstrained level of first-period labor supply, (b) the rationing function never demands more labor from a household than its unconstrained level (c) the rationing constraint is only an upper bound on labor supply, (d) household preferences are are GHH – i.e. $u_n^t(c, l) = v_n^t(c - w_n^t(l))$ – and sufficiently concave, so that households supply as much labor as they are rationed and so that their unconstrained levels of first-period labor supply do not respond to transfers.}

### B.3. Firms

In each period $t$, the representative producer of each good $i \in I^t$ offers a wage $w^t_i$ and produces gross output $Q^t_i$ using intermediate inputs $X^t_i$ and labor $L^t_i$ in order to maximize profits. The firm’s problem in period $t$ is therefore

$$\max_{\tilde{Q}_i^t, \tilde{X}_i^t, \tilde{L}_i^t} (p^t)^T\tilde{Q}_i^t - (p^t)^T\tilde{X}_i^t - w^t\tilde{L}_i^t$$

subject to

$$\tilde{Q}_i^t = F_i^t(\tilde{X}_i^t, \tilde{L}_i^t, \tilde{z}_i^t) \quad (A39)$$

### B.4. Market clearing and rationing

Goods markets and clear in the usual fashion. In any equilibrium,

$$Q^t_i = \sum_{j \in I^t} X^t_{ji} + \sum_{n \in N} \mu_n c^t_{ni} + G^t_i \quad (A40)$$
Labor markets also must clear.

\[ \sum_{i \in \mathbb{Z}^1} L_i^1 = \sum_{n \in N} \mu_n L_n^1 \]  

(A41)

However, recall that when the lower bound on first-period wages binds, household labor supply is determined by rationing rather than free labor supply.

**B.5. Equilibrium comparative statics with rationing**

We study first-order comparative statics in this model when the downward wage rigidity constraint binds. More formally, we start from an equilibrium at which—given any infinitesimal shock to fundamentals—the downward wage rigidity constraint still binds, and then consider local changes in equilibrium outcomes.

Crucially, this focus ensures that—locally—first-period wages are equal to \( w_1 \) and second-period wages are equal to \( w_1 \pi = w_1 \); by the no-substitution theorem, prices are similarly fixed. Moreover, each household’s labor supply is determined by the rationing function—which in some instances diverges from what households would freely choose. Together, these facts significantly simplify our general equilibrium analysis.

Toward an expression for aggregate goods market clearing, let \( c^1(y_n^1, \tau_n) \) denote the solution of each household’s \( n \)’s problem, given its rationed income \( y_n^1 = w_1 R_n^1((L_t^1)_{i \in \mathbb{Z}^1}) \) and the taxes \( \tau_n = (\tau_n^1, \tau_n^2) \) it faces. Note that by additive separability, this consumption is independent of government purchases. We therefore have the first-period goods market clearing condition:

\[ Q^1 = \sum_{j \in \mathbb{Z}^1} X_j^1 + \sum_{n \in N} \mu_n c_n^1 (w_1 R_n^1((L_t^1)_{i \in \mathbb{Z}^1}), \tau_n) + G^1 \]  

(A42)

Moreover, by Corollary 1, \( L^1 = \hat{L}^1 Q^1 \).

Letting \( C^1 \equiv \sum_{n \in N} \mu_n c_n^1 \), and normalizing \( w_1 = 1 \), we now totally differentiate with respect to a small change in taxes \( d \tau \) and or government purchases \( dG \):

\[
\frac{dQ^1}{dY^1} = \hat{X}^1 dQ^1 + C_{y^1}^1 R_{L^1}^1 \hat{L}^1 dQ^1 + C_{\tau^1}^1 d\tau + dG^1 \\
\left( I - \hat{X}^1 \right) \frac{dQ^1}{dY^1} = C_{y^1}^1 R_{L^1}^1 \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \left( I - \hat{X}^1 \right) \frac{dQ^1}{dY^1} + C_{\tau^1}^1 d\tau + dG^1 \\
\frac{dY^1}{dY^1} = \left( I - C_{y^1}^1 R_{L^1}^1 \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \right) \frac{dY^1}{dY^1}
\]  

(A43)

This income multiplier expression coincides with that of Proposition 1.
B.5.1. Extension to multiple labor types and/or labor markets

The microfoundation above can be extended to accommodate many labor types (firm preferences over workers) and many labor markets (worker preferences over firms) under a few additional assumptions. The first key assumption is that each household-type \( n \) × firm-type pair \((n, i)\) has a wage \( w^1_{n,i} \) which is constrained by a lower bound \( w^1_{n,i} \). We restrict our analysis to the case in which all of these bounds bind in a neighborhood around the initial equilibrium. Second, we assume that not only aggregate but also relative inflation expectations are common and fixed, i.e. for each \((n, i) \in N \times I^2\) there exists \( \pi_{n,i} \) so that \( w^2_{n,i} = w^1_{n_1,i_1} \pi_{n,i} \). This assumption, together with the binding downward rigidity constraints ensure that all wages—and therefore all prices—are exogenously fixed local to the initial equilibrium. Finally, the rationing function in this case takes as input firm-and-household-specific demands \( L^1_{i,n} \), and returns constraints on each household’s labor supply to each firm. These constraints bind in the case where all lower bounds on wages are binding. Following analogous steps to in the single-labor-market case, we again obtain the multiplier of Proposition 1.
C. Additional Results

Here we present a provide results on properties (including existence) of rationing equilibrium (C.1), provide comparative statics for the multiplier (C.2), provide a further network reinterpretation of the multiplier at the zero lower bound (C.3), generalize our decomposition results to account also for supply shocks (C.4), analyze benchmark cases in which the network adjustments to the Keynesian multiplier are zero (C.5), provide first order conditions for optimal policy (C.6).

C.1. Equilibrium Properties

In this Appendix, we ensure our analysis of the multiplier is well-posed and eliminate any nuisance terms that unnecessarily complicate the analysis. To this end, we first provide a no-substitution theorem that ensures prices are technologically determined – and thus independent of demand – and, second, prove the existence of a rationing equilibrium.

The following technical conditions on production technologies and household preferences are sufficient for the no-substitution theorem. Assumption 2 provides basic technical conditions on production and Assumption 3 imposes a simple positivity condition on demand such that there is demand for all goods.

Assumption 2. For all $i, t$ and $z^t_i$, production $F^t_i(X^t_i, L^t_i, z^t_i)$ is continuous, weakly increasing, strictly quasi-concave, and homogeneous of degree one in $(X^t_i, L^t_i)$. Further, labor is essential in production, i.e. $F^t_i(X^t_i, 0, z^t_i) = 0$, and production is strictly increasing in labor. Finally, there exists some $\bar{p}^t \in \mathbb{R}^T_i$ and $\{X^t_i, L^t_i\}_{i \in I^t}$ s.t. for all $i$, $F^t_i(X^t_i, L^t_i, z^t_i) \geq 1$ and $\bar{p}^t X^t_i + L^t_i \leq \bar{p}_i^{t, 50}$

Assumption 3. For any $\varrho, y^t, \tau, \theta$: for each good $i$, some household type $n$ has $c^t_{ni} > 0$.

Under these two rather weak assumptions, we can show that:

Proposition 5. Under Assumptions 2 and 3, for a given $z^t$, there exists a unique $q^t \in \mathbb{R}^T_i$ consistent with rationing equilibrium, independent of demand.

Proof. We follow closely the proof technique used in Acemoglu and Azar (2020). We will prove the result for an economy with arbitrary time horizon for maximum applicability. Fix a time period $t$ vector of productivity parameters $z^t$. For each $i$, define the unit cost function:

$$\kappa^t_i(p) = \min_{F^t_i(X^t_i, L^t_i, z^t_i) \geq 1, X^t_i, L^t_i \geq 0} p X^t_i + L^t_i$$

(A44)

A sufficient but not necessary condition is that every good can be produced using only labor.
The minimum is well-defined owing to Assumption 2, which states that $F$ is strictly increasing in labor, CRS, and strictly quasiconcave.

We now establish properties of the unit cost function on the domain $p \in \mathbb{R}_+^{I_t}$. First, since labor is necessary for production, $\kappa'_i(0) > 0$ for all $i$. Second, by the last part of Assumption 2, there exists $\bar{p}$ such that $\kappa'_i(\bar{p}) \leq \bar{p}_i$ for all $i$. Finally, $\kappa'_i(p)$ is weakly increasing in $p$ by inspection. These three properties establish that $\kappa'_i(p) \equiv (\kappa'_1(p), ..., \kappa'_{I_t}(p))$ maps $\bigotimes \equiv \times_{i=1}^{I_t} [0, \bar{p}_i] \to \bigotimes$ and is weakly increasing. Moreover, $\bigotimes$ is a complete lattice with respect to the following operators:

$$p \land q \equiv (\min(p_1, q_1), ..., \min(p_{I_t}, q_{I_t})) \quad \text{and} \quad p \lor q \equiv (\max(p_1, q_1), ..., \max(p_{I_t}, q_{I_t}))$$  \hspace{1cm} (A45)

By Tarski’s fixed point theorem, the set of fixed points $\{p \in \mathbb{R}_+^{I_t} \mid \kappa^t(p) = p\}$ is therefore a complete lattice.

In order for $p$ to be consistent with either our flexible-wage or rationing equilibrium, all operating firms must make zero profits. Assumption 3 implies that all firms operate in equilibrium, so $p = \kappa^t(p)$ is a necessary condition for any equilibrium. It therefore remains to show that $\kappa^t$ has a unique fixed point. To this end, we first show that each $\kappa'_i$ is concave. For price vectors $p$ and $q$ and $\lambda \in (0, 1)$, we construct the price vector:

$$p^\lambda = \lambda p + (1 - \lambda)q$$  \hspace{1cm} (A46)

By cost minimization,

$$\kappa'_i(p) \leq pX^i_t(p^\lambda) + L^i_t(p^\lambda) \quad \text{and} \quad \kappa'_i(q) \leq qX^i_t(p^\lambda) + L^i_t(p^\lambda)$$  \hspace{1cm} (A47)

It follows that:

$$\kappa'_i(p^\lambda) = p^\lambda X^i_t(p^\lambda) + L^i_t(p^\lambda) \geq \lambda \kappa'_i(p) + (1 - \lambda)\kappa'_i(q)$$  \hspace{1cm} (A48)

establishing that each $\kappa'_i$ is a concave function.

Toward a contradiction, suppose $\kappa^t$ has more than one fixed point. Then since the set of fixed points is a complete lattice, there must exist distinct fixed points $p^*, p^{**}$ with $p_i^* \leq p_i^{**}$ for all $i$. Now take $\lambda$ to be given by the following:

$$\lambda = \min_{i \in I_t} \frac{p_i^*}{p_i^{**}}$$  \hspace{1cm} (A49)

Note that $\lambda \in (0, 1)$ since $p >> 0$ for all fixed points $p$, since $\kappa'_i(0) > 0$ for all $i$ and $\kappa^t$ is weakly increasing. We have that $p_i^* \geq \lambda p_i^{**}$ for all $i \in I_t$ with equality for at least one $j$ by
construction. For this $j$ such that $p_j^* = \lambda p_j^{**}$, we then have

$$0 = \kappa_j^i(p^*) - p_j^* \geq \kappa_j^i(\lambda p^{**}) - \lambda p_j^{**} \geq (1 - \lambda)\kappa_j^i(0) + \lambda \kappa_j^i(p^{**}) - \lambda p_j^{**} = (1 - \lambda)\kappa_j^i(0) > 0 \quad (A50)$$

where the first line follows from the zero profit condition, the second line follows from the fact that $\kappa^i_j$ is weakly increasing and $\lambda \in (0, 1)$, the third line follows from concavity of $\kappa^i_j$, the fourth line follows again from the zero profit condition, and the final line follows from positivity of costs. This is a contradiction. Hence, there must be a unique fixed point at all times $t$. This implies the stated result and also makes the no-substitution theorem applicable to Appendix D.3 where we extend the baseline model to allow for multiple time periods. ⎕

The existence of unique, positive prices $p^1(z^1), p^2(z^2) \in \mathbb{R}_{+}^{T_i}$ consistent with equilibrium allows us to reduce the number of endogenous price variables in considering comparative statics that keep $z^1$ and $z^2$ fixed, allowing us to keep track of just the real interest rate. Implicit in this no-substitution economy is the assumption that good prices respond instantaneously to changes in technology, which is irrelevant in the case of demand shocks.

Moreover, combining Proposition 5 with constant returns to scale technology implies a simple form for aggregate input and labor demands. Formally:

**Corollary 1.** The aggregate input demand $X^t(p^t, Q^t)$ and labor demand $L^t(p^t, Q^t)$ vectors are given by:

$$X^t = \hat{X}^t(z^t)Q^t \quad L^t = \hat{L}^t(z^t)Q^t \quad (A51)$$

where $\hat{X}^t(z^t)$ is the matrix with $i$th column $\hat{X}_i^t(z^t)$ and $\hat{L}^t(z^t)$ is the diagonal matrix with $i$th entry $\hat{L}_i(z^t)$.

**Proof.** Fixing a period $t$ and a technology vector $z^t$, by Proposition 5, there exists a unique price vector $p^t$ consistent with equilibrium. The unit input demands for any firm $i$ at this price solve the following program:

$$(\hat{X}_i^t(z^t), \hat{L}_i^t(z^t)) = \arg \min_{(X_i^t, L_i^t) \text{ s.t. } F_i^t(X_i^t, L_i^t, z_i^t) = 1} p^t(z^t)X_i^t + L_i^t \quad (A52)$$

CRS then implies that for a firm producing $Q_i^t$ units in equilibrium,

$$X_i^t = Q_i^t\hat{X}_i^t(z^t) \quad L_i^t = Q_i^t\hat{L}_i^t(z^t) \quad (A53)$$

Stacking these equations over $T_i$ gives

$$X^t = \hat{X}^t(z^t)Q^t \quad L^t = \hat{L}^t(z^t)Q^t \quad (A54)$$
Proposition 5 implies two additional, useful results. First, the Leontief-inverse matrix always exists. Second, one can use the Leontief-inverse to obtain a useful closed-form expression for the demand-independent prices. This is stated formally in the following corollary:

**Corollary 2.** For any $t$, $z^t$ the Leontief-inverse matrix $(I - \hat{X}^t(z^t))^{-1}$ exists. Moreover, prices are given uniquely by the following expression:

$$p'(z^t) = \left(I - \hat{X}^t(z^t)^T\right)^{-1} \hat{L}^t(z^t)\mathbb{1}$$

(A55)

**Proof.** We first prove that the matrix $(I - \hat{X}^t(z^t))$ is invertible. The zero-profit condition for all $i$ implies that:

$$p'(z^t)X^t_i + L^t_i = p'_i(z^t)Q^t_i$$

(A56)

Normalizing by the quantity yields:

$$p'(z^t)\hat{X}^t_i(z^t) + \hat{L}^t_i(z^t) = p'_i(z^t)$$

(A57)

Stacking this equation yields the matrix equation:

$$\hat{L}^t(z^t)\mathbb{1} + \hat{X}^t(z^t)^T p'(z^t) = p'(z^t)$$

(A58)

This allows us to solve for the unit labor demands as the unique diagonal matrix such that:

$$\hat{L}^t(z^t)\mathbb{1} = (I - \hat{X}^t(z^t)^T) p'(z^t)$$

(A59)

Iterating this equation $k \in \mathbb{N}$ times yields:

$$p'(z^t) = \left(1 + \hat{X}^t(z^t)^T + ... + (\hat{X}^t(z^t)^T)^k \right) \hat{L}^t(z^t)\mathbb{1} + (\hat{X}^t(z^t)^T)^{k+1} p'(z^t)$$

(A60)

Recall that $\hat{X}^t(z^t)$ is non-negative, $\hat{L}^t(z^t)\mathbb{1}$ is strictly positive because labor is essential, and $p'(z^t)$ is positive. A necessary condition for $p'(z^t)$ to exist is therefore that $(\hat{X}^t(z^t)^T)^k \to 0$ as $k \to \infty$. This implies that $\hat{X}^t(z^t)^T$ (and therefore also $\hat{X}^t(z^t)$) has modulus strictly less than unity. It is immediate that the inverse $(I - \hat{X}^t(z^t))^{-1}$ exists. (A59) then implies the result.

Throughout the paper we will write $\hat{X}$, $\hat{L}$ for $\hat{X}^t(z^t), \hat{L}^t(z^t)$ when $z^t$ is fixed. We write $\hat{X}$ and $\hat{L}$ for the block-diagonal matrices composed of $\hat{X}^1$ and $\hat{X}^2$, and $\hat{L}^1$ and $\hat{L}^2$ respectively.
We now proceed to establish that the analysis of equilibrium is well posed by providing regularity conditions under which equilibria exist. To this end, we assume basic continuity properties of demand and that household consumption in the first period is bounded away from fully consuming first period income as income grows large.

**Assumption 4.** The primitives satisfy the following properties:

1. The consumption and labor functions \( c_n \) and \( l_1 \) are continuous in \( r^1 \) and \( y^1 \).
2. For all \( n, \varrho, \tau_n, \theta_n \), \( p^1 c_n(\varrho, y^1_n, \tau_n, \theta_n) \) is weakly increasing in \( y^1_n \).
3. For any \( p, \tau, \theta \): there exists some \( \overline{y} \in \mathbb{R}_+ \) and \( \overline{c} < 1 \) such that for all \( n \in N \), \( r^1 \in [r, \overline{r}] \), and \( y^1_n > \overline{y} \), we have that \( p^1 c_n(\varrho, y^1_n, \tau_n, \theta_n) \leq \overline{c} y^1_n \).
4. Interest rates have an upper and lower bound, i.e. \( r^1(Q) \in [r, \overline{r}] \) and \( r \) is differentiable.

This assumption is extremely mild and satisfied by virtually all standard household problems of which we are aware.\(^5\) With this additional structure we are now able to prove the existence of rationing equilibria for the economy under consideration.

**Proposition 6.** Under assumptions 2, 3, and 4, there exists a rationing equilibrium.

**Proof.** Fix all exogenous parameters. Note that by Proposition 5, prices \( p^1 \) and \( p^2 \) are pinned down by technology and so can be taken as given as well.

The outline of the proof is as follows. First, for any interest rate \( r^1 \), we construct a function \( \Psi_{r^1} \) that maps vectors of first-period income to vectors of first-period income and show that any fixed point of this map corresponds to an equilibrium with constant \( r^1 \). Second, we extend this map to construct a second function \( \Psi \) that takes as inputs both a vector of incomes and an interest rate, and we show that any fixed point of this extended map corresponds to an equilibrium of the model. We then apply Brouwer’s fixed point theorem to \( \Psi \) to show that such a fixed point exists.

First, by Assumption 4 we have the following two facts:

1. For any \( p^1, p^2, \tau, \theta \): \( p^1 c_n(\varrho, y^1_n, \tau_n, \theta_n) \) is weakly increasing in \( y^1_n \) for any \( n \), \( r^1 \in [r, \overline{r}] \)
2. For any \( p^1, p^2, \tau, \theta \): there exists some \( \overline{y} \in \mathbb{R}_+ \) and some \( \overline{c} < 1 \) such that \( p^1 c_n(\varrho, y^1_n, \tau_n, \theta_n) \leq \overline{c} y^1_n \) for all \( n \), \( y^1_n > \overline{y} \), \( r^1 \in [r, \overline{r}] \)

Thus, for any vector of incomes \( y^1 \), first period consumption spending \( p^1 C^1 \) is bounded above:

\[
p^1 C^1 \leq \overline{c} y^1 + \overline{c} \overline{y}^T y^1
\]  

\(^5\)It is easy to see how Assumption 4 holds if households are utility maximizers whose utility functions satisfy various standard assumptions. Existence and continuity of the consumption and labor functions follow from continuity and quasiconcavity of utility, and from Berge’s theorem. Satisfying the lifetime budget constraint follows from non-satiation. Consumption being asymptotically bounded away from first-period income follows from sufficiently decreasing marginal utility.
Thus, aggregate spending is bounded above by:

\[
p^1C^1 + p^1G^1 \leq \tau(\mathbb{Y} + 1^T y^1) + \max_{r \in [\underline{r}, \bar{r}]} p^1G^1(p^1, p^2, r^1, \tau, \theta_G) \tag{A62}
\]

where this maximum exists by continuity of \(G^1(\cdot)\) in \(r^1\) and compactness of \([\underline{r}, \bar{r}]\). Since \(\tau < 1\), it follows that there exists \(\mathbb{Y}\) such that if \(y^1 \in Y^1 = \{y^1 \in \mathbb{R}^N_+ \mid 1^T y^1 \leq \mathbb{Y}\}\), then aggregate spending—and so, as all spending flows to wages, also the resulting aggregate income—is weakly less than \(\mathbb{Y}\). Formally:

\[
\forall r^1 \in [\underline{r}, \bar{r}], y^1 \in Y^1 : R^1 \left( \mathbb{L}^1 (1 - \hat{X}^1)^{-1} \left( C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \in Y^1 \tag{A63}
\]

This observation allows us to define, for any \(r^1 \in [\underline{r}, \bar{r}]\), a function \(\Psi_{r^1} : Y^1 \rightarrow Y^1\) given by:

\[
\Psi_{r^1}(y^1) = R^1 \left( \mathbb{L}^1 (1 - \hat{X}^1)^{-1} \left( C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \tag{A64}
\]

where recall \(\varrho\) denotes \((p^1, p^2, r^1)\) and where the previous argument establishes that \(\Psi_{r^1}(y^1)\) is indeed contained in \(Y^1\). Moreover, continuity of \(R^1(\cdot), C^1(\cdot)\) and \(G^1(\cdot)\) establishes that \(\Psi_{r^1}\) is a continuous function.

Second, we define an extended function \(\Psi : Y^1 \times [\underline{r}, \bar{r}] \rightarrow Y^1 \times [\underline{r}, \bar{r}]\) by setting:

\[
\Psi(y^1, r^1) = (\Psi_{r^1}(y^1), r^1(Q)) \tag{A65}
\]

where \(Q = (Q^1, Q^2)\) is given by \(Q^t = (1 - \hat{X}^1)^{-1} \left( C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \) and where \(r^1(\cdot)\) is the monetary policy function, which recall selects an interest rate in \([\underline{r}, \bar{r}]\).

Third, we now claim that \(\Psi\) has a fixed point \((y^1, r^1)\). This follows from Brouwer’s theorem: \(Y^1 \times [\underline{r}, \bar{r}]\) is a compact, convex domain, and \(\Psi\) is continuous because \(R^1(\cdot)\) and \(r^1(\cdot)\) are continuous, \(c_n(\varrho, y^1_n, \tau_n, \theta_n)\) is continuous in \(y^1_n\) and \(r^1\), and \(G^t(\varrho, \tau, \theta_G)\) is continuous in \(r^1\).

Finally, given a fixed point \((y^1, r^1)\) of \(\Psi\), we can construct a rationing equilibrium as follows: Let \(p^t\) be the no-substitution-theorem prices implied by \(z^t\). Let \(c_n^t, l_n^t,\) and \(G^t\) be given by the relevant functions taking in prices \(p^t\), real rate \(r^1\), and incomes \(y^1\). Let production in each period be:

\[
Q^t = (I - \hat{X}^1)^{-1} (G^t + C^t) \tag{A66}
\]

The definition of the consumption, labor supply, and government purchases function ensure that household and government budget constraints hold. The construction of \(Q^t\) ensures that
each goods market clears. Because \((y^1, r^1)\) is a fixed point, first period income is consistent with the rationing function and the first period labor market clears; also because \((y^1, r^1)\) is a fixed point, the interest rate \(r^1 = r^1(Q)\) is consistent with central bank policy. Finally, the second period labor market clears by Walras’ law.

As we have established conditions under which an equilibrium exists, our analysis of equilibria going forward will be well-posed. While the fixed-point theorems we use are familiar, we employ a somewhat different strategy to usual existence proofs in (i) leveraging the structure of no-substitution and (ii) clearing markets intertemporally and then constructing intratemporal market clearing from the resulting fixed point interest rate. This may be useful to other authors proving equilibrium existence in economies with labor rationing.

C.2. Comparative Statics for the Multiplier

We use the structure of the multiplier from Proposition 1 to provide comparative statics of the multiplier in the various objects that contribute towards it. To this end, define the matrix:

\[
\mathcal{M} = C^{y^1}_{y^1} l^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1}
\]

which serves the role of a generalized MPC in our multiplier expression. We first consider the effect of arbitrary changes in this object on the response of value added to an arbitrary shock.

**Proposition 7.** Consider a change in the economy such that \(\mathcal{M}\) is replaced with \(\mathcal{M}' = \mathcal{M} + \varepsilon \mathcal{E}\). The effect on \(dY^1\) of this change is given to first order in \(\varepsilon\) by:

\[
\frac{d}{d\varepsilon} dY^1|_{\varepsilon=0} = (I - \mathcal{M})^{-2} \mathcal{E} \partial Q^1
\]

where recall \(\partial Q^1\) generalizes \(\partial Y^1\) to the case with supply shocks, see Footnote 48.

**Proof.** We start from the multiplier derived in Proposition 1:

\[
dY^1 = \left( I - C^{y^1}_{y^1} l^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \right) \partial Q^1
\]

We we have that:

\[
(I - \mathcal{M}')^{-1} = I + \sum_{n=1}^{\infty} (\mathcal{M} + \varepsilon \mathcal{E})^n = I + \sum_{n=1}^{\infty} \sum_{k=0}^{n} \binom{n}{k} \mathcal{M}^{n-k} (\varepsilon \mathcal{E})^k
\]

\[
= (I - \mathcal{M})^{-1} + (I - \varepsilon \mathcal{E})^{-1} - I + \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} \binom{n}{k} \mathcal{M}^{n-k} (\varepsilon \mathcal{E})^k
\]
The change in the impact is:

\[
\delta \varepsilon (\varepsilon) = \left[ (I - \mathcal{M}^{-1}) - (I - \mathcal{M})^{-1} \right] \partial Q^1 = \left[ \sum_{n=1}^{\infty} \sum_{k=1}^{n} \binom{n}{k} \mathcal{M}^{n-k}(\varepsilon \varepsilon)^k \right] \partial Q^1
\] (A71)

Thus, we have that:

\[
\delta' \varepsilon (\varepsilon) = \frac{\partial}{\partial \varepsilon} \left[ \sum_{n=1}^{\infty} \sum_{k=1}^{n} \binom{n}{k} \mathcal{M}^{n-k}(\varepsilon \varepsilon)^k \right] \partial Q^1 = \left[ \sum_{n=1}^{\infty} \sum_{k=1}^{n} k \varepsilon^{k-1} \binom{n}{k} \mathcal{M}^{n-k} \varepsilon \varepsilon \right] \partial Q^1
\] (A72)

It follows that:

\[
\delta' \varepsilon (0) = 1^T \left[ \sum_{n=1}^{\infty} \binom{n}{1} \mathcal{M}^{n-1} \varepsilon \right] \partial Q^1 = \left[ \sum_{n=1}^{\infty} n \mathcal{M}^{n-1} \varepsilon \right] \partial Q^1 = (1 - \mathcal{M})^{-2} \varepsilon \partial Q^1
\] (A73)

Thus, for changes in network structure given by any \( \mathcal{E} \), we can compute how the multiplier from any shock changes. In particular, suppose that all MPCs in the economy increase proportionately by \( \lambda \). We have that \( \frac{\partial M}{\partial \lambda} |_{\lambda=1} = \mathcal{M} \). Thus, taking \( \mathcal{E} = \mathcal{M} \), we obtain that:

\[
\frac{d}{d\lambda} dY^1 |_{\lambda=1} = (I - \mathcal{M})^{-2} \mathcal{M} \partial Q^1
\] (A74)

Which is of course positive for all positive shocks when all MPCs are positive. Consider now an exercise in which the labor share increases proportionately be \( \lambda \) and all residual income is drawn from an MPC zero household (owners of capital). The effect is given exactly by A74.

While the general formulae above permit exact computation of the effects on the full vector of value added, given the potentially unrestricted network structures that we allow, it is hard to draw qualitative conclusions. For the remainder of this analysis, we report comparative statics of total value added in the empirically relevant case that we have some baseline incidence \( \partial y^* \) around which the bias and homophily adjustments are zero for all possible \( \partial y^1 \). In this case, the proof of Proposition 3 shows that we may write:

\[
\mathbf{1}^T dY^1 \times K + \mathbf{1}^T m \partial y^1
\] (A75)

where \( K \) depends only on the vector of MPCs and \( \partial y^* \). Thus, to compute the impact of various shocks on total value added in the absence of bias and homophily effects, we need
consider only how changes in the economy affect:

\[ \mathbb{1}^T m R_{L_1}^1 \tilde{L}^1 \left( I - \tilde{X}^1 \right)^{-1} \partial Q^1 \]  
\[ (A76) \]

It is then immediate that a rationing matrix that places higher entries on higher MPC households increases the output effect of any uniformly positive or negative shock.

To consider the impact of the input-output network, consider two economies with input output networks \( \tilde{X}^1 \) and \( \tilde{X}^{1'} \), such that \( \tilde{X}^{1'} \geq \tilde{X}^1 \). For both economies to have constant returns to scale, this requires \( \tilde{L}^{1'} \leq \tilde{L}^1 \). The change in the multiplier is then:

\[ \mathbb{1}^T m R_{L_1}^1 \left( \tilde{L}^{1'} \left( I - \tilde{X}^{1'} \right)^{-1} - \tilde{L}^1 \left( I - \tilde{X}^1 \right)^{-1} \right) \partial Q^1 \]  
\[ (A77) \]

Considering each of the unit demand shocks to each industry \( \partial Q_i = \dot{e}_i \), we then obtain that the effect is proportionate to:

\[ \mathbb{1}^T m R_{L_1}^1 \left( \tilde{L}^{1'} \left( I - \tilde{X}^{1'} \right)^{-1} - \tilde{L}^1 \left( I - \tilde{X}^1 \right)^{-1} \right)_{,i} \]  
\[ (A78) \]

### C.3. A Network Interpretation of the Multiplier

The multiplier formula in Proposition 1 that forms the backbone of our analysis in this paper also appears in the regional economics literature on social accounting matrices dating back to Miyazawa (1976). Our result therefore provides the first formal economic analysis that provides a microfoundation for this formula which receives widespread use in the regional economics literature and applied work to compute purchases multipliers (such as the BEA’s RIMS II system). This relationship motivates yet another way to understand the multiplier formula at the zero lower bound. One can think of households as though they are simply additional nodes in the production network, with the restriction that they exchange goods and labor only with firm nodes, and not with other households.

Formally this corresponds to an input-output matrix given by:

\[ \tilde{X}^1 = \begin{bmatrix} \tilde{X}^{I{T}_1} & \tilde{X}^{I{T}_1,N} = C^1_{y^1} \\ \tilde{X}^{N{T}_1} = R_{L_1}^{1} \tilde{L}^1 & 0 \end{bmatrix} \]  
\[ (A79) \]
The multiplier at the zero lower bound can then be expressed as:

\[
\begin{bmatrix}
\frac{dQ^1_{T1}}{dQ^1_N}
\end{bmatrix} = \left( I - \begin{bmatrix}
\hat{X}^1_{T1T1} & \hat{X}^1_{T1N} \\
\hat{X}^1_{NT1} & 0
\end{bmatrix} \right)^{-1} \begin{bmatrix}
\hat{Q}^1_{T1} \\
0
\end{bmatrix}
\]

\[
= \left( I - \hat{X}^1_{T1T1} - \hat{X}^1_{T1N} \hat{X}^1_{NT1} \right)^{-1} \begin{bmatrix}
\hat{Q}^1_{T1} \\
0
\end{bmatrix}
\]  

(A80)

where recall \( \hat{Q}^1 \) generalizes \( \hat{Y}^1 \) to the case with supply shocks, see Footnote 48.

One sees immediately that this recovers our generalized Keynesian cross of Proposition 1. We can therefore think of households as firms who, in order to supply labor, demand a consumption bundle as inputs. On top of the assumption that households only interact through firms, this representation also relies on the assumption that households do not choose their labor supply in the first period; this makes them analogous to firms, who must meet market demand.

C.4. Network Decompositions for Supply Shocks

We now derive network decompositions of the multiplier as in Section 3.2 that are valid for both demand and supply shocks, extending the earlier analysis. To this end, we see that changes in GDP when we consider a supply shock have two distinct components:

\[
d(GDP) \equiv d(p^{IT}Y^1) = \underbrace{p^{IT}dY^1}_{\text{Change in Product}} + \underbrace{dp^{IT}Y^1}_{\text{Change in Price Index}}
\]  

(A81)

Where it is without loss to redefine units of consumption goods and evaluate at an initial equilibrium with \( p^{IT} = \mathbb{1} \). Propositions 2 and 9 decompose the first term \( \mathbb{1}^{T}dY^1 \). To achieve our decomposition for supply shocks, we therefore need only compute \( dp^{IT}Y^1 \). To this end, we can employ Corollary 2, which shows prices are a closed-form function of \( z \):

\[
p^1(z) = (1 - \hat{X}^1(z)^T)^{-1} \hat{L}^1(z) \mathbb{1}
\]  

(A82)

It follows that the change in GDP can then be decomposed as before but with a new term which depends only on the IO matrix and labor shares and not labor rationing or household consumption. This is stated formally below:

**Proposition 8.** The total change in first-period GDP due to a shock with unit-magnitude labor income incidence \( \hat{y}^1 \) can be approximated as:
\[
\begin{align*}
d(p^1 Y^1) = & \frac{1}{1 - \mathbb{E}_g [m_n]} \left( 1 + \mathbb{E}_{\tilde{y}^1} [m_n] - \mathbb{E}_g [m_n] + \mathbb{E}_{\tilde{y}^1} [m_n] (\mathbb{E}_{\tilde{y}^1} [m_{\text{next}}] - \mathbb{E}_g [m_n]) \right) \\
& + \text{Cov}_{y^1} [m_n, m_{\text{next}}] \\
& + d \left[ \left( 1 - \hat{X}^1 (z)^T \right) \hat{L}^1 (z) \mathbb{I} \right]^T Y^1 + O^3(|m|)
\end{align*}
\]

(A83)

where \( y^* \) is any reference income weighting of unit-magnitude and \( m_{\text{next}} \) is the average MPC of households who receive as income \( i \)'s marginal dollar of spending.

Proof. Recall that we have:

\[
d(p^1 Y^1) = \frac{p^1 \text{d} Y^1}{\text{Change in Product}} + \frac{d p^1 Y^1}{\text{Change in Price Index}}
\]

which we can always take as \( d(p^1 Y^1) = \mathbb{I}^T d Y^1 + dp^1 Y^1 \) through an appropriate renormalization of the initial units of the goods.

By Proposition 2, \( 1^T d Y^1 \) consists of all terms in (A83) except the price effect. We now need only compute the term \( dp^1 Y^1 \). To this end, from Corollary 2 we have that:

\[
p^1 (z) = \left( 1 - \hat{X}^1 (z)^T \right)^{-1} \hat{L}^1 (z) \mathbb{I} \implies dp^1 Y^1 = d \left[ \left( 1 - \hat{X}^1 (z)^T \right)^{-1} \hat{L}^1 (z) \mathbb{I} \right]^T Y^1
\]

(A85)

Adding the two terms yields the claimed expression and completes the proof.

\[\Box\]

C.5. Special Cases Where Network Effects in Propagation Vanish

In the main text, we briefly discussed two important cases where network effects in shock propagation vanish. Here, we state and more formally discuss these results.

Proposition 9. The following statements are true:

1. (No incidence or bias effects) Suppose that consumption preferences and labor rationing are homothetic, that no households are net borrowers in period 1, and that there are no government purchases.\(^{52}\) Then, for a GDP-proportional, unit-magnitude demand shock, the incidence and bias effects are zero, so that we have:

\[
\mathbb{I}^T d Y^1 = \frac{1}{1 - \mathbb{E}_g [m_n]} \left( 1 + \text{Cov}_{y^1} [m_n, m_{\text{next}}] \right) + O^3(|m|)
\]

(A86)

\(^{52}\)By homothetic labor rationing, we mean that marginal and average rationing of income are equal. Formally, if we let \( L^1 = \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} Y^1 \) be the vector of first-period firm-level labor bills, then we require that \( y^1 = R_L^1 L^1 \).
where $y^1$ is the vector of first-period incomes.

2. (No incidence, bias, or homophily effects) Suppose that all industries have a common rationing-weighted average MPC, $m$.\footnote{Formally, $\sum_{n \in N} (R^1_{L, n})_n m_n = m$ for all $i \in I^1$.} Then the incidence, bias, and homophily effects are zero, so that for any reference weighting $y^*$ that can be induced by a demand shock, the change in GDP corresponding to any unit-magnitude demand shock is:\footnote{Formally, saying that $y^*$ can be induced by a demand shock says that there exists a $\partial Q^*$ such that:}

$$1^T dY^1 = \frac{1}{1 - E_{y^*}[m_n]} = \frac{1}{1 - m} \quad (A88)$$

Proof. We prove the two claims separately:

1. Recalling that $(m^\text{next})^T = m^T G$, and the shock satisfies $\partial y^1 \times y^1$, the following are equivalent:

$$m^T G y^1 - m^T y^1 = 0 \iff E_{\partial y^1}[m^\text{next}_n - m_n] = 0 \quad (A89)$$

It therefore suffices to show that $G y^1 = y^1$.

Plugging in the definition of $G$, we have $G y^1 = R^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} C^1_y y^1$. Since each household saves zero on net, $y^1$ is equal to total spending. Homotheticity of consumption implies that $C^1_y y^1$, then, is the vector of total consumption of goods; since there are no government purchases, this equals aggregate output net of inputs, i.e. $Y^1$. Finally, homotheticity of rationing implies that $R^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} Y^1 = y^1$.

2. Recall by Proposition 1 that when either $C_{r_1} + G_{r_1} = 0$ or $r^{1}_{Q_1} = 0$, the general equilibrium effect on income of a partial equilibrium shock is given by:

$$dY^1 = \left( I - C^1_y R^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \right)^{-1} \partial Y^1 \quad (A90)$$

We wish to investigate whether there exists some $m \in (0, 1)$ such that the following holds for all $\partial Y$:

$$1^T dY^1 = \frac{1}{1 - m} 1^T \partial Y^1 \quad (A91)$$

First, we note a simple fact of linear algebra. Suppose an invertible matrix $M$ has columns summing to some constant $m$. This is equivalent to:

$$1^T M v = m 1^T v, \quad \forall v \quad (A92)$$
It is then true that for any $v$:

$$m\mathbb{1}^T(M^{-1}v) = \mathbb{1}^TM(M^{-1}v) = \mathbb{1}^Tv$$  \hspace{1cm} (A93)

Thus, $M^{-1}$ has columns summing to $\frac{1}{m}$.

Second, note that the desired result (A91) holds if and only if

$$\left( I - C^1_{y^1}R^1_{L^1}\hat{L}^1(1 - \hat{X}^1)^{-1} \right)^{-1}$$  \hspace{1cm} (A94)

sums to $\frac{1}{1-m}$. This is equivalent, by the first observation, to the fact that each column of

$$C^1_{y^1}R^1_{L^1}\hat{L}^1(1 - \hat{X}^1)^{-1}$$  \hspace{1cm} (A95)

sums to $m$.

It remains to show that this claim is equivalent to the condition provided in the statement of the Proposition. Namely, we must show that

$$\mathbb{1}^TC^1_{y^1}R^1_{L^1}\hat{L}^1(1 - \hat{X}^1)^{-1} = m\mathbb{1}^T \iff \mathbb{1}^TC^1_{y^1}R^1_{L^1} = m\mathbb{1}^T$$  \hspace{1cm} (A96)

Multiplying each side by $(I - \hat{X}^1)(\hat{L}^1)^{-1}$—which exists since labor is essential in production—reveals that (A96) holds if $(I - \hat{X}^1)(\hat{L}^1)^{-1}$ has columns summing to one. By our earlier linear algebra observation, this holds if and only if $\hat{L}^1(1 - \hat{X}^1)^{-1}$ has columns summing to one. This can be seen by recalling the no-profit condition

$$p^1 = (I - (\hat{X}^1)^T)^{-1}\hat{L}^1\mathbb{1},$$  \hspace{1cm} (A97)

using our normalization $p = 1$, and taking the transpose of both sides.

The first part of the proposition shows how, even in a “homothetic economy,” heterogeneity in household consumption baskets and sectoral employment can generate network effects through homophily. This happens even at the same time as homotheticity eliminates the bias effect by ensuring that each household’s marginal consumption is proportional to its initial consumption, so that the income-weighted average of marginal consumption is proportional to the GDP vector. Still, when households with different MPCs direct their spending toward different goods, the households employed to produce the goods consumed by higher-MPC households experience a greater change in income – not from the initial, uniform shock, but from the economy’s response to it. Insofar as these households have different MPCs from
the average, homophily is still possible. This mechanism generates non-neutrality for the multiplier, even if the economy and the shock considered are “neutral” in all other aspects. Aggregate neutrality requires (to second order in MPCs) that the economy feature exactly zero correlation between households’ MPCs and the MPCs of the households they spend on.

The second part of the proposition imposes that each firm’s marginal employees have the same average MPC as one another. This eliminates the incidence, bias, and homophily effects, leaving only the classical Keynesian multiplier. That is, wherever in the economy a shock strikes, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $R_{1L}^1 = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm’s marginal employees have the population average MPC.

C.6. Optimal Policy at a Global Optimum

In the main text, we focused primarily on small changes in welfare corresponding to small changes in policy and without instrumental value of government purchases. In this section, we specialize to the case of small changes in policy at an optimum where households may also value government purchases. To do so, we consider the following planner’s problem: The full version of the planner’s problem, Equation 24, is

$$
\max_{\{c^t_n, t_n, Q^t_l, G^t_i, \tau^t_i\}_{t \in \{1, 2\}, n \in N, i \in I}} W \equiv \sum_{n \in N} \mu_n \lambda_n \sum_{t = 1, 2} \beta_n^{t-1} \left[ u^t_n(\tilde{c}^1) - u^t_n(\tilde{r}) + w^t_n(G^t) \right]
$$

s.t. $(c^1_n, c^2_n, l^2_n)$ solves Equation 23 given $l^1_n$

$$
Q^t = \mu^T c^t + \tilde{X}^t(z^t)Q^t + G^t
$$

$$
\mu^{l^1} = R^1(\tilde{L}^1Q^1), \mu^{l^2} = I^T \tilde{L}^2(z^t)Q^2
$$

$$
1 = p^t = \left( I - \tilde{X}^t(z^t) \right)^{-1} \tilde{L}^t(z^t)1
$$

$$
1^TG^1 + \frac{1^TG^2}{1 + r^1} + \mu^T \tau^1 + \frac{\mu^T \tau^2}{1 + r^1} = 0
$$

(A98)

Our first result decomposes the first-order condition for optimal government purchases and transfers into five distinct mechanisms. This is closely related to Proposition 3 in the main text, which considers the change in welfare away from the global optimum.

**Proposition 10.** Suppose taxes $\tau^{1*}, \tau^{2*}$ and purchases $G^{1*}, G^{2*}$ solve the planner’s problem.
Now consider a change in policy \( \tau^t = \tau^t* + \varepsilon \tau^t_\varepsilon \), \( G^t = G^t* + \varepsilon G^t_\varepsilon \), indexed by \( \varepsilon \). The following first-order condition holds:

\[
0 = \left( \hat{\lambda}^T \mu WTP^1 - (\gamma^T + \hat{\lambda}^T \Delta \Gamma^1) \right) G^t_\varepsilon + \left( \hat{\lambda}^T \mu (I - \hat{\phi}) WTP^2 - \gamma^T \right) G^2_\varepsilon \quad \frac{1}{1 + r^1}
\]

\[
- (\hat{\lambda} - \gamma^T) \mu \left( \tau^t_\varepsilon + \frac{\tau^2_\varepsilon}{1 + r^1} \right) + \hat{\lambda}^T \hat{\phi} \mu \tau^t_\varepsilon \frac{1}{1 + r^1}
\]

\[
- \hat{\lambda}^T \Delta \Gamma^1 \left( I - C^1_{y^1} \Gamma^1 \right)^{-1} C^1_{y^1} \left( \Gamma^1 G^t_\varepsilon - \mu \tau^t_\varepsilon - \frac{1}{1 + r^1} \right)
\]

\[
\text{Keynesian stimulus (alleviation of involuntary unemployment)}
\]

where \( \gamma \) is the marginal value of public funds and \( \Gamma^1 \equiv R^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \).

**Proof.** The planner takes prices and—locally—the interest rate as given. Goods and labor market clearing and first-period rationing determine the change in first-period employment as a function of \( G^t_\varepsilon \) and \( \tau^t_\varepsilon \). We are left with the following first-order condition:

\[
0 = dW + \gamma \left[ \mu^T \tau^t_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1 + r^1} - \mu^T G^t_\varepsilon - \frac{1}{1 + r^1} \right] \quad \text{(A100)}
\]

where \( dW \) is as in Equation 25. This gives an expression for the change in welfare in terms of \( \tau^t_\varepsilon \), \( G^t_\varepsilon \), and \( l^1_\varepsilon \), the change in first-period employment. By Equation 11, \( \mu_\varepsilon^t = \Gamma^1 \left( I - C^1_{y^1} \Gamma^1 \right)^{-1} \partial Y^1 \), where \( \Gamma^1 \equiv R^1_L \hat{L}^1 \left( I - \hat{X}^1 \right)^{-1} \) and \( \partial Y^1 = G^t_\varepsilon - C^1_{y^2} \mu_\varepsilon^t - C^1_{y^2} \mu_\varepsilon^t \). For borrowing-constrained households, \( C^1_{y^2} = 0 \); they would already like to substitute additional consumption toward the first period but are constrained not to do so. Other households are Ricardian, implying \( C^1_{y^2} = \frac{C^1_{y^2}}{1 + r^1} \). Plugging in for \( dW \), and using matrix notation, we have

\[
0 = \hat{\lambda}^T \left[ - \Delta \Gamma^1 (I - C^1_{y^1} \Gamma^1)^{-1} \left( G^t_\varepsilon - C^1_{y^1} \mu \left( \tau^t_\varepsilon + \frac{1}{1 + r^1} \right) \right) - \left( \mu \tau^t_\varepsilon + \frac{\mu^T \mu \tau^2_\varepsilon}{1 + r^1} \right) \right] + \gamma \left[ \mu^T \tau^t_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1 + r^1} - \mu^T G^t_\varepsilon - \frac{1}{1 + r^1} \right] \quad \text{(A101)}
\]
Now, observe that the term on the first line can be rewritten:

\[
\Gamma^1(I - C^1_y \Gamma^1)^{-1} \left( G^1 - C^1_y \mu \left( \tau^1 + \frac{1_{\phi_n = 0} \tau^2}{1 + r^1} \right) \right) = \Gamma^1 \left( \sum_{k=0}^{\infty} (C^1_y \Gamma^1)^k \right) \left( G^1 - C^1_y \mu \left( \tau^1 + \frac{1_{\phi_n = 0} \tau^2}{1 + r^1} \right) \right)
\]

\[
= \left( \Gamma^1 G^1 + \left( \sum_{k=0}^{\infty} (C^1_y \Gamma^1)^k \right) C^1_y \Gamma^1 G^1 \right) - \Gamma^1 \left( \sum_{k=0}^{\infty} (C^1_y \Gamma^1)^k \right) C^1_y \mu \left( \tau^1 + \frac{1_{\phi_n = 0} \tau^2}{1 + r^1} \right)
\]

\[
= \Gamma^1 G^1 + \Gamma^1 (I - C^1_y \Gamma^1)^{-1} C^1_y \left( \Gamma^1 G^1 - \mu \tau^1 - \mu \frac{1_{\phi_n = 0} \tau^2}{1 + r^1} \right)
\]

(A102)

Substituting this back in and rearranging, we obtain Equation A99.

The opportunistic government purchases term is as in Werning (2011) and Baqaee (2015). It augments the standard first-order condition for government purchases with a labor-wedge term, reflecting the fact that the social cost of additional government purchases is lower than the market cost when they are produced using underemployed labor. The second term is also an augmented version of the standard expression for government purchases—this time in the second period. The borrowing wedge reflects the fact that households with binding borrowing constraints implicitly discount the future at a higher-than-market rate; the planner must account for this when deciding whether to make purchases on their behalf.

The third term of Equation A99 is a standard, pure redistribution term, weighing the private benefits of transfers against the social cost (the MVPF). The fourth term augments this, when there are borrowing constraints. In particular, taxes in the second period are less costly to borrowing-constrained households since they discount the future more heavily than the market rate indicates.

Finally, the last line captures the value of stimulus brought on by changes in income—those corresponding to pure income transfers via taxes and labor market income earned by employees producing government purchases.\(^{55}\)

### C.7. Evaluating Optimality of Fiscal Policy

We now use Proposition 10 to more fully characterize optimal fiscal policies in two benchmark cases where the planner’s indifference between transfers to each household and/or purchases in each sector leads to optimality conditions that can be evaluated without knowledge of the rich interconnections between households.

**Proposition 11.** The following two statements are true:

\(^{55}\)If second period purchases are held constant, then the net income transfer is zero, i.e. this term operates solely through redistribution to different households (who may spend differently).
1. (Optimal transfer policy) Suppose that the marginal social dis-utility of labor supply is constant across all households rationed to on the margin at the optimum. Then $dW = 0$ with respect to marginal changes in first-period transfers if and only if, for all $n \in N$, 

$$
\gamma = \tilde{\lambda}_n \left( 1 + \frac{m_n}{1 - m_n}(-\Delta_n) \right)
$$

(A103)

where $\gamma$ is the marginal value of public funds.

2. (Optimal purchases policy) Suppose that the social gains from first-period government purchases are equal to some $\tilde{\nu}$ across goods and constraints bounding purchases above zero do not bind. Then $dW = 0$ with respect to marginal changes in first-period purchases if and only if, for all $i \in I^1$,

$$
\gamma = \bar{\nu} + \frac{1}{1 - \tilde{m}_i} \left( -\hat{\lambda}\Delta_i \right)
$$

(A104)

where $\tilde{m}_i$ is the rationing-weighted average MPC in the production of good $i$ and $\hat{\lambda}\Delta_i$ is the rationing-and-welfare-weighted average rationing wedge in the production of good $i$.57

Proof. The proof of this result relies on material in Appendix C.6 on characterizing optimal fiscal policy; consult this section and the results therein before proceeding with this proof.

We first prove the result for first-period transfers. At any optimum, we know that Equation A99 must hold for all policy variations $\tau^1_\varepsilon \in \mathbb{R}^N$ that only vary first-period transfers, keeping other instruments fixed. Taking $\tau^1_\varepsilon = e_n$, the $n$th basis vector, we see that:

$$
\left( \tilde{\lambda}^T - \gamma \mathbb{1} \right)^T_n = \left( \tilde{\lambda}^T \Delta \Gamma^1 \left( I - C^1_{y^1} \Gamma^1 \right)^{-1} C^1_{y^1} \right)_n
$$

(A105)

Stacking these equations over $n$, we obtain:

$$
\left( \tilde{\lambda} - \gamma \mathbb{1} \right)^T = \tilde{\lambda}^T \Delta \Gamma^1 \left( I - C^1_{y^1} \Gamma^1 \right)^{-1} C^1_{y^1}
$$

(A106)

Since $\{e_n\}$ is a basis and Equation A99 is linear, this equation fully encompasses the optimality condition of Proposition 10 with respect to first period transfers.

56Formally, if $\Gamma^1 C^1_{y^1} \downarrow \left[ n, 0 \right] \neq \emptyset$ then $\tilde{\lambda}_n (1 + \Delta_n) = \text{const}$, where $\Gamma^1 \equiv \left[ l^1 \mathbf{1} \right] \left( I - \hat{X}^1 \right)^{-1}$.

57Formally, $\tilde{m}_i \equiv (m^T \Gamma^1)_i$ and $\hat{\lambda}\Delta_i \equiv \left( \tilde{\lambda}^T \Delta \Gamma^1 \right)_i$. 

29
We can simplify this system of equations. First, see that:

$$\Gamma^1(I - C_{y1}^1 \Gamma^1)^{-1} C_{y1}^1 = \sum_{k=0}^{\infty} \Gamma^1(C_{y1}^1 \Gamma^1)^k C_{y1}^1 = \sum_{k=1}^{\infty} \Gamma^1 C_{y1}^1 \quad (A107)$$

Adding $\tilde{\lambda}^T \Delta$ to both sides of Equation A106, we therefore obtain:

$$\left(\tilde{\lambda}(1 + \Delta) - \gamma \mathbb{1}\right)^T = \tilde{\lambda}^T \Delta \left(I - \Gamma^1 C_{y1}^1\right)^{-1} \implies \left(\tilde{\lambda}(1 + \Delta) - \gamma \mathbb{1}\right)^T \left(I - \Gamma^1 C_{y1}^1\right) = \tilde{\lambda}^T \Delta$$

(A108)

Now, express $\Gamma^1 C_{y1}^1 = \Gamma^1 \bar{C}_{y1}^1 m$. Recognizing that all columns of the spending-to-income matrix $\Gamma^1 \bar{C}_{y1}^1$ sum to one, as total spending is equal to total factor income, and—by assumption—that $\tilde{\lambda}_n(1 + \Delta_n)$ is constant across all households $n$ except for those for which the $n^{th}$ row of $\Gamma^1 C_{y1}^1$ is zero, (A108) can be rewritten as:

$$\left(\tilde{\lambda}(1 + \Delta) - \gamma \mathbb{1}\right)^T (I - m) = \tilde{\lambda}^T \Delta$$

(A109)

We therefore have all, for all $n$, that

$$\tilde{\lambda}_n(1 + \Delta_n) - \gamma = \frac{1}{1 - m_n} \tilde{\lambda}_n \implies \gamma = \tilde{\lambda}_n \left(1 + \frac{m_n}{1 + m_n}(-\Delta_n)\right) \quad (A110)$$

We prove the result for first-period government purchases in an analogous way. To begin, consider Equation A99 for policy variations $G_{\varepsilon}^1 \in \mathbb{R}^{T_1}$ that only vary first period purchases. Again considering each basis vector of $\mathbb{R}^{T_1}$ and stacking we obtain:

$$0 = \tilde{\lambda}^T WTP^1 - (\gamma \mathbb{1}^T + \tilde{\lambda}^T \Delta \Gamma^1) = \tilde{\lambda}^T \Delta \Gamma^1 (I - C_{y1}^1 \Gamma^1)^{-1} C_{y1}^1 \Gamma^1 \quad (A111)$$

This can be rewritten as:

$$\tilde{\lambda}^T WTP^1 - \gamma \mathbb{1}^T = \tilde{\lambda}^T \Delta \Gamma^1 (I - C_{y1}^1 \Gamma^1)^{-1} \quad (A112)$$

From the assumption that the social gains from government purchases equal $\tilde{v}$, we have that $\tilde{\lambda}^T WTP^1 = \tilde{v}$. Moreover, by definition $\tilde{\lambda} \Delta^T = \tilde{\lambda}^T \Delta \Gamma^1$. Hence (A112) can be rewritten as:

$$\tilde{v} \mathbb{1}^T - \gamma \mathbb{1}^T = \tilde{\lambda} \Delta^T \left(I - C_{y1}^1 \Gamma^1\right)^{-1} \quad (A113)$$

Next, define $\tilde{m}_i \equiv (m^T \Gamma^1)_i$ to be the rationing-weighted average MPC in the production of good $i$ and let $\tilde{m}$ be the corresponding matrix with $\tilde{m}_i$ on the diagonal. Moreover, define
\[ C_{ji} \equiv (C_{y,1}^{1}, \Gamma^{1})_{ji}/\tilde{m}_i \] to be the average direction of consumption of workers producing \( i \), weighted by their MPC and marginal rationing in \( i \)'s production.\(^{58}\) Crucially, note that \( C_{\tilde{m}}^{\tilde{m}} = C_{y,1}^{1, \Gamma^{1}} \) by construction and that \( \mathbb{1}^T C_{\tilde{m}}^{\tilde{m}} = \mathbb{1}^T \tilde{m} \):

\[
\mathbb{1}^T C_{\tilde{m}}^{\tilde{m}} = \mathbb{1}^T C_{y,1}^{1, \Gamma^{1}} = m^T \Gamma^{1} = \tilde{m}^T \tag{A114}
\]

The first order condition for purchases (A113) is therefore equivalent to:

\[
(\tilde{v} - \gamma)\mathbb{1}^T (I - C_{y,1}^{1, \Gamma^{1}}) = (\tilde{v} - \gamma)\mathbb{1}^T (I - \tilde{m}) = \tilde{\lambda} \Delta^{T} \iff \gamma = \tilde{v} + \frac{1}{1 - \tilde{m}_i} (-\tilde{\lambda} \Delta_i) \quad \forall i \in \mathcal{I}^{1}. \tag{A115}
\]

Proposition 11 says that the planner may verify whether the current policy is optimal despite having very partial knowledge about the economy. In the transfer case, the planner only needs information on household-level welfare weights (\( \tilde{\lambda}_n \)), rationing wedges (\( \Delta_n \)), and MPCs (\( m_n \)) — not the network of marginal spending flows between households. In the purchases case, the planner needs to know the average MPC and welfare-weighted rationing wedge by industry; these require knowledge of the rationing function linking output to incomes, but not the directed consumption matrix.

The main idea underlying Proposition 11 is that—at an optimum—the social value of additional spending by any household is independent of how that spending is directed. This is clearest in the case of transfers: For any household employed in order to produce marginally-demanded goods, the social value of their employment is equal to the value of a transfer to that household, less the dis-utility of labor. Since (by assumption) the dis-utility of labor is constant across households, and since—at an optimum—the value of transfers must also be constant across households, it follows that the social value of additional employment is constant across households. Since the planner is indifferent over the direction of household spending, she targets solely based on the magnitude of that spending—i.e. household MPCs—as well as household welfare weights. A similar argument applies in the case of government purchases.

\(^{58}\)For any \( i \) with \( \tilde{m}_i = 0 \), define \( C_{ji} \) in any way satisfying \( \sum_j C_{ji} = 1 \).
D. Model Extensions and Results

In this appendix, we extend the baseline model to allow for imperfect competition with fixed markups (D.1), consider policy in this environment (D.2), and extend the baseline model to many periods (allowing for an infinite horizon) (D.3).

D.1. Imperfect Competition

In this section we show how to incorporate imperfect competition in the form of fixed markups on marginal costs. Now, instead of each sector being populated by a continuum of perfectly competitive firms, we suppose that for all \( i \in I \) there is a single monopolist producing each good, charging a fixed markup of \( m_t^i \) over their marginal cost and making (and distributing) profits \( \pi_t^i \).\(^{59}\) Despite this, we argue that a no substitution theorem still holds and we can obtain analogous multiplier formulae once we augment labor income rationing with profit rationing. To do this, we have to slightly modify Assumption 2:

**Assumption 5.** For each \( t \) there exists some \( \bar{p}^t \in \mathbb{R}^{1+I} \) and \( \{X_t^i, L_t^i\}_{i \in I} \) such that for all \( i \),
\[
F_t^i(X_t^i, L_t^i, z_t^i) \geq 1 \quad \text{and} \quad (1 + m_t^i)(\bar{p}^t X_t^i + L_t^i) \leq \bar{p}^t
\]

Under this modified assumption, we can state and prove the modified no-substitution theorem with markups:

**Proposition 12.** Under Assumptions 5 and 3, for a given \( z^t \) and \( m^t \), there exists a unique \( p^t \) consistent with both flexible-wage and rationing equilibrium, independent of demand.

**Proof.** We modify the proof of proposition 5 to accommodate markups. Each firm \( i \) in period \( t \) now sets a price \( p_t^i = (1 + m_t^i)\kappa_t^i(p^t) \), where \( \kappa_t^i \) is \( i \)'s unit cost function in period \( t \). That is, \( i \) prices goods as though it were a competitive firm with production function \( \frac{1}{1+m_t^i}F_t^i(X_t^i, L_t^i, z_t^i) \). Consider now a modified economy without markups and production functions given by the previously-stated markup-adjusted production functions. Assumption 5 implies that Assumption 2 holds in this modified economy. The result then follows by application of Proposition 5. \( \square \)

We assume that profits from each firm are distributed to households according to an exogenous profit rationing function \( \Pi^t : \mathbb{R}^I \to \mathbb{R}^N \) satisfying \( \sum_{i \in I} \pi_t^i = \sum_{n \in N} \Pi^t(\pi^t)_n \) for all \( \pi^t \in \mathbb{R}^I \). We let \( d_n^t = \Pi^t(\pi^t)_n \) represent household \( n \)'s total dividend income in period \( t \). With profits, household income is comprised of rationed first-period labor income, chosen

\(^{59}\)One microfoundation for constant markups is that industries are comprised of a continuum of firms, with each other firm’s and household’s demands having the same CES aggregator for these firms’ varieties.
second-period labor income, and (not chosen) dividend income in both periods. We therefore allow household consumption and labor supply functions to depend on $d\theta_n$ directly.

We can now state a profit-inclusive Keynesian cross. Note that the only difference to Proposition 4 comes from the need to account for changes in profits, how these are distributed to households as dividends, and their directed MPCs out of dividends.\footnote{For the sake of generality, we distinguish between aggregate MPC out of dividend and labor income, i.e. $C_d' \neq C_y'$. Of course, for utility-maximizing households, these will be the same provided the income arrives in the same period.}

**Proposition 13.** For any small shock to parameters there exist a pair of rationing equilibria production $Q$ and $Q + dQ$ before and after the shock. If the shock induces a partial equilibrium change in production $\partial Q$,\footnote{Recall $\partial Q^1$ generalizes $\partial Y^1$ to the case with supply shocks, see Footnote 48.} the general equilibrium change $dQ$ is given to first order by:

$$

\frac{dQ}{\partial \omega} = \hat{X}dQ + (C_r + G_r)r_QdQ + C_y R^1_L \hat{L}^1dQ^1 + C_\pi \hat{\Pi}dQ + \partial Q

$$

where $C_\pi$ is the matrix of household directed MPCs out of profit income, $\hat{\Pi}$ is the block diagonal matrix composed of $\hat{\Pi}^1$ and $\hat{\Pi}^2$, and where $\hat{\Pi}^t$ is the diagonal matrix with $i^{th}$ entry $m^t_i p_i$, and all quantities are evaluated at the initial equilibrium.

**Proof.** This proof simply modifies the proof of Proposition 4. It is stated in full for clarity. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 6, we know that for each $\omega_n$ there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit $\omega$. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Toward a contradiction, suppose that the set of equilibria is not UHC in the parameters, i.e. $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at $e$: household budget balance, government budget balance or market clearing. But by Assumption 4, continuity of the fiscal rule, continuity of the interest rate rule, continuity of the rationing function and continuity of the profit allocation function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contradiction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$

dr^1 = r^1_{Q_1}dQ^1 + r^1_{Q_2}dQ^2 = r^1_{Q}dQ

$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market...
equilibrium conditions:

\[ dQ = \hat{X}dQ + \hat{X}_zdzQ + C_p p_dz + C_{y^1}d\tau^1 + C_{\tau}d\theta + C_{\theta}d\theta^1 \\
+ G_p p_dz + G_{\tau}d\tau^1 + G_{\theta}d\theta^1 + C_{\pi}\hat{\Pi}dQ \]

Plugging in for \( dr^1 \) and \( dy^1 = R^1_{L^1} \hat{L}^1 dQ^1 + R^1_{L^1} d\hat{L}^1 Q^1 \)

\[ dQ = \hat{X}dQ + C_{y^1} R^1_{L^1} \hat{L}^1 dQ^1 + (C_{\tau^1} + G_{\tau^1}) r^1_Q dQ + C_{\pi}\hat{\Pi}dQ + \hat{\pi}Q \]

where here \( \hat{\pi}Q = (C_p + G_p)p_dz + \hat{X}_zdzQ + C_{y^1} R^1_{L^1} \hat{L}^1 dzQ^1 + (C_{\tau} + G_{\tau})d\tau + C_{\theta}d\theta + G_{\theta^1}d\theta^1 \).

\[ \square \]

**D.2. Optimal policy with imperfect competition**

In this section, we extend the optimal policy results of section 6 to the more general environment with constant, non-zero markups. As is section 6 we normalize prices \( p^1_t \) to one throughout, without loss of generality.

To highlight as clearly as possible the parallels to the case without profits, we make two important assumptions. First—although in the first period, profit-creation is uninternalized by households—we assume that the government incentivizes second-period profit-creation with Pigouvian subsidies funded lump-sum by shareholders.

**Assumption 6.** There is an ad-valorem subsidy \( s^2_i \) on the purchase of \( i \) (for consumption or production), set equal to the profit rate \( m^2_i \). It is funded directly by an additional lump-sum, second-period tax \( \hat{\tau}^2_n \) defined by \( \mu_n \hat{\tau}^2_n = \sum_{i \in I} \left( \frac{\hat{\Pi}^2_{n_i}}{\sum_{n' \in N} \hat{\Pi}^2_{n_i}} \right) s^2_i Q^2_i \).

Second, we assume that the MPC out of future profits is zero. This is a rather weak assumption, as the MPC out of even current capital income is small empirically.

**Assumption 7.** For all households \( n \), \( C^1_{y^2} = 0 \).

**D.2.1. Planner’s problem**

We begin by defining the household’s problem. It is the same as Equation 23 in section ??, except that households now also receive profit income, so that the budget constraint becomes \( \bar{l}^1 + \pi^1_n - p^1 \cdot \hat{c}^1 - \tau^1_n \geq \underline{s}^1_n \). Note that this microfoundation implies \( C_y = C_{\pi} \). That is, additional income from rationed labor has the same effects on consumption as additional income from profits.
As in section 6, we study the policy problem of a planner at the zero lower bound. Formally, the planner’s problem is the same as in Equation 24 except that household behavior solves Equation 23 with the profit-inclusive budget constraint and aggregate variables evolve according to Equation A116 with \( r_Q = 0 \).

D.2.2. Policy changes away from the optimum

This section considers changes in welfare due to small changes in not-necessarily-optimal policies, as in section ???. The only difference now is the presence of profits.

With this setup in mind, we now consider the change in welfare induced by changes in transfers and government purchases, analogously to Proposition 3.

**Lemma 3.** Under assumptions 6 and 7, the change in welfare \( dW \) due to a small change in taxes and government purchases—at a constant interest rate—can be expressed as:

\[
dW = \sum_{n \in N} \tilde{x}_n \mu_n \left[ -\Delta_n dl_n^1 + d\pi_n^1 - \left( d\tau_n^1 + (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) \right] + \left( WTP_n^1 dG^1 + (1 - \phi_n) \frac{WTP_n^2}{1 + r^1} dG^2 \right)
\]

where \( \tilde{x}_n \) is the value the planner places on the marginal transfer of first-period wealth to a household of type \( n \), \( \Delta_n \) and \( \phi_n \) are \( n \)'s implicit first-period labor wedge and borrowing wedge, and \( WTP_n^i \) is the vector of \( n \)'s marginal willingness to pay for period \( t \) government purchases on each good, in period \( t \) dollars. The changes in first-period employment and profits are in turn given by

\[
\mu dl^1 = R^1_L \hat{L} \left( 1 - \hat{X} \right)^{-1} dY^1, \quad \mu d\pi^1 = \hat{\Pi} \left( 1 - \hat{X} \right)^{-1} dY^1,
\]

\[
dY^1 = \left( I - C^1_y \left( R^1_L \hat{L} + \hat{\Pi} \right) \left( I - \hat{X} \right)^{-1} \right)^{-1} \hat{\Pi} \hat{\Pi} \hat{\Pi} d\tilde{G}^1
\]

**Proof.** We follow the same steps as the proof of Proposition 3 (see Appendix A.3) up to the substitution of the budget constraint, which now includes profits. With profits, differentiating the household’s lifetime budget constraint (at constant \( r^1 \)) gives:

\[
p^1 dc_n^1 - dl_n^1 - d\pi_n^1 + \frac{p^1 dc_n^2 - dl_n^2}{1 + r^1} = -d\tau_n^1 + \frac{d\pi_n^2 - d\pi_n^2 - d\pi_n^2}{1 + r^1}
\]

Note that since \( \sum_{n' \in N} \hat{\Pi}_{n'i}^2 = m_i^2 = s_i^2 \):

\[
d\tau_n^2 = \frac{1}{\mu_n} \sum_{i \in I} \left( \hat{\Pi}_{ni}^2 / \sum_{n' \in N} \hat{\Pi}_{n'i}^2 \right) s_i^2 dQ_i^2 = \frac{1}{\mu_n} \hat{\Pi}_{ni}^2 dQ_i^2 = d\pi_n^2
\]
Substituting in the change in the differentiated budget constraint, we have:

\[
\begin{align*}
dW &= \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[ -\Delta_n d\lambda_n^1 + \phi_n \left( p^1 dc_n^1 - dl_n^1 \right) + (1 - \phi_n) \left( d\pi_n^1 - dr_n^1 - \frac{dr^2_n}{1 + r^1} \right) + \left( \frac{u_n^1 G}{L_n^1} dG^1 + \left( \frac{1 - \phi_n}{1 + r^1} \right) \frac{u_n^2 G}{R_n^2} dG^2 \right) \right]
\end{align*}
\]  

(A124)

For households with non-strictly-binding borrowing constraints, \( \phi_n = 0 \). For households with \( \phi_n > 0 \), the borrowing constraint \( \tilde{\lambda}_n^1 = l_n^1 + \pi_n^1 - r_n^1 - p^1 c_n^1 \) implies \( p^1 dc_n^1 + dr_n^1 = dl_n^1 + d\pi_n^1 \). Defining the within-period willingnesses to pay \( \text{WTP}^t = \frac{u_n^1 G}{\kappa_n^1} \), we arrive at the final expression:

\[
\begin{align*}
dW &= \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[ -\Delta_n d\lambda_n^1 + \left( d\pi_n^1 - dr_n^1 - (1 - \phi_n) \frac{dr^2_n}{1 + r^1} \right) + \left( \text{WTP}^1 dG^1 + (1 - \phi_n) \frac{\text{WTP}^2}{1 + r^1} dG^2 \right) \right]
\end{align*}
\]  

(A125)

Finally, the expressions for \( dl, d\pi, dY \) come from rearranging Equation A116 under assumption 7 and using \( dY = (1 - \tilde{X}) dQ \).

Studying Equation A120 reveals a key insight: Under assumptions 6 and 7, the change in welfare due to a change in taxes and purchases is the same as in an as-if economy without profits but where share-holders supply labor with a wedge \(-1\). This labor supply wedge corresponds to complete under-employment; share-holders—who experience no marginal disutility of holding shares—would continue to be willing to hold shares until profits-per-revenue reached zero. Just like labor suppliers, share-holders do not choose their income but rather take it as given. This as-if representation of profits as under-employed labor allows us to carry over all of the results from Section 6 with minimal alterations.

**Proposition 14.** Under assumptions ??, 6, and 7, the welfare change from a change in purchases is proportional to the resulting change in GDP, whereas the welfare change from a change in transfers is proportional to the resulting change in income. Formally,

\[
\begin{align*}
dW = \frac{1}{1+r^1} d\pi^1 \frac{dG^1}{d\pi^1} \left( -\mu dr^1 - \frac{\mu dr^2}{1 + r^1} \right)
\end{align*}
\]  

(A126)

where \( \frac{d\pi^1}{dG^1} = (1 - C^1_{y^1} \Gamma^1)^{-1} \) and \( \frac{d\pi^1}{dG^2} = 0 \) are first-period purchases multipliers and \( \frac{d(l + \pi^1)}{dy^1} = (1 - \Gamma^1 C^1_{y^1})^{-1} \) is the first-period transfers multiplier; here \( \Gamma^1 = \left( TL^1_{\tilde{L}^1} + \tilde{H}^1 \right) (I - \tilde{X}^1)^{-1} \). Moreover, if relative to some income incidence \( y^* \), \( m^\text{next}_n = E_{y^*}[m^\text{next}_n] \) for all \( n \), where \( m^\text{next}_n \equiv \ldots \)
\[(m \Gamma^1 C^1_y, m^{-1})_i\] then under assumptions ??, 6, and 7,

\[
dW = \left(1 + \frac{1}{1 - \mathbb{E}_y[m_n]}m\right)^T \left(\Gamma^1 dG^1 - \mu d\tau^1 - \frac{\mu d\tau^2}{1 + r^1}\right)
\]

(A127)

**Proof.** Reinterpret profit income as labor supply with wedge \(-1\), as discussed above. The proof then follows from Online Appendices ?? and ??.

Key here is that assumption ??’s imposition that all marginal labor supplies have a labor supply wedge of \(-1\) matches with the shareholders’ implicit labor supply wedge of \(-1\) – both are indifferent to supplying more of their factor. Thus, there is zero social cost to any marginal employment, so the optimal policy maximizes GDP. As without markups, the GDP-maximizing policy targets MPC when bias and homophily effects are absent.

**D.2.3. First-order conditions for optimal policy**

The same as-if representation of profits as under-employed labor also allows us to carry over results from section C.6 to the case of imperfect competition.

**Proposition 15.** Suppose taxes \(\tau^{1*}, \tau^{2*}\) and purchases \(G^{1*}, G^{2*}\) solve the planner’s problem. Now consider a change in policy \(\tau^t = \tau^{t*} + \varepsilon \tau^t, G^t = G^{t*} + \varepsilon G^t\), indexed by \(\varepsilon\). Then, under assumptions 6 and 7, the following first-order condition holds:

\[
0 = \left(\tilde{\lambda}^T \mu WTP^1 - (\gamma^1 + \tilde{\lambda}^T \Delta \tilde{\Gamma}^1)\right) G^1_{\varepsilon} + \left(\tilde{\lambda}^T (I - \hat{\phi}) WTP^2 - \gamma^1 \right) G^2_{\varepsilon} \]

\[
- \left(\tilde{\lambda} - \gamma^1\right)^T \mu \left(\tau^1 + \frac{\tau^2}{1 + r^1}\right) + \tilde{\lambda}^T \hat{\mu} \tau^2 \frac{1}{1 + r^1}
\]

\[
- \tilde{\lambda}^T \Delta \tilde{\Gamma} \left(I - \tilde{C}_y^1 \tilde{\Gamma}^1\right)^{-1} \tilde{C}_y^1 \left(\tilde{\Gamma}^1 G^1_{\varepsilon} - \mu \tau^1_{\varepsilon} - \frac{1}{1 + r^1}\right)
\]

(A128)

where \(\gamma^1\) is the marginal value of public funds, \(\tilde{\Gamma}^1 = \left[R^1_{\tilde{\delta}} \tilde{X}^1\right] \left(I - \tilde{X}^1\right)^{-1}\), \(\tilde{C}_y^1 = \left[C_y^1 C_y^1\right]\), and \(\tilde{\Delta}\) is the \(N \times 2N\) matrix with entries \(\tilde{\Delta}_{n,n} = \Delta_n, \tilde{\Delta}_{n,N+n} = -1,\) and zeros elsewhere.

As in the main text, this condition corresponds to the the bias and homophily effects (now profit inclusive) being zero for all purchases and transfers shocks.
Proof. This follows from reinterpreting profit income as labor supply with wedge $-1$ and then following the proof of Proposition 10.

Intuitively, the planner targets “profit-wedges” just like labor supply wedges. These reduce the social cost of government purchases and motivate Keynesian stimulus.

Finally, a similar network irrelevance result holds as in the case without profits.

**Proposition 16.** Impose Assumptions 6 and 7. Now, suppose that all households rationed to on the margin at the optimum have no marginal labor disutility, i.e. if $(\Gamma^1 C_{y1}^1)_{n-} \neq \bar{0}$ then $\Delta_n = 0$. Then Equation A128 holds with respect to variations in first-period transfers if and only if, for all $n \in N$,

$$\gamma = \frac{\tilde{\lambda}_n}{1 - m_n}$$ (A129)

Alternatively, suppose that the social gains from first-period government purchases are equal to some $\tilde{v}$ across goods and constraints bounding purchases above zero do not bind. Then Equation A99 holds with respect to first-period purchases variations if and only if, $\forall i \in I$,

$$\gamma = \tilde{v} + \frac{1}{1 - \tilde{m}_i} \left( -\tilde{\lambda} \tilde{\Delta}_i \right)$$ (A130)

where $\tilde{m}_i \equiv (m^T \Gamma^1)^{-1}_i$ is the rationing-weighted average MPC in the production of good $i$ and $\tilde{\lambda} \tilde{\Delta}_i \equiv \left( \tilde{\lambda}^T \tilde{\Delta} \tilde{\Gamma}^1 \right)^{-1}_i$ is the rationing-and-welfare-weighted average rationing wedge in the production of good $i$, for $\Gamma^1$ is as in Proposition 14 and $\tilde{\Gamma}^1$ and $\tilde{\Delta}$ are as in Proposition 15.

Proof. This follows from reinterpreting profit income as labor supply with wedge $-1$, following Appendix ??, and using $\Delta_n = 0$ for marginal labor-suppliers in the transfer case.

D.3. Multiple Time Periods

Consider the benchmark model from Section 2 but instead suppose that $t \in \mathbb{T} = \{1, ..., T\}$, where $T \in \mathbb{N} \cup \{\infty\}$. That is, in each period $t$, firms $i$ use a vector of intermediates $X^t_i$, labor $L^t_i$ and a CRS production technology $F^t_i(X^t_i, L^t_i, z^t_i)$. The households have consumption $c^t_n$ and labor supply $l^t_n$ functions that satisfy the dynamic budget constraint:

$$\sum_{t \in \mathbb{T}} \frac{l^t_n}{\prod_{i \leq t} 1 + r^i} = \sum_{t \in \mathbb{T}} \frac{p^t c^t_n + r^t_n}{\prod_{i \leq t} 1 + r^i}$$ (A131)
The government chooses a sequence of lump-sum taxes and spending \( \{\tau_n^t\}_{n \in N}, \{G_i^t\}_{i \in I} \) subject to its lifetime budget constraint:

\[
\sum_{n \in N} \mu_n \left( \sum_{t \in T} \frac{1}{\prod_{s \leq t} (1 + r_s^t)} \tau_n^t \right) = \sum_{t \in T} \frac{1}{\prod_{s \leq t} (1 + r_s^t)} p^t G^t
\]  
(A132)

The key difference in defining equilibrium here is the need to specify a rule that decides in which periods we have labor rationing. To this end define a set \( \mathcal{T}(\omega) \subseteq \mathcal{T} \) which specifies time indices for which the economy is in a state of labor rationing, where \( \omega \) is a vector of all exogenous parameters of the model. In periods with rationing \( t \in \mathcal{T}(\omega) \), instead of labor market clearing, we have that \( l_n^t = l_n^t((L_i^t)_{i \in I}) \). An equilibrium of the model is then given by:

**Definition 2.** (Dynamic rationing equilibrium) Given parameters \( \omega \), a dynamic rationing equilibrium is a set of agent- and market-level variables \( \{s_i^t, \{c_{ni}^t\}_{i \in I}, \{l_n^t\}_{n \in N, t \in T} \} \) and \( \{\tau_t, \{p_i^t, \{X_i^t\}_{i \in I}, \{p_i^t, \{L_i^t, C_i^t, G_i^t\}_{i \in I} \}_{t \in T} \} \) that satisfy the following conditions: (1) Each household \( n \) consumes according to its consumption function \( c_{ni}^t(\cdot) \) in all periods and supplies labor according to \( l_n^t(\cdot) \) in all non-rationing periods i.e. \( t \in \mathcal{T}/\mathcal{T}(\omega) \); (2) Firms choose \( (X_i^t, L_i^t) \) to maximize profits for all \( t \in \mathcal{T} \); (3) The market for all goods clears for all \( t \in \mathcal{T} \); (4) The labor market clears in periods \( t \in \mathcal{T}/\mathcal{T}(\omega) \) and is determined by rationing in all periods in periods \( t \in \mathcal{T}(\omega) \), i.e. \( l_n^t = l_n^t((L_i^t)_{i \in I}) \); (5) The government spends according to its government purchases function \( G^t(\cdot) \).

For our dynamic equilibrium, we can again achieve an analogous Keynesian cross representation to our two period model, as the no-substitution theorem continues to hold. The dynamic fixed point equation for production is given by:

\[
Q^t = \hat{X}^t Q^t + G^t(\{r^t\}_{t \in T}) + C^t(\{r^t, Q^t\}_{t \in T})
\]  
(A133)

Taking a first-order approximation following a partial equilibrium shock \( \partial Q \) for both rationing and flexible periods yields:

\[
dQ^t = \hat{X}^t dQ^t + \sum_{\tau \geq 1} \left[ (G^t_{r^t} + C^t_{r^t}) d r^t \right] + \sum_{\tau \in \mathcal{T}(\omega)} \left[ C^t_{\hat{Y}^t} L^t \hat{L}^t dQ^t \right] + \partial Q^t
\]  
(A134)

\(^{63}\)For example, \( \mathcal{T} \) can represent the set of periods in which the effective zero lower bound on real interest rates binds. Insofar as \( \omega \) is sufficient to determine whether the zero lower bound binds, it is sufficient for it to determine \( \mathcal{T} \).

\(^{64}\)Recall \( \partial Q^1 \) generalizes \( \partial Y^1 \) to the case with supply shocks, see Footnote 48.
Stacking these relations yields the Keynesian cross representation:

\[ dQ = \hat{X}dQ + (G_r + C_r)dr + C_yL^\top\hat{J}_{T(\omega)}dQ + \partial Q \]  

(A135)

where \( J_{T(\omega)} \) is a diagonal matrix with ones on the diagonal.

Interestingly, via an appropriate relabelling, there is an heuristic isomorphism between the 2-period model and the \( T \)-period model whenever \( T(\omega) = \{ t \}_{t=1}^{T_1} \), i.e. there is rationing for the first \( T_1 \) periods and non-rationing for the subsequent \( T_2 = T - T_1 \) periods. That is, in the \( T \)-period model, the rationing spell maps to the rationing period in the 2-period model. To this end, the formula in Proposition 8 corresponds to a dynamic generalization of the Miyazawa special case.

**Proposition 17.** (Dynamic multipliers at the zero lower bound) Suppose that \( r^t = \bar{r}^t \) for all \( t \in T \). Then the general equilibrium effect on value added \( dY \) of a partial equilibrium shock \( \partial Q \) is generically given by

\[ dY^T = \left( I - C_y^{T^\top} \hat{L}_T^\top \hat{X}^T \left( I - \hat{X}^T \right)^{-1} \right)^{-1} \partial Q^T \]  

(A136)

where \( dY^T \) and \( dQ^T \) are \( T \times T \)-length vectors, \( \hat{L}_T^T \) and \( \hat{X}_T^T \) are diagonal matrices with entries corresponding to each rationing period, and where \( C_y^{T^\top} \) is the \( (T \times \mathcal{I}) \times (\mathcal{T} \times \mathcal{N}) \) matrix of intratemporal marginal propensities to consume, which maps changes in the household income distribution during rationing periods to changes in the consumption of each good during rationing periods.

**Proof.** The dynamic fixed point equations for market clearing are given in matrix form as:

\[ Q^t = \hat{X}^t Q^t + G^t(\{r^t\}_{t \in T}) + C^t(\{r^t, Q^t\}_{t \in T}) \]  

(A137)

Taking a first-order approximation following a partial equilibrium shock \( \partial Q \) for both rationing and flexible periods yields:

\[ dQ^t = \hat{X}^t dQ^t + \sum_{\tau \geq 1} \left[ (G_{r^\tau}^t + C_{r^\tau}^t)dr^\tau \right] + \sum_{\tau \in T(\omega)} \left[ C_{y^\tau}^t L_{\tau^\top} \hat{L}_T^\top dQ^\tau \right] + \partial Q^t \]  

(A138)

Stacking these relations yields the Keynesian cross representation:

\[ dQ = \hat{X}dQ + (G_r + C_r)dr + C_yL^\top\hat{J}_{T(\omega)}dQ + \partial Q \]  

(A139)

where \( J_{T(\omega)} \) is a diagonal matrix with ones on the diagonal. Imposing \( r^t = \bar{r}^t \) for all \( t \in T \)
simplifies this to:

\[ dQ = \tilde{X}dQ + C_y l_L \hat{L} J_{T(\omega)} dQ + \partial Q \]  \hspace{1cm} (A140)

Inverting this system to solve for the total change in production and solving for value added:

\[ dY = \left( I - C_y l_L \hat{L} J_{T(\omega)} \left( I - \tilde{X} \right)^{-1} \right) \partial Q \]  \hspace{1cm} (A141)

Applying the selection matrix \( J_{T(\omega)} \) and taking the first \( T \times I \) rows:

\[ dY^T = \left( I - C_y^T l_L^T \hat{L}^T \left( I - \tilde{X}^T \right)^{-1} \right)^{-1} \partial Q^T \]  \hspace{1cm} (A142)

which is the required expression.

However, there is a subtle difference in the intuition behind the results in the two cases. In the \( T \)-period case, the shocks in each rationing period can influence the level of output in all other periods. As a result, it is no longer sufficient to consider the directed MPC of households, but rather the directed \textit{intertemporal} MPC of households that represents marginal changes in consumption across goods and time. Indeed, if we set the response of the rationing function, the unit labor demands and the input-output matrix to the identity, we recover a \( T \)-period version of the multiplier formula provided by Auclert \textit{et al.} (2018):

\textbf{Corollary 3} (Intertemporal Keynesian Cross). \textit{In the environment of Proposition 17, if the rationing matrix and the input-output matrix compose to the identity matrix, i.e.

\[ I = l_L^T \hat{L}^T \left( I - \tilde{X}^T \right)^{-1} \]  \hspace{1cm} (A143)

then the general equilibrium effect on value added \( dY^T \) in response to a partial equilibrium shocks \( \partial Q^T \) is given by:

\[ dY^T = \left( I - C_y^T \right)^{-1} \partial Q^T \]  \hspace{1cm} (A144)

\textit{Proof.} Simply imposing the given condition on Equation A136 yields the stated result. \hspace{1cm} \Box
E. Measuring Rationing Wedges

In this Appendix, we describe how to recover rationing wedges in the data and how we estimate the counterfactual welfare effects of fiscal stimulus in the Great Recession. We present two microfoundations for the same, particularly simple form of the rationing wedge in terms of the demographic level percentage change in unemployment from before the Great Recession to during the Great Recession. In particular, we provide two microfoundations for the following expression for the change in welfare induced by fiscal stimulus $dG^1$:

$$dW = \sum_{n \in N} \frac{l_n^2 - l_n^1}{l_n^2} \times \left( \Gamma^1(I - C^1y^1\Gamma^1)^{-1}dG^1 \right)_n$$

(A145)

E.1. Intensive margin microfoundation

Our first microfoundation assumes that all households within each demographic group can be treated as having the same quantity of labor supply. This is equivalent to the assumption that all labor supply adjustment happens on the intensive margin (hours worked), and that workers within any demographic group experience the same change in hours.

By optimality of second period labor supply (and noting that $w^t = 1$):

$$v_n^{2t} = \kappa_n^2$$

(A146)

where $\kappa_n^t = u_n^{\prime \prime}$. The rationing wedge is defined as the wedge in the first period intratemporal Euler equation:

$$v_n^{1t} = \kappa_n^1(1 + \Delta_n)$$

(A147)

See that $\Delta_n < 0$ corresponds to involuntary underemployment, $\Delta_n = 0$ is consistent with optimal labor supply and $\Delta_n > 0$ corresponds to involuntary overemployment. The intertemporal Euler equation is given by:

$$\kappa_n^1 = \beta_n \frac{1 + r^1}{1 - \phi_n} \kappa_n^2$$

(A148)

where $\phi_n \geq 0$ is a wedge stemming from the potentially-binding borrowing constraint. Combining these equations yields:

$$v_n^{1t} = \beta_n \frac{1 + r^1}{1 - \phi_n} v_n^{2t}(1 + \Delta_n)$$

(A149)
Thus, the rationing wedge is given by:

$$\Delta_n = \frac{1 - \phi_n}{\beta_n(1 + r^1)} \frac{v_n^{1\prime}}{v_n^{2\prime}} - 1 \quad (A150)$$

We now assume that (i) all households have slack borrowing constraints $\phi_n = 0$ and that (ii) $\beta_n(1 + r^1) = 1$, which is empirically justifiable with standard estimates for discount factors and the real interest rate in the US during the Great Recession. Alternatively, it follows exactly from (i) and (A148) in the special case where consumption utility is linear. Under these assumptions, the rationing wedge is given by:

$$\Delta_n = \frac{v_n^{1\prime}}{v_n^{2\prime}} - 1 \quad (A151)$$

Under the assumption that labor disutility is time invariant and isoelastic, we have that:

$$v_n^t(l) = \xi \frac{l^{1+\psi}}{1 + \psi} \quad (A152)$$

The rationing wedge is then given by:

$$\Delta_n = \left( \frac{l_n^1}{l_n^2} \right)^\psi - 1 \quad (A153)$$

Intuitively, whenever the household is working less than that steady state value, they are underemployed. This is because wages are not changing and their preferences and interest rate are such that they apply no dollar discount to future disutility. A standard calibration allows us to set $\psi = 1$. In this case, the rationing wedge is the percentage gap in labor supply from the steady state:

$$\Delta_n = \frac{l_n^1 - l_n^2}{l_n^2} \quad (A154)$$

It follows by Proposition 3 that – in the absence of direct willingness to pay for government purchases – the change in welfare induced by a change in first period government purchases $dG^1$ is given by:

$$dW = - \sum_{n \in N} \tilde{\lambda}_n \Delta_n \left( \Gamma^1(I - C^1_{y^1})^{-1} dG^1 \right)_n \quad (A155)$$

Assuming no distributional motive $\tilde{\lambda}_n = 1$, we then obtain that the welfare benefits of stimulus spending $dG^1$ are given by:

$$dW = \sum_{n \in N} \frac{l_n^2 - l_n^1}{l_n^2} \left( \Gamma^1(I - C^1_{y^1})^{-1} dG^1 \right)_n \quad (A156)$$
E.2. Extensive margin microfoundation

Our second microfoundation focuses on the polar case in which, before a change in government purchases, all households within a demographic group are either fully employed or fully unemployed. We assume that, within each demographic group \( n \), a mass \( 1 - f_n \) of households never supplies labor. The complementary mass \( f_n \) supplies labor inelastically up to some level \( l_n^\ast \) after which their marginal dis-utility of labor supply sharply—but continuously—increases, so that they always supply close to \( l_n^\ast \) in equilibrium.

At the initial equilibrium, a total mass \( \zeta_n \leq f_n \) are employed at the efficient level in the initial equilibrium; the remainder are unemployed. This implies that initially employed households supply \( l_n^1 \) in both periods, whereas initially unemployed households supply 0 in the first period and \( l_n^2 \) in the second period. Total first- and second-period labor supplies by group \( n \) are therefore approximately \( l_n^1 \approx \zeta_n l_n^\ast \) and \( l_n^2 \approx f_n l_n^\ast \), respectively.\(^{65}\)

Now consider a change in labor demand induced by government purchases. We assume that each firm rations the same expected amount of marginal labor to each member of the employable subpopulation \( f_n \). However, in the case of employed workers, they do this by rationing infinitesimally more labor to a continuum of workers, whereas in the case of unemployed workers, they do this by hiring workers at their efficient level of labor supply. The former has only second-order welfare consequences. The latter increases welfare by

\[
\Delta_n \equiv \max_{1 \tau c^1 + \frac{1}{1+\tau} \tau l_n^1 + \frac{1}{1+\tau} \tau l_n^2 < l_n^1 + \frac{1}{1+\tau} l_n^2} u_n^1(c^1) - v_n^1(l_n^1) + \beta_n \left[ u_n^2(c^2) + v_n^2(l_n^2) \right] \\
- \max_{1 \tau c^1 + \frac{1}{1+\tau} \tau l_n^1 + \frac{1}{1+\tau} \tau l_n^2 \leq l_n^1 + \frac{1}{1+\tau} l_n^2} u_n^1(c^1) - v_n^1(0) + \beta_n \left[ u_n^2(c^2) + v_n^2(l_n^2) \right] 
\]

per newly employed worker.

Finally, note that the fraction of the \( f_n \)-sized subgroup which is initially unemployed—and therefore experiences the welfare gain \( \Delta_n \) if employed—is equal to

\[
\frac{f_n - \zeta_n}{f_n} = \frac{(f_n - \zeta_n) l_n^\ast}{f_n l_n^\ast} \approx \frac{l_n^2 - l_n^1}{l_n^2} 
\]

The expected welfare gain per marginal dollar rationed to \( n \) in the first period is therefore

\[
\approx \tilde{\lambda}_n \Delta_n \frac{l_n^2 - l_n^1}{l_n^1} 
\]

\(^{65}\)These approximations are exact in the limit where labor disutility is kinked at \( l_n^\ast \).
Finally, we assume that the planner puts weight \( \tilde{\lambda}_n = \tilde{\Delta}_n^{-1} \) on each group \( n \), i.e. she values change in utility from employment equally across demographic groups. Combining this assumption with our earlier formula for the change in employment in each industry, we recover

\[
dW \approx \sum_{n \in N} \frac{l^2_n - l^1_n}{l^2_n} \left( \Gamma^1 (I - C^1 y, \Gamma^1)^{-1} dG^1 \right)_n
\]

(E160)

E.3. Estimation

We have already estimated \( \Gamma^1 \) and \( C^1 y \), so to compute the welfare effects of stimulus in any given episode, we require only estimates of the demographic-level gap in labor income from the steady state at any point in time. For our Great Recession analysis, we compute this in the ACS by taking the percentage change in labor hours worked from 2005-06 to 2009-10 in each of our state-by-demographic bins. When there are no observations in any given bin, we assume that the change in labor hours is given by the state-level average.

For robustness, we compute a version of the state-by-demographic level rationing wedge by imposing that each demographic group’s rationing wedge is the change in hours for that demographic group nationwide compared to the average multiplied by the average change in hours across demographics at the state level. The results are very similar, with the \( R^2 \) of multipliers in explaining welfare changes dropping slightly to 54% from 78%.
F. Validating the Model

The model that we develop and estimate in this paper makes stark predictions about the propagation of industry- and region-specific shocks. In this section, we attempt to empirically validate those quantitative predictions. Specifically, Proposition 1 provides an expression relating total value added to spending in a state $s$ and industry $i$:

$$\text{d}Y^1 = \left( I - C_{y^1} R_{L^1} \hat{L}^1 (I - \hat{X}^1)^{-1} \right)^{-1} \delta Q^1 = M \delta Q^1$$  \hspace{1cm} (A161)

where $M$ is the generalized multiplier matrix, and recall $\delta Q^1$ generalizes $\delta Y^1$ to the case with supply shocks (see Footnote 48). The $m_{s,r}$ entry gives the total change in value added in state $s$ when there is a one-unit partial equilibrium shock to state $r$ distributed across industries in proportion to their share of total value added in state $r$. Any identified partial equilibrium shock $G$ will be some component of the many partial equilibrium shocks hitting the economy, which we can express as $\delta Q^1 = G + U$, where $U$ is the partial equilibrium effect on demand of the unobserved shocks hitting the economy. Plugging this in, we arrive at the foundation for our estimating equation:

$$\text{d}Y_t = M(G + U) = \beta MG_t + \epsilon_{i,t}$$  \hspace{1cm} (A162)

where $G$ is the vector of identified industry-by-region shocks and $M$ is our estimated generalized multiplier. The strict prediction of our model is that $\beta = 1$, meaning that we have perfectly predicted the heterogeneous effects of the shocks on value added growth. Note that the matrix $M$ includes not only heterogeneity in the response to a shock in one’s own market, but also how each market will respond to other markets through spillovers arising from spending network effects. Therefore, in addition to testing $\beta = 1$, we also test separately for the existence of spillovers of the nature predicted by the model. More specifically, we run the following regression:

$$\text{d}Y_t = M(G + U) = \alpha_0 (M_{\text{diag}}) G_t + \alpha_1 (M_{\text{offdiag}}) G_t + \epsilon_{i,t}$$  \hspace{1cm} (A163)

where $M_{\text{diag}}$ is the diagonal entries of the multiplier matrix (i.e. all other entries are set to 0) and $M_{\text{offdiag}}$ are the off-diagonal entries of the multiplier matrix. $\alpha_0$ captures the degree to which the multiplier accurately captures the effect of a direct shock and $\alpha_1$ captures the degree to which the model accurately captures the nature of the spillovers across regions and industries.

In the following sections, we will use two different identified shocks for the demand shock $G$ – state-level military spending shocks from Nakamura and Steinsson (2014) and a
growth in industry imports from Autor et al. (2013). Of course, bringing this to the data presents several identification challenges particular to the shock in question. We address the challenges particular to each shock below as we slightly modify Equation A162 to fit the particular setting.


The first shock that we consider is the local government purchases shock developed by Nakamura and Steinsson (2014) to estimate the local purchases multiplier. We refer the reader to that paper for the details on the construction of the shock. We closely follow their original specification, using data on US states from 1966-2006. We restrict our attention to variation across states and our dependent variable is the 2-year change in state GDP per capita, divided by the level of state GDP lagged 2 periods. The state spending shock is the 2-year change in military spending per capita, also divided by the level of state GDP lagged 2 periods. Specifically, we run the following regression

$$\frac{y_{s,t} - y_{s,t-2}}{y_{s,t-2}} = \beta \frac{(MG)_{s,t} - (MG)_{s,t-2}}{y_{s,t-2}} + \gamma_s + \gamma_t + \epsilon_{s,t}$$  \hspace{1cm} (A164)

where $\gamma_s$ and $\gamma_t$ are state and year fixed effects, respectively. The central concern is that military spending is not random and may be directed towards states based on their economic performance. Therefore, we follow Nakamura and Steinsson (2014) and instrument the state changes in spending with state dummies interacted with national changes in military spending. Table A1 shows the results. First, Column 1 shows the replication of the result in Nakamura and Steinsson (2014), which is the equivalent of imposing that $M$ has 1 on the diagonals but is 0 elsewhere (call this $M_1$). Column 2 shows the estimate of Equation A164. The estimates are noisy, but two small pieces of evidence suggest that including the multiplier provides a better fit for the data than the simple specification. First, while we cannot reject that the coefficient on either $M_1G$ or $MG$ is 1, the coefficient on $MG$ is closer to 1 than the coefficient on $M_1G$, suggesting that the heterogeneity embedded in $M$ is getting us closer to capturing all of the variation in the data. Second, the r-squared in Column 2 is slightly higher than that in Column 1. However, the estimates are noisy and largely inconclusive.

The remaining columns of Table A1 show the estimates separating the own and spillover effects as in Equation A163. A finding that the coefficient on the spillover term was positive and close to 1 would suggest that our measure was accurately picking up the experienced spillovers. Here, the estimates are also too noisy to be conclusive.
Baseline Robustness

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<th>Baseline</th>
<th>Robustness</th>
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<tbody>
<tr>
<td></td>
<td>No State FE</td>
<td>post-1980</td>
</tr>
<tr>
<td>State Spending (M₁G)</td>
<td>1.474***</td>
<td>(0.373)</td>
</tr>
<tr>
<td>Model Prediction (MG)</td>
<td>1.189***</td>
<td>(0.299)</td>
</tr>
<tr>
<td>Model Prediction (M\text{diag}G)</td>
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<td>(0.355)</td>
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<tr>
<td>Model Prediction (M\text{nodiag}G)</td>
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<td>(3.367)</td>
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<tr>
<td>Constant</td>
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<td></td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.316</td>
<td>0.319</td>
</tr>
</tbody>
</table>

Table A1: Reduced Form Validation: Government Spending from Nakamura and Steinsson (2014)

F.2. Chinese Import Shocks from Autor et al. (2013)

We also explore the predictions of our model using import shocks constructed as in Autor et al. (2013). While the government purchases shocks were primarily at the level of the state, import shocks are primarily at the level of the industry. Thus, as in Autor et al. (2013), we construct the state-level exposure to the China shock (ΔIP\text{,s,t}) using the industry distribution in each state as:

\[
ΔIP\text{,s,t} = \sum_j \frac{L_{sj,t}}{L_{st}} \frac{ΔImports_{j,t}}{L_{i,1991}} \tag{A165}
\]

where \(j\) is the industry, \(s\) is the state, \(L\) is employment, and \(Imports_{j,t}\) are the imports from China to the US. Variation across states in import exposure stems from differences across states in their industry distribution. We assume that there are no imports to non-manufacturing industries. Using this measure as our state-level demand shock, we estimate

\[
ΔlogY_{s,t} = β_1 M ΔIP_{s,t} + γ_s + γ_t + ϵ_{st} \tag{A166}
\]

where \(Y_{s,t}\) is state GDP and \(γ_s\) and \(γ_t\) are state and year fixed effects, respectively. We use stacked 5-year changes and utilize data from 1991-2011. The central concern is that imports grow most in areas that are already shrinking or growing, and therefore we instrument the China shock ΔIP\text{,s,t} with the imports from China in eight other developed countries as in Autor et al. (2013).

Table F.2 shows the results. Column 1 first shows the baseline estimate where \(M = M_1\), where \(M_1\) is a diagonal matrix of ones. As predicted given the results in Autor et al. (2013),
Table A2: Reduced Form Validation: China Shock from Autor et al. (2013)

<table>
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<tr>
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<th>Robustness</th>
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</thead>
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<td>China Shock G)</td>
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<td>-1.481**</td>
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<tr>
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<td>(0.596)</td>
</tr>
<tr>
<td>Model Prediction (MG)</td>
<td>-1.485**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.596)</td>
<td></td>
</tr>
<tr>
<td>Model Prediction (M_{diag}G)</td>
<td>-1.322*</td>
<td>-1.481**</td>
</tr>
<tr>
<td></td>
<td>(0.684)</td>
<td>(0.638)</td>
</tr>
<tr>
<td>Model Prediction (M_{nodiag}G)</td>
<td>-2.638</td>
<td>-1.819</td>
</tr>
<tr>
<td></td>
<td>(1.814)</td>
<td>(1.637)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>204</td>
<td>204</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.482</td>
<td>0.481</td>
</tr>
<tr>
<td></td>
<td>153</td>
<td>561</td>
</tr>
<tr>
<td></td>
<td>0.409</td>
<td>0.485</td>
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states with a larger growth in imports experienced lower GDP growth rates. The following columns test for the ability of our estimated multiplier to predict the magnitude of the effect as well as the direction of the spillovers. The coefficient on \( M_{nodiag} \) is generally negative and similar in magnitude to the coefficient on \( M_{diag} \). This means that we generally find that the model correctly predicts the direction of the spillovers. However, the results are too noisy to draw any firm conclusions.
G. Additional Tables and Figures

Fig. A1. Heterogeneity in estimated MPCs for total consumption across demographic groups.

Fig. A2. Estimated Directed MPCs Vs. CEX basket-weighted MPCs
Fig. A3. Earnings elasticity to GDP shocks scattered against estimated MPC. See Patterson (2019) for more details.

Fig. A4. Histograms of the bias term (left) and homophily term (middle) and overall error terms (right) from the decomposition in Proposition 2. For all subfigures, the distribution reflects a unit demand shock to each of the 2805 sector-region pairs, with baseline $y^*$ given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

Fig. A5. The left panel shows the scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume. The right panel shows a scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume.
Fig. A6. Left panel: The x-axis gives the purchases multiplier for a dollar of government purchases targeting each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government purchases targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession. Right panel: The x-axis gives the population-weighted Great Recession rationing wedge of employees in each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government purchases targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession.

Fig. A7. Scatter plot of bias and homophily for a GDP-proportional shock in alternative models where we compare all combinations of models with and without: input-output linkages, regional trade, and heterogeneous income rationing by MPC and location. The orange dot corresponds to the baseline model. Here, the reference incidence $y^*$ is that induced by a GDP-proportional shock.
Fig. A8. Scatter plot of purchases multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which all households have homogeneous consumption baskets in proportion to aggregate consumption (y-axis).

Fig. A9. Consumption basket weights for each demographic group (each line is a demographic group) across each CEX consumption category.
Fig. A10. Histogram of the fraction of consumer demand resulting in income for labor within the same state for each state-demographic pair.

Fig. A11. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the share of income from production that goes directly to labor (as opposed to capital, foreigners, or inputs).
Fig. A12. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair against the ultimate labor share accounting for labor employed in the production of intermediates.

Fig. A13. Sorted change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households’ labor income being scaled in proportion to their labor income.
Fig. A14. Scatter plot of the change in GDP for each industry-region pair according to a one dollar demand shock in each pair. Full model is the baseline. Uniform rationing corresponds to all households’ labor income being scaled in proportion to their income.

Fig. A15. Income-weighted average MPC by state.
Fig. A16. Scatter plot of purchases multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which there is no intermediate goods use by firms (y-axis).
<table>
<thead>
<tr>
<th>Component</th>
<th>Incidence multiplier</th>
<th>Bias</th>
<th>Homophily</th>
<th>Total</th>
<th>Error</th>
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<tbody>
<tr>
<td>Magnitude</td>
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<td>-0.002</td>
<td>-0.001</td>
<td>1.298</td>
<td>0.000</td>
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</table>

Table A3: Homophily decomposition for a shock to demand proportional to 2012 GDP across sectors. “Incidence multiplier” includes the first and second terms in Proposition 2. “Bias” is the bias correction and “homophily” is the homophily correction. Error is the difference between the sum of these terms and the exact multiplier.

<table>
<thead>
<tr>
<th></th>
<th>No Directed MPC</th>
<th>Directed MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Rationing</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>MPC Rationing</td>
<td>1.28</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Table A4: Multiplier of a GDP-proportional purchases shock across model specifications. In this table, we eliminate regional structure and instead have 55 industries at the national level. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.

<table>
<thead>
<tr>
<th></th>
<th>No Directed MPC</th>
<th>Directed MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Rationing</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>MPC Rationing</td>
<td>1.30</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table A5: Multiplier of a GDP-proportional purchases shock across model specifications. In this table, everything is as in the baseline except we eliminate regional trade and assume that all consumption and intermediate goods use is within each state. Directed MPC and MPC rationing are as in the baseline. No Directed MPC corresponds to a case where all households direct their consumption in proportion to aggregate consumption. Uniform rationing assumes that all households are rationed to in each industry in proportion to their share of income in that industry.
Fig. A17. Multipliers for state-level and industry level shocks. Formally, we take the shock for each state $r$ as $\tilde{\mathcal{Q}}_r = \left( \mathbb{I}[s = r] \frac{y_{rj}}{\sum_k y_{rk}} \right)_{j}^{s}$, where $y_{rj}$ is BEA gross output for sector $j$ in state $r$ and each industry $j$ as $\tilde{\mathcal{Q}}_j = \left( \mathbb{I}[k = j] \frac{y_{rk}}{\sum_r y_{rk}} \right)_{r}^{k}$. That is, we marginalize across each dimension according to value added shares.

Fig. A18. Multipliers for state-level and demographic level transfers shocks. Formally, for the state-level shock, we transfer each state one dollar, in proportion to the demographic composition of that state. For the demographic-level shock, we transfer each demographic group one dollar, in proportion to the distribution of that demographic across states.
Fig. A19. Labor shares of revenue, by industry, in 2000 vs. 2012. Most industries experience a modest decline in labor share. The most dramatic decline is in the sector labelled “data processing, internet publishing, and other information services.” The most dramatic increase is in the sector labelled “apparel and leather and allied products.”

Fig. A20. Scatter plot of purchases multipliers in 2000 vs. 2012, by state-industry pair.
Fig. A21. Scatter plot of MPC vs. income for each age-sex-race demographic group.